Physics 820 homework IV, due Mon Sep 22

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Reading: Chapter 2, 3.1-6

Problems:

1. Goldstein, Problem 2-2.

2. Goldstein, Problem 2-4.

3. A particle is free to move on the surface of a sphere of unit radius under the influence of no forces other than those that constrain the particle to the sphere. It starts at a point $q_1$ and ends at another point $q_2$ (without loss of generality, both points may be taken to lie on a meridian of longitude).

(a) Show that there are many physical paths (i.e. $q(t)$ functions) the particle can take in going from $q_1$ to $q_2$ in a given time $\tau$. How many? Under what conditions are there uncountable many?

(b) Calculate the action for each of two possible paths the particle can take and show that they are not in general equal. Now construct two new, nonphysical paths close to the original ones, going from $q_1$ to $q_2$ in the same time $\tau$. This can be done by adding to each physical path a small distortion of the form $\eta_k(t)$ such that $\eta_k(0) = \eta_k(\tau) = 0$. Show that for each of the nonphysical paths the action is greater than it is on the neighboring physical one, thus demonstrating that each physical path minimizes the action locally.

4. Goldstein, Problem 2-14.

5. A ladder of length $L$ and mass $M$ rests against a smooth wall and slides without friction on the wall and the floor. Assume that the ladder is initially at rest at an angle $\alpha_0$ with respect to the floor. Utilize the method of Lagrange undetermined multipliers to find the angle $\alpha_1$ at which the ladder leaves the wall.