PHY-851 QUANTUM MECHANICS I
Homework 2, 30 points
September 10 - 17, 2003
Wave function. Reading: *Messiah*, Chapters 1-3.

1. /8/ (a) Starting from the Maxwell distribution function of speed of molecules in a classical gas, find the distribution function of the de Broglie wave length and the most probable de Broglie wave length of hydrogen molecules at room temperature.

   (b) Many researchers in various laboratories study the Bose-Einstein condensation of identical atoms in atomic traps. This phenomenon starts when, as temperature decreases, the de Broglie wavelength of thermal motion of the atoms grows up to the average distance between atoms. Estimate the necessary density of atoms with the mass number $A = 100$ in the trap, if the condensation temperature is $T = 10^{-7}$K.

2. /6/ A particle is moving along the x-axis in the potential field $U(x) = \alpha|x|^s$, where α and s are positive constants. Using the Bohr-Sommerfeld quantization rule, find the energy spectrum of bound states E_n.

3. /8/ In systems with free moving charge carriers (metals, plasmas), electric charge Ze of a nucleus or an impurity is *screened* by a cloud of free charges of the opposite sign. The size of the cloud, called the *Debye radius* r_D, becomes smaller as the density of free carriers increases. The resulting electrostatic potential (*Yukawa potential*),

$$\phi(r) = \frac{Ze}{r} e^{-\kappa r}, \quad \kappa = \frac{1}{r_D}, \tag{1}$$

   in contrast to the pure Coulomb potential ($\kappa \to 0,\ r_D \to \infty$), exponentially falls off at distances greater than r_D where the system "center + cloud" looks neutral. It is observed that in hot hydrogen plasmas spectral lines gradually disappear with the increasing electron density. Explain this phenomenon by showing (with the aid of the Bohr-Sommerfeld quantization rule) that there is only a *finite* number of quantum bound states supported by the screened potential (1).

4. /8/ An electron with energy E moves over a metallic strip of width a that can be modelled by a potential well of depth W.

   (a) Find the reflection and transmission coefficients.

   (b) Find the energy values corresponding to the full transmission (*resonances*) and explain the wave mechanism of this phenomenon.

   (c) For given values of E and W find the width a that gives the maximum reflection coefficient and explain the wave mechanism for this case.