PHY-851 QUANTUM MECHANICS I
Homework 3, 30 points
September 17-24, 2003.
One-dimensional motion.
Reading: *Messiah*, Chapter III, §§1-7.

1. /8/ Consider the potential

$$U(x) = \begin{cases} U_1, & x < 0, \\ 0, & 0 < x < a, \\ U_2, & x > a. \end{cases} \tag{1}$$

   Here $U_1 \geq U_2 > 0$. Find the condition for the existence of bound states. Check your results for the limiting cases (a) $U_1 \to \infty$ and (b) $U_1 = U_2$.

2. /10/ A particle is placed in a potential well of finite depth W. The width a of the well is fixed in such a way that the particle has only one bound state with binding energy $\epsilon = W/2$. Calculate the probabilities of finding the particle in classically allowed and classically forbidden regions.

3. /12/ Consider the potential barrier of height U_0 and width a.

   a. Calculate three lowest values of electron energy $E > U_0$ corresponding to the full transparency of the barrier. Give numbers assuming $U_0 = 10\,\mathrm{eV}$ and $a = 5\,\text{Å}$.

   b. Calculate the transmission coefficient through a barrier ($U_0 = 10\,\mathrm{eV}$, $a = 1\,\text{Å}$) for the electron and for the proton with $E = 5\,\mathrm{eV}$. Show that under the barrier the exponentially increasing exponent has, in the limit of a high and broad barrier, an exponentially small amplitude.

   c. Determine the transmission coefficient for a given particle and energies near $E = U_0$ as a limit from above the barrier and as a limit from below the barrier and compare the results.

   d. Assuming $U_0 = 8\hbar^2/ma^2$, draw a plot of the transmission coefficient T as a function of $\epsilon = E/U_0$ in the interval $0 \leq \epsilon \leq 7$; specifically determine $T(\epsilon = 1)$.

   e. Present graphically a qualitative behavior of the probability density $\rho(x) = |\psi(x)|^2$ for $E < U_0$.

   f. For the electron coming from the left with energy $E = U_0/2$, find the ratio $\rho(a)/\rho(0)$ of probability densities for finding the particle near the edges of the barrier; calculate this ratio for the parameters U_0 and a of point (b).