1. /8/ Consider the potential

\[ U(x) = \begin{cases} 
  U_1, & x < 0, \\
  0, & 0 < x < a, \\
  U_2, & x > a.
\end{cases} \]  

Here \( U_1 \geq U_2 > 0 \). Find the condition for the existence of bound states. Check your results for the limiting cases (a) \( U_1 \to \infty \) and (b) \( U_1 = U_2 \).

2. /10/ A particle is placed in a potential well of finite depth \( W \). The width \( a \) of the well is fixed in such a way that the particle has only one bound state with binding energy \( \epsilon = W/2 \). Calculate the probabilities of finding the particle in classically allowed and classically forbidden regions.

3. /12/ Consider the potential barrier of height \( U_0 \) and width \( a \).

a. Calculate three lowest values of electron energy \( E > U_0 \) corresponding to the full transparency of the barrier. Give numbers assuming \( U_0 = 10 \text{ eV} \) and \( a = 5 \text{ Å} \).

b. Calculate the transmission coefficient through a barrier (\( U_0 = 10 \text{ eV}, a = 1 \text{ Å} \)) for the electron and for the proton with \( E = 5 \text{ eV} \). Show that under the barrier the exponentially increasing exponent has, in the limit of a high and broad barrier, an exponentially small amplitude.

c. Determine the transmission coefficient for a given particle and energies near \( E = U_0 \) as a limit from above the barrier and as a limit from below the barrier and compare the results.

d. Assuming \( U_0 = 8\hbar^2/ma^2 \), draw a plot of the transmission coefficient \( T \) as a function of \( \epsilon = E/U_0 \) in the interval \( 0 \leq \epsilon \leq 7 \); specifically determine \( T(\epsilon = 1) \).

e. Present graphically a qualitative behavior of the probability density \( \rho(x) = |\psi(x)|^2 \) for \( E < U_0 \).

f. For the electron coming from the left with energy \( E = U_0/2 \), find the ratio \( \rho(a)/\rho(0) \) of probability densities for finding the particle near the edges of the barrier; calculate this ratio for the parameters \( U_0 \) and \( a \) of point (b).