PHY-851 QUANTUM MECHANICS I

Homework 6, 40 points

October 8 - 15, 2003

Operators.

Reading: *Messiah*, Chapter VII.

1. /8/ *Messiah*, Problem 3, p. 293.

2. /7/ *Messiah*, Problem 4, p. 293.

3. /8/ Consider the following operators $\mathcal{O}$ acting on the coordinate functions $\psi(x)$, $-\infty < x < +\infty$:

   a. inversion $\mathcal{P}$, $\mathcal{P}\psi(x) = \psi(-x)$;

   b. shift (displacement) $\mathcal{D}(a)$, $\mathcal{D}(a)\psi(x) = \psi(x-a)$;

   c. scale transformation $\mathcal{M}(\alpha)$, $\mathcal{M}(\alpha)\psi(x) = \sqrt{\alpha}\psi(\alpha x), \quad \alpha > 0$;

   d. complex conjugation $\mathcal{K}$, $\mathcal{K}\psi(x) = \psi^*(x)$.

   For all these operators establish if they are linear; find the transpose operators $\mathcal{O}^T$, complex conjugate $\mathcal{O}^*$, Hermitian conjugate $\mathcal{O}^\dagger$, and inverse $\mathcal{O}^{-1}$.

4. /7/ Calculate the operator product

$$\hat{F}(a) = \exp[-(i/\hbar)a\hat{p}]\,\hat{x}\,\exp[(i/\hbar)a\hat{p}], \tag{1}$$

   where a is an arbitrary parameter of dimension of length.

5. /10/ Construct the vector operator

$$\hat{\mathbf{l}} = [\hat{\mathbf{r}} \times \hat{\mathbf{p}}] \tag{2}$$

   of the orbital momentum in three-dimensional space (x, y, z).

   a. Is the property

$$[\hat{\mathbf{r}} \times \hat{\mathbf{p}}] = -[\hat{\mathbf{p}} \times \hat{\mathbf{r}}] \tag{3}$$

   valid for quantum operators?

   b. Calculate commutators of components $(i, j = x, y, z)$

$$[\hat{l}_i, \hat{x}_j], \quad [\hat{l}_i, \hat{p}_j], \quad [\hat{l}_i, \hat{l}_j]. \tag{4}$$

   c. Show that the components of $\hat{\mathbf{l}}$ commute with any function of $r^2 = x^2 + y^2 + z^2$ or $p^2 = p_x^2 + p_y^2 + p_z^2$.