Astronomy 304, STARS
Homework #1, Answers

1. Carroll & Ostlie: problem 3.7

The average person has 1.4 m$^2$ of skin at a skin temperature of roughly 92 F (306 K). Consider the average person to be an ideal radiatro standing in a room at a temperature of 68 F (293 K).

(a) Calculate the energy per second radiated by the average person in the form of blackbody radiation. Express the answer in both units of ergs s$^{-1}$ and in watts.

\[ L = S\sigma T^4 \]  \hspace{1cm} (1)
\[ S = 1.4 \text{ m}^2 \]  \hspace{1cm} (2)
\[ \sigma = 5.67 \times 10^{-5} \text{ erg cm}^{-2} \text{ s}^{-1} \]  \hspace{1cm} (3)
\[ T = 306 \text{ K} \]  \hspace{1cm} (4)

Hence,

\[ L = 1.4 \times 10^4 \times 5.67 \times 10^{-5} \times (3.06 \times 10^2)^4 \]  \hspace{1cm} (5)
\[ = 6.96 \times 10^9 \text{ ergs}^{-1} = 696 \text{ w}. \]  \hspace{1cm} (6)

(b) Determine the peak wavelength $\lambda_{\text{max}}$ of the blackbody radiation emitted by the average person. In what region of the spectrum is this wavelength found?

\[ \lambda_{\text{max}}T = 0.29 \text{ cmK}, \]

so

\[ \lambda_{\text{max}} = \frac{0.29}{306} = 9.48 \times 10^{-4} \text{ cm} = 9.48 \times 10^{-6} \text{ m} = 9.48 \mu \text{ m}, \]

which is in the IR part of the spectrum.

(c) A blackbody also absorbs energy from its environment, in this case the room a 293 K. Calculate the energy absorbed per second by the average person, expressed in both units of erg s$^{-1}$ and watts.

\[ L = S\sigma T^4 \]  \hspace{1cm} (7)
\[ L = 1.4 \times 10^4 \times 5.67 \times 10^{-5} \times (2.93 \times 10^2)^4 \]  \hspace{1cm} (8)
\[ = 5.85 \times 10^9 \text{ erg} \text{ s}^{-1} = 585 \text{ w}. \]  \hspace{1cm} (9)
(d) Calculate the net energy per second lost by the average person due to blackbody radiation.

Net Loss = emission - absorption = 111 w.

2. Carroll & Ostlie: problem 3.8

Consider a model of a star as a spherical blackbody with a surface temperature of 28,000 K and a radius of \(5.16 \times 10^{11}\) cm, located at a distance of 180 pc from Earth. Determine its:

(a) Luminosity

\[
L = 4\pi R^2 \sigma T^4
\]

\[
= 4\pi (5.16 \times 10^{11})^2 \times 5.67 \times 10^{-5} \times (2.8 \times 10^4)^4
\]

\[
= 1.17 \times 10^{38} \text{ erg s}^{-1} = 3.06 \times 10^4 L_\odot
\]

(b) Absolute bolometric magnitude

\[
M_B = M_\odot - 2.5 \log \left( \frac{L}{L_\odot} \right)
\]

\[
M_\odot = 4.76
\]

\[
M_B = 4.76 - 2.5 \times 4.49 = -6.45
\]

(c) Apparent bolometric magnitude

\[
m = M + 5 \log \left( \frac{d}{10 \text{pc}} \right) = -6.45 + 5 \log \left( \frac{180}{10} \right) = -6.45 + 6.28 = -0.17
\]

(d) Distance modulus

\[
\text{distance modulus} = m - M = 5 \log \left( \frac{d}{10 \text{pc}} \right) = 6.28
\]

(e) Flux at stars surface

\[
F_* = \sigma T^4 = 5.67 \times 10^{-5} (2.8 \times 10^4)^4 = 3.49 \times 10^{13} \text{ erg cm}^{-2} \text{ s}^{-1}
\]

(f) Flux at Earth
\[ F_{\text{earth}} = F_* \left( \frac{R_*}{D} \right)^2 = 3.49 \times 10^{13} \left( \frac{5.16 \times 10^{11}}{180 \times 3.086 \times 10^{18}} \right)^2 = 3.01 \times 10^{-5} \text{erg cm}^{-2} \text{s}^{-1} \]

The solar constant = 1.36 \times 10^6 \text{erg cm}^{-2} \text{s}^{-1}, so

\[ \frac{F_{*\text{(earth)}}}{F_{\odot\text{(earth)}}} = \frac{3.01 \times 10^{-5}}{1.36 \times 10^6} = 2.2 \times 10^{-11} \]

(g) Peak wavelength, \( \lambda_{\text{max}} \),

\[ \lambda T = 0.29 \text{ cm K} \]

so

\[ \lambda = \frac{0.29}{2.8 \times 10^4} = 1.04 \times 10^{-5} \text{ cm} = 0.104 \mu\text{m}, \]

which is in the UV.

3. Carroll & Ostlie: problem 9.2

(a) Derive the expression for the blackbody number density of photons, \( n_\lambda \).

\[ n_\lambda = \frac{E_\lambda}{hc/\lambda} \quad (17) \]

\[ E_\lambda = \frac{4\pi c}{c} B_\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \quad (18) \]

\[ n_\lambda = \frac{8\pi}{\lambda^4} \frac{1}{e^{hc/\lambda kT} - 1} \quad (19) \]

(b) Find the total number of photons inside an oven with volume 1 m\(^3\) at 400 F.

\[ n = \int n_\lambda d\lambda = 8\pi \int \frac{\lambda^{-4} d\lambda}{e^{hc/\lambda kT} - 1}. \]

Change the variable to \( x = \hbar d/\lambda kT \), so \( \lambda = \hbar c/x kT \) and \( d\lambda = -\lambda/x \, dx \), so

\[ n = 8\pi \left( \frac{kT}{hc} \right)^3 \int_0^\infty \frac{x^2 \, dx}{e^x - 1} \]

The value of the definite integral is 2.4 (as found from a book of definite integrals), so

\[ n = 8\pi \left( \frac{kT}{hc} \right)^3 \times 2.4 = 20.2T^3 \text{ photons cm}^{-3}. \]
The temperature of the oven is $T = 400$ F = 204 C = 477 K, so the number of photons in the oven is

$$N_{\text{oven}} = 10^6 \text{cm}^3 \times 20.2 \times (477)^3 \text{photons cm}^{-3} = 2.2 \times 10^{15} \text{photons.}$$


(a) Calculate the total number density of blackbody photons of all wavelengths and the average energy per photon at a temperature $T$.

$$N = 2.4 \times 8\pi \left(\frac{kT}{hc}\right)^3 \quad (20)$$

$$E = aT^4 = \frac{8\pi^5}{15} \left(\frac{kT}{hc}\right)^3 kT \quad (21)$$

$$\frac{E}{N} = \frac{\pi^4 kT}{2.4 \times 15} = 2.7kT. \quad (22)$$

(b) Calculate the average energy per blackbody photon at the center of the Sun, where $T = 1.58^7$ K, and at the solar surface, where $T = 5770$ K. Express the answer in eV.

$$k = 1.38 \times 10^{-16} \text{ergK}^{-1} = 8.617 \times 10^{-5} \text{eVK}^{-1}$$

$$\left(\frac{E}{N}\right) = 2.33 \times 10^{-4} T \text{eV} \quad (23)$$

$$\left(\frac{E}{N}\right)_{\text{center}} = 3.7 \times 10^3 \text{eV} \quad (24)$$

$$\left(\frac{E}{N}\right)_{\text{surface}} = 1.34 \text{eV} \quad (25)$$