Astronomy 304, STARS

Homework # 1, Answers

1. Carroll & Ostlie: problem 3.7

The average person has 1.4 m^2 of skin at a skin temperature of roughly 92 F (306 K). Consider the average person to be an ideal radiatro standing in a room at a temperature of 68 F (293 K).

(a) Calculate the energy per second radiated by the average person in the form of blackbody radiation. Express the answer in both units of ergs s−1 and in watts.

$$\begin{aligned} L &= S\sigma T^4 && (1)\\ S &= 1.4\,\mathrm{m}^2 && (2)\\ \sigma &= 5.67\times10^{-5}\,\mathrm{erg\,cm^{-2}\,s^{-1}} && (3)\\ T &= 306\,K && (4)\\ & && (5) \end{aligned}$$

Hence,

$$\begin{aligned} L &= 1.4\times10^4\times5.67\times10^{-5}\times(3.06\times10^2)^4 && (6)\\ &= 6.96\times10^9\,\mathrm{ergs^{-1}} = 696\,\mathrm{w}. && (7) \end{aligned}$$

(b) Determine the peak wavelength λ_{max} of the blackbody radiation emitted by the average person. In what region of the spectrum is this wavelength found?

$$\lambda_{\mathrm{max}}T = 0.29\,\mathrm{cmK},$$

so

$$\lambda_{\mathrm{max}} = \frac{0.29}{306} = 9.48\times10^{-4}\mathrm{cm} = 9.48\times10^{-6}\mathrm{m} = 9.48\,\mu m,$$

which is in the IR part of the spectrum.

(c) A blackbody also absorbs energy from its environment, in this case the room a 293 K. Calculate the energy absorbed per second by the average person, expressed in both units of erg s^{-1} and watts.

$$\begin{aligned} L &= S\sigma T^4 && (8)\\ L &= 1.4\times10^4\times5.67\times10^{-5}\times(2.93\times10^2)^4 && (9)\\ &= 5.85\times10^9\,\mathrm{erg\,s^{-1}} = 585\,\mathrm{w}. && (10) \end{aligned}$$

(d) Calculate the net energy per second loast by the average person due to blackbody radiation.

Net Loss = emission - absorption = 111 w.

2. Carroll & Ostlie: problem 3.8

Consider a model of a star as a shperical blackbody with a surface temperature of 28,000 K and a radius of 5.16×10^{11} cm, located at a distance of 180 pc from Earth. Determine its:

(a) Luminosity

$$\begin{aligned} L &= 4\pi R^2 \sigma T^4 && (11) \\ &= 4\pi(5.16 \times 10^{11})^2 \times 5.67 \times 10^{-5} \times (2.8 \times 10^4)^4 && (12) \\ &= 1.17 \times 10^{38}\,\mathrm{erg\,s^{-1}} = 3.06 \times 10^4\, L_\odot && (13) \end{aligned}$$

(b) Absolute bolometric magnitude

$$\begin{aligned} M_\mathrm{B} &= M_\odot - 2.5\log\left(\frac{L}{L_\odot}\right) && (14) \\ M_\odot &= 4.76 && (15) \\ M_\mathrm{B} &= 4.76 - 2.5 \times 4.49 = -6.45 && (16) \end{aligned}$$

(c) Apparent bolometric magnitude

$$m = M + 5\log\left(\frac{d}{10\mathrm{pc}}\right) = -6.45 + 5\log\left(\frac{180}{10}\right) = -6.45 + 6.28 = -0.17$$

(d) Distance modulus

$$\mathrm{distance\,modulus} = m - M = 5log\left(\frac{d}{10\mathrm{pc}}\right) = 6.28$$

(e) Flux at stars surface

$$F_* = \sigma T^4 = 5.67 \times 10^{-5}(2.8 \times 10^4)^4 = 3.49 \times 10^{13}\mathrm{erg\,cm^{-2}\,s^{-1}}$$

(f) Flux at Earth

$$F_{\rm earth} = F_* \left(\frac{R_*}{D}\right)^2 = 3.49 \times 10^{13} \left(\frac{5.16 \times 10^{11}}{180 \times 3.086 \times 10^{18}}\right)^2 = 3.01 \times 10^{-5} {\rm erg\,cm^{-2}\,s^{-1}}$$

The solar constant $= 1.36 \times 10^6\,{\rm erg\,cm^{-2}\,s^{-1}}$, so

$$\frac{F_{*({\rm earth})}}{F_{\odot({\rm earth})}} = \frac{3.01 \times 10^{-5}}{1.36 \times 10^6} = 2.2 \times 10^{-11}$$

(g) Peak wavelength, $\lambda_{\rm max}$,

$$\lambda T = 0.29\,{\rm cm\,K}$$

so

$$\lambda = \frac{0.29}{2.8 \times 10^4} = 1.04 \times 10^{-5}\,{\rm cm} = 0.104\,\mu{\rm m},$$

which is in the UV.

3. Carroll & Ostlie: problem 9.2

(a) Derive the expression for the blackbody number density of photons, n_λ.

$$n_\lambda = \frac{E_\lambda}{hc/\lambda} \qquad (17)$$

$$E_\lambda = \frac{4\pi}{c} B_\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \qquad (18)$$

$$n_\lambda = \frac{8\pi}{\lambda^4} \frac{1}{e^{hc/\lambda kT} - 1} \qquad (19)$$

(b) Find the total number of photons inside an oven with volume 1 m^3 at 400 F.

$$n = \int n_\lambda d\lambda = 8\pi \int \frac{\lambda^{-4} d\lambda}{e^{hc/\lambda kT} - 1}.$$

Change the variable to $x = hd/\lambda kT$, so $\lambda = hc/xkT$ and $d\lambda = -(\lambda/x)dx$, so

$$n = 8\pi \left(\frac{kT}{hc}\right)^3 \int_0^\infty \frac{x^2 dx}{e^x - 1}$$

The value of the definite integral is 2.4 (as found from a book of definite integrals), so

$$n = 8\pi \left(\frac{kT}{hc}\right)^3 \times 2.4 = 20.2 T^3\,{\rm photons\,cm^{-3}}.$$

The temperature of the oven is T = 400 F = 204 C = 477 K, so the number of photons in the oven is

$$N_{\rm oven} = 10^6 {\rm cm}^3 \times 20.2 \times (477)^3 \,{\rm photons\,cm}^{-3} = 2.2 \times 10^{15}\,{\rm photons.}$$

4. Carroll & Ostlie: problem 9.3

(a) Calculate the total number density of blackbody photons of all wavelengths and the average energy per photon at a temperature T.

$$N = 2.4 \times 8\pi \left(\frac{kT}{hc}\right)^3 \tag{20}$$

$$E = aT^4 = \frac{8\pi^5}{15}\left(\frac{kT}{hc}\right)^3 kT \tag{21}$$

$$\frac{E}{N} = \frac{\pi^4 kT}{2.4 \times 15} = 2.7kT. \tag{22}$$

(b) Calculate the average energy per blackbody photon at the center of the Sun, where $T = 1.58^7$ K, and at the solar surface, where $T = 5770$ K. Express the answer in eV.

$$k = 1.38 \times 10^{-16}\,{\rm ergK}^{-1} = 8.617 \times 10^{-5}\,{\rm eVK}^{-1}$$

$$\left(\frac{E}{N}\right) = 2.33 \times 10^{-4} T\,{\rm eV} \tag{23}$$

$$\left(\frac{E}{N}\right)_{\rm center} = 3.7 \times 10^3\,{\rm eV} \tag{24}$$

$$\left(\frac{E}{N}\right)_{\rm surface} = 1.34\,{\rm eV} \tag{25}$$