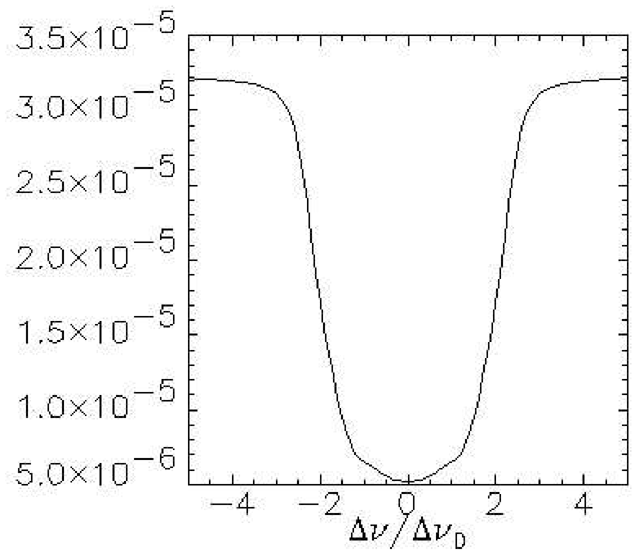
Astronomy 304, STARS

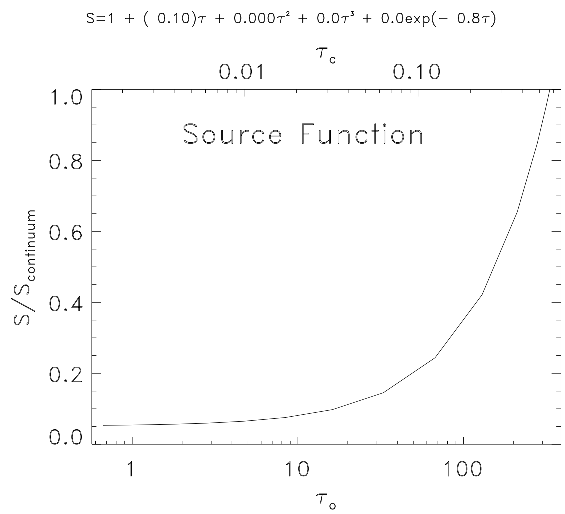
Homework # 7, Due Wednesday, March 12, 2003

Show All Your Work

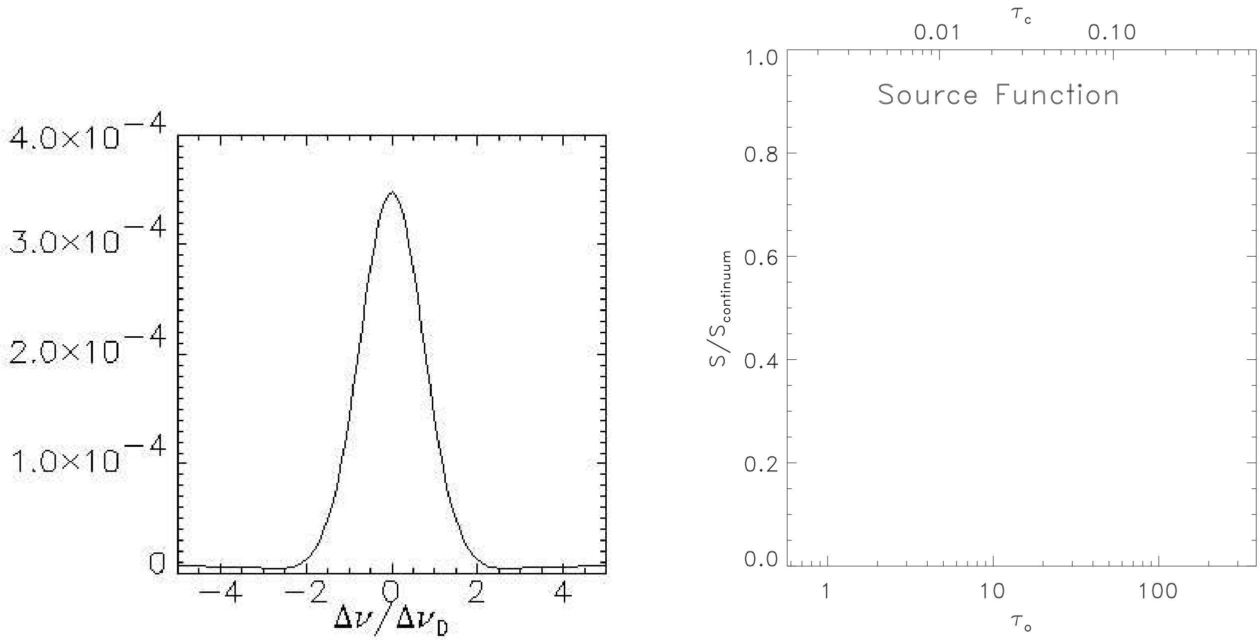
1. Use the Eddington-Barbier relations to sketch the optical depth dependence of the source function (in the blank graph on the right) that produces the line profile shown in the graph on the left, for the three cases (a), (b), (c). Here $\Delta\nu$ is the frequency offset from the center of the line, $\Delta\nu_D$ is the Doppler width of the line, τ_o is the line center optical depth and τ_c is the continuum optical depth. Note: the absorption coefficient in a line decreases approximately as a Gaussian with increasing offset from the line center frequency.

   (a)

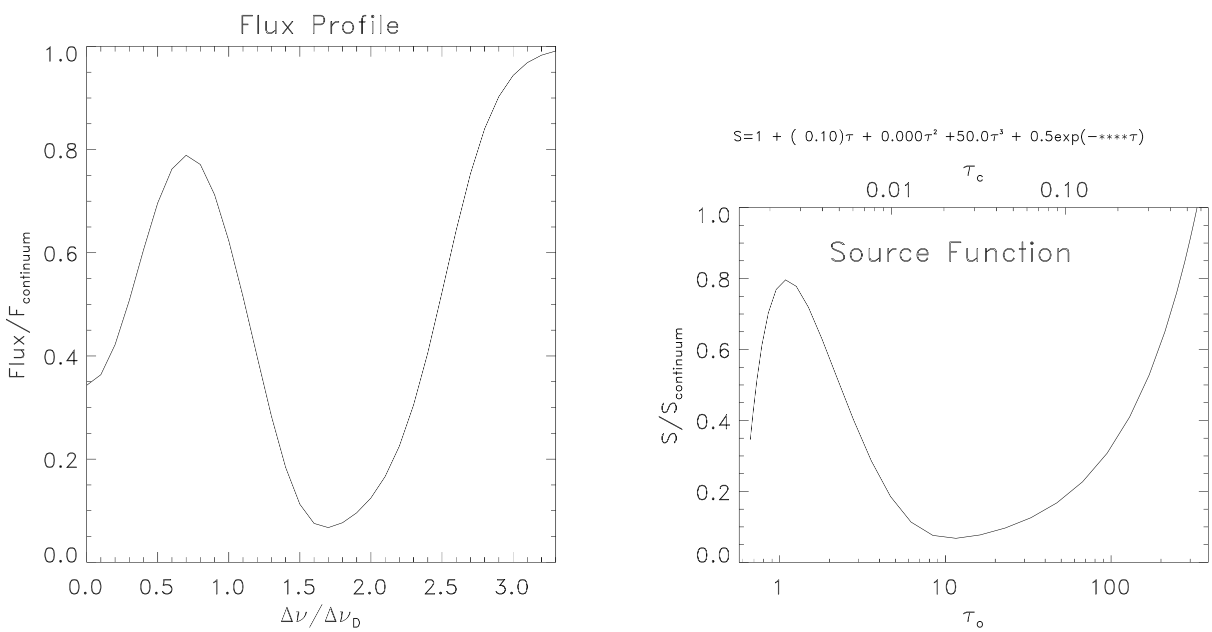




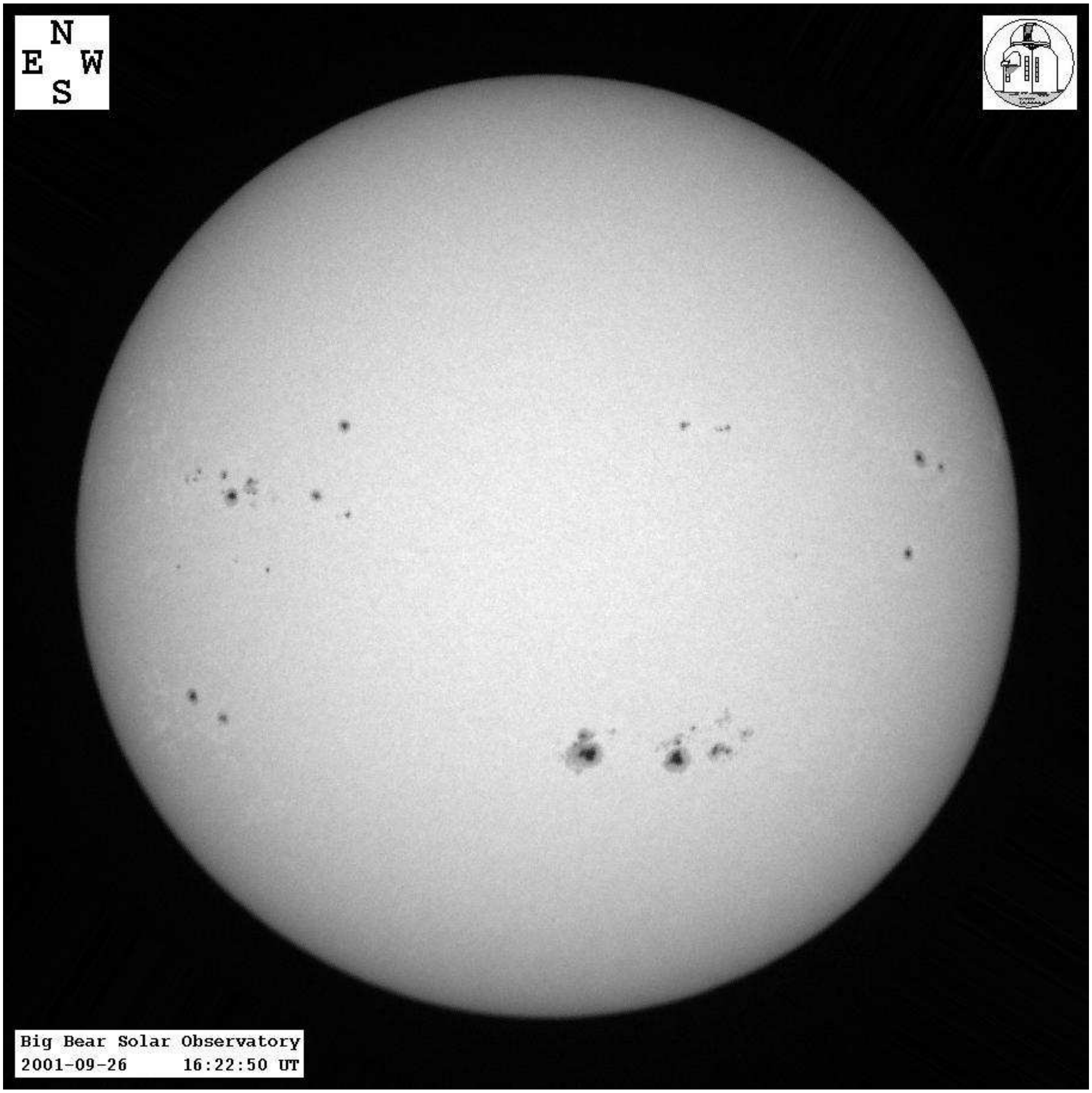
(b)


4.0×10⁻⁴
3.0×10⁻⁴
2.0×10⁻⁴
1.0×10⁻⁴
0
−4
−2
0
2
4
Δν/Δν_D
τ_c
0.01
0.10
Source Function
S/S_continuum
1.0
0.8
0.6
0.4
0.2
0.0
1
10
100
τ_o

(c)


Flux Profile
Flux/F_continuum
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Δν/Δν_D
S=1 + (0.10)τ + 0.000τ² +50.0τ³ + 0.5exp(−****τ)
τ_c
0.01
0.10
Source Function
S/S_continuum
1
10
100
τ_o

2. Below is an image of the Sun taken in the continuum (a broad wavelength range corresponding roughly to visible light). Based on this image, what can you tell about the variation of the source function (in this case equal to the Planck function) with height above the level of optical depth = 1 looking vertically down on the center of the Sun?



First notice that there are dark sunspots and also that the Sun appears darker towards its limb than at disk center. Darker means less intensity which means a smaller source function = emission per unit volume times mean free path at optical depth = one along the light ray. Sunspots are darker because they are cooler than their surroundings so even looking

straight down on them the source function (approximately equal to the Planck function $\approx \sigma/\pi T^4$) is smaller. Looking toward the limb of the Sun, the angle to the vertical gets larger (cos θ gets smaller) so optical depth along the ray =1 at higher layers as the limb is approached. Since the intensity is getting smaller toward the limb and $I_\nu = S_\nu(\tau_\nu = \mu)$ it means the source function (= the Planck function) is getting smaller at higher layers in the photosphere, which is due to the temperature decreasing with height in the photosphere.

3. Carroll & Ostlie: problem 11.2

   (a) At what rate is the Sun's mass decreasing due to nuclear reactions? Express your answer in solar masses per year.

$$\dot{M} = \frac{L}{c^2} = \frac{3.826 \times 10^{33}}{(3 \times 10^{10})^2} = 4.25\times10^{12} g/s = \frac{4.25 \times 10^{12} \times 3.14 \times 10^7}{1.989 \times 10^{33}} = 6.71\times10^{-14} M_\odot/yr$$

   (b) Compare your answer to part (a) with the mass loss rate due to the solar wind

   From example 11.1 the solar mass loss rate is $M_\odot = 3 \times 10^{-14}$ $M_\odot$ yr^{-1}.

   (c) Since the age of the Sun is about 4.5×10^9 yrs, the total mass lost would be only a negligible $10^{-4} M_\odot$.

4. Carroll & Ostlie: problem 11.3

   Using the Saha equation calculate the ratio of the number of H^- ions to neutral hydrogen atoms in the Sun's photosphere. Take the temperature to be the effective temperature and assume that the electron pressure is 15 dyne cm^{-2}. Note that the Pauli exclusion principle requires that only one state can exist for the ion because its two electrons must have opposite spins.

   The Saha equation (Eq. 8.7) is

$$\frac{N_{i+1}}{N_i} = \frac{N_H}{N_{H^-}} = \frac{2kTZ_H}{P_e Z_{H^-}} \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_i/kT} .$$

   Looking up H^- in the index, we find that its ionization potential is $\chi = 0.754 eV$. Substitute all the numerical values,

$$\begin{aligned}\frac{n_H}{n_{H^-}} &= \frac{2 \times 1.38 \times 10^{-16} \times 5.77 \times 10^3 \times 2}{15 \times 1} \left[\frac{2\pi \times 9.11 \times 10^{-28} \times 1.38 \times 10^{-16}}{(6.63 \times 10^{27})^2}\right]^{3/2} \\ &\quad \times (5.77 \times 10^3)^{3/2} exp\left[-\frac{0.754 \times 1.6 \times 10^{-12}}{1.38 \times 10^{-16} \times 5.77 \times 10^3}\right] \\ &= 2.12 \times 10^{-13} \times 2.41 \times 10^{15} \times 4.38 \times 10^5 \times e^{-1.52} = 4.92 \times 10^7 .\end{aligned}$$

   Thus only 1 in 10^7 hydrogen atoms attaches an electron to become an H^- ion.