Astronomy 304, Spring 2003

Midterm Exam

Name: _______________________

Student number: ________________

Show All Your Work! Remember units!
1. (10 points) Suppose your house is rectangular and has a flat roof. Suppose its dimensions are 5m wide $\times$ 10m long $\times$ 5m high. Outside it is 14 F = 263 K. The outside house walls have a uniform temperature of 52 F = 283 K. What is the net rate of heat loss from the house?

   The luminosity is the flux $\times$ the area and the flux is $\sigma T^4$, so

   $$ L = A \sigma T^4 $$

   The surface area of the rectangular solid (two ends, 2 sides and top, not bottom) is

   $$ A = 2 \times 5^2 + 3 \times 5 \times 10 = 200 \text{ m}^2 = 2 \times 10^6 \text{ cm}^2 $$

   Note:

   $$ T^4_2 - T^4_1 \neq (T_2 - T_1)^4 $$

   The emitted radiation from the wall is

   $$ L_{\text{emitted}} = A \sigma T^4_{\text{wall}} = 2 \times 10^6 \times 5.7 \times 10^{-5} \times 283^4 = 7.31 \times 10^{11} \text{ erg/s} $$

   The radiation absorbed by the wall is

   $$ L_{\text{absorbed}} = A \sigma T^4_{\text{air}} = 2 \times 10^6 \times 5.7 \times 10^{-5} \times 263^4 = 5.45 \times 10^{11} \text{ erg/s} $$

   The net rate of energy loss is

   $$ L_{\text{emitted}} - L_{\text{absorbed}} = A \sigma \left( T^4_{\text{wall}} - T^4_{\text{air}} \right) = 2 \times 10^6 \times 5.7 \times 10^{-5} \times (283^4 - 263^4) = 1.86 \times 10^{11} \text{ erg/s} $$
2. (10 points) Describe two methods that can be used to determine the surface temperature of a star.

(a) Color
   • color index
   • wavelength of peak emission

(b) Spectrum
   • strength of spectral lines
   • spectral class

(c) Flux + distance + radius
   (for eclipsing binaries)

\[ F_\star = F_{\text{obs}} \left( \frac{D}{R_\star} \right)^2 = \sigma T_{\text{eff}}^4 \]
3. (10 points) Derive the expression for the pressure of colliding particles,

\[ P = \frac{1}{3} n(mv)v \, . \]

Use it to derive the relation between pressure density and temperature for an ideal gas.

(a) Pressure is the force per unit area due to colliding particles.
P = force exerted per collision \times number of collisions per unit area per second
The force exerted per collision is the change in momentum

\[ \Delta p = 2mv \]

The number of collisions per unit area per unit time is

flux particles = \( \frac{n}{6} \)

Hence,

\[ P = \frac{1}{3} n(mv)v \]

(b) Equation of State for an ideal gas

\[ P = \frac{1}{3} n(mv)v \]

and

\[ \frac{1}{2}mv^2 = \frac{3}{2}kT \]

so

\[ P = nkT \]
4. (10 points) (a) What is the overall reaction for the fusion of hydrogen into helium, that is, all the initial particles → all the final particles?
(b) Calculate the rate at which the Sun is losing mass due to this fusion of hydrogen into helium.
(a) $4H \rightarrow 1He + 2\nu_e + 2e^+$
(b) The rate of mass loss is
$$\frac{dM}{dt} = \frac{L}{c^2}$$
since $E = mc^2$, so
$$\frac{dM}{dt} = \frac{4 \times 10^{33}}{9 \times 10^{20}} = 4.4 \times 10^{12} \text{ g/s} = 7 \times 10^{-27} \text{ M}_\odot/\text{yr}$$
5. (10 points) Draw a theoretical HR diagram.

- Include the axis scales in units of absolute magnitude $M_V$ and $\log T_{\text{eff}}$. The absolute bolometric magnitude of the Sun is 4.75.
- Sketch the location of the Main Sequence in this diagram.
- Mark the location of the Sun in the diagram and label it “Sun”.
- Mark the location of a star (labeled A) with the same spectral type of the Sun, but with three times larger radius.
- Outline the region occupied by White Dwarfs and label it.
- Mark the location of a main sequence star (labeled B) with luminosity $100 \ L_\odot$. 
6. (10 points) Star X has a parallax of 0.002 seconds of arc and an apparent magnitude \( m = 20 \).

(a) What is the distance of star X?
(b) What is the absolute magnitude of star X, assuming no interstellar absorption?

(a) Distance

\[
d(\text{pc}) = \frac{1}{\pi} = \frac{1}{0.002} = 500 \text{ pc}
\]

(b) Absolute Magnitude

\[
m - M = 5 \log_{10} \left( \frac{d}{10\text{pc}} \right)
\]

so

\[
M = m - 5 \log_{10} \left( \frac{d}{10\text{pc}} \right) = 20 - 5 \log 50 = 20 - 5 \times 1.7 = 20 - 8.49 = 11.5
\]
7. (10 points) Estimate the time it would take the Sun to cool down if all thermonuclear fusion reactions were to cease.

There are two approaches
1. The thermal timescale

\[ t = \frac{E_{\text{thermal}}}{L} \]

2. The random walk timescale for the photons

\[ t = N\ell, \quad N = \left(\frac{R}{\ell}\right)^2, \quad \ell = 1/\rho \kappa. \]

Method 1:

\[ E_{\text{thermal}} = N k < T > \]

Best way to get \( E_{\text{thermal}} \) is from the Virial Theorem

\[ 2E_{\text{thermal}} + W_{\text{gravity}} = 0 \]

where

\[ W_{\text{gravity}} \approx -\frac{3GM^2}{5R} \]

so

\[ E_{\text{thermal}} = \frac{3GM^2}{10R} = \frac{3 \times 6.7 \times 10^{-8} \times (2 \times 10^{33})^2}{7 \times 10^{10}} = 1.15 \times 10^{48}\text{erg} \]

and

\[ \ell = 4 \times 10^{33} \]

so

\[ t = \frac{1.15 \times 10^{48}}{4 \times 10^{33}} = 2.87 \times 10^{14} \text{s} = 9 \times 10^6 \text{yrs} \]

Method 2:

\[ t = \frac{R^2}{\ell c} = \frac{\rho \kappa R^2}{c} \]

The hard part is knowing what to take for the typical interior conditions for the Sun.
8. Below are spectra of 7 standard stars, types M-O and 4 stars of unknown spectral type for you to classify. Write down the spectral types of the unknown stars with as great an accuracy as you can.

Figure 1: Standard Stars

![Figure 1: Standard Stars](image)

Figure 2: Unknown Stars

![Figure 2: Unknown Stars](image)

<table>
<thead>
<tr>
<th>Star</th>
<th>Spectral Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>G6-8</td>
</tr>
<tr>
<td>2</td>
<td>F6-7</td>
</tr>
<tr>
<td>3</td>
<td>K5</td>
</tr>
<tr>
<td>4</td>
<td>O5</td>
</tr>
</tbody>
</table>
CONSORTS

\[ D_{\text{earth-sun}} = AU = 1.5 \times 10^{13} \text{ cm} \]
\[ M_{\text{earth}} = 6 \times 10^{27} \text{ g} \]
\[ M_{\text{sun}} = 2 \times 10^{33} \text{ g} \]
\[ L_{\text{sun}} = 4 \times 10^{33} \text{ erg s}^{-1} \]
\[ T_{e\odot} = 1.6 \times 10^{7} \text{ K} \]
\[ X_{e\odot} = 0.34 \]
\[ M_{\text{moon}} = 7 \times 10^{25} \text{ g} \]
\[ D_{\text{earth-moon}} = 3.8 \times 10^{10} \text{ cm} \]
\[ c = 3 \times 10^{10} \text{ cm s}^{-1} \]
\[ k = 1.4 \times 10^{-16} \text{ erg K}^{-1} \]
\[ a = 7.6 \times 10^{-15} \text{ erg cm}^{-3} \text{ K}^{-4} \]
\[ m_{H} = 1.7 \times 10^{-24} \text{ g} \]
\[ m_{e} = 9.1 \times 10^{-28} \text{ g} \]
\[ 1 \text{pc} = 2.06 \times 10^{5} \text{ AU} = 3.26 \text{ LY} \]
\[ Q_{H} = 6 \times 10^{18} \text{ erg g}^{-1} \]
\[ yr = 3.1 \times 10^{7} \text{ sec} \]
\[ R_{\text{earth}} = 6.4 \times 10^{8} \text{ cm} \]
\[ R_{\text{sun}} = 7 \times 10^{10} \text{ cm} \]
\[ \rho_{e\odot} = 162 \text{ g cm}^{-3} \]
\[ Y_{e\odot} = 0.64 \]
\[ R_{\text{moon}} = 1.7 \times 10^{8} \text{ cm} \]
\[ P_{\text{moon}} = 27.3 \text{ days} \]
\[ G = 6.7 \times 10^{-8} \text{ g}^{-1} \text{ cm}^{3} \text{ s}^{-2} \]
\[ h = 6.6 \times 10^{-27} \text{ erg sec} \]
\[ \sigma = 5.7 \times 10^{-5} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ K}^{-4} \]
\[ 1 \text{ rad} = 2 \times 10^{5} \text{ arcsec} \]
\[ 1 \text{LY} = 9.5 \times 10^{18} \text{ cm} \]
\[ Q_{He} = 6 \times 10^{17} \text{ erg g}^{-1} \]
\[ \theta(\text{rad}) = \text{Arc/Radius} \]
\[ m_1 - m_2 = -2.5 \log_{10} \left( \frac{F_1}{F_2} \right) \]
\[ a = \frac{F}{m} \]
\[ V \sim \frac{L}{t} \]
\[ t^2 \simeq \frac{L^2}{GM} \]
\[ M_1 + M_2 = \left( \frac{4\pi^2}{G} \right) \left( \frac{a^3}{P^2} \right) \]
\[ = \frac{Pv^3}{2\pi G} = \frac{v^2 R}{G} \]
\[ KE + PE = E_{\text{total}} \]
\[ 3(\gamma - 1)U + \frac{PE_{\text{gravity}}}{v^2} = 0 \]
\[ KE = \frac{1}{2}mV^2 \]
\[ P \simeq n(mV)V = nkT \]
\[ \Delta x \Delta (mv) \approx \hbar \]
\[ n_j^{i+1}/n_j^{i} = \left( \frac{2Z_j^{i+1}/Z_j^{i}}{2\pi m_e k T / h^2} \right)^{3/2} e^{-\chi/kt} \]
\[ \sigma(E) = S(E)/Ee^{x \left[ -2\pi Z_1 Z_2 e^2 / h (m/2E)^{1/2} \right]} \]
\[ \lambda = cP \]
\[ \epsilon_{\text{photon}} = h\nu = h c / \lambda \]
\[ \lambda_{\text{peak}} \simeq \frac{hc}{kT} = 0.3 / T(0K) \text{ cm} \]
\[ B_\nu = \frac{2h\nu^3}{c^2} \]
\[ U_R \simeq \left( \frac{k^4}{h^4 c^8} \right) T^4 = aT^4 \]
\[ P = \frac{dE}{dt} = FA = I\Omega A \]
\[ dI_\nu / d\tau_\nu = \epsilon_\nu \ell_\nu - I_\nu \]
\[ J_\nu = \frac{1}{4\pi} \int I_\nu d\Omega \]
\[ L = F \times A \]
\[ F = -\frac{1}{\ell^2} n < v > \ell \frac{dQ}{dz} \]
\[ \ell = 1 / n\sigma \]
\[ \kappa_\alpha = 0.2 (1 + X) \text{ cm}^2 \text{ g}^{-1} \]
\[ \kappa_\beta = 1.4 \times 10^{25} (1 + X) Z \rho T^{-3.5} \text{ cm}^2 \text{ g}^{-1} \]
\[ R_* = 2 \times 10^5 \text{AU} / \theta_{\text{arcsec}} = (pc) / \theta_{\text{arcsec}} \]
\[ m - M = 5 \log_{10} \left( \frac{d}{10 \text{pc}} \right) \]
\[ F_{\text{gravity}} = GMm/R^2 \]
\[ a \sim \frac{L}{t^2} \]
\[ Work = F \Delta x = -\Delta PE \]
\[ PE_{\text{gravity}} = -GMm/R \]
\[ P \times A = F_G \]
\[ \lambda = h / mv \]
\[ (2\pi m_e k / h^2)^{3/2} = 2.4 \times 10^{16} \]
\[ S(E) \sim 657 \text{ keV barns} \]
\[ P = 1 / \nu \]
\[ E \simeq kT \]
\[ n_\gamma \simeq \lambda^{-3} \]
\[ B_\lambda = \frac{2hc^3 / \lambda^5}{c^7 / \lambda^8} \]
\[ F = U_{Re} \simeq \frac{(kT/\hbar c)^3 kTc}{\hbar c^4} = \frac{\phi_T}{4} \frac{T^4}{T^4} = \sigma T^4 \]
\[ dE_\nu = I_\nu (r, n, t) dA \cos \theta d\Omega dtd\nu \]
\[ d\tau_\nu = ds / \ell_\nu \]
\[ F_\nu = \int I_\nu \cos \theta d\Omega \]
\[ \theta_{\text{min}}(\text{radians}) = \lambda / d \]
\[ L_{\text{rms}} = N^{1/2} \ell \]
\[ R = n_f r_f < \sigma v > \]
\[ \kappa_{H-} = 10^{-29} \rho^{1/2} T^{8.5} \text{ cm}^2 \text{ g}^{-1} \]
\[ \kappa_{ff} = 3.7 \times 10^{22} (1 + X)(X + Y) \rho T^{-3.5} \text{ cm}^2 \text{ g}^{-1} \]