Astronomy 304, Spring 2003

Midterm Exam

Name: ______________________________

Student number: ______________________

Show All Your Work! Remember units!

1. (10 points) Suppose your house is rectangular and has a flat roof. Suppose its dimensions are 5m wide × 10m long × 5m high. Outside it is 14 F = 263 K. The outside house walls have a uniform temperature of 52 F = 283 K. What is the net rate of heat loss from the house?

The luminosity is the flux × the area and the flux is σT^4, so

$$L = A\sigma T^4$$

The surface area of the rectangular solid (two ends, 2 sides and top, not bottom) is

$$A = 2 \times 5^2 + 3 \times 5 \times 10 = 200\,\mathrm{m}^2 = 2 \times 10^6\,\mathrm{cm}^2$$

Note:

$$T_2^4 - T_1^4 \neq (T_2 - T_1)^4$$

The emitted radiation from the wall is

$$L_{\mathrm{emitted}} = A\sigma T_{\mathrm{wall}}^4 = 2 \times 10^6 \times 5.7 \times 10^{-5} \times 283^4 = 7.31 \times 10^{11}\,\mathrm{erg/s}$$

The radiation absorbed by the wall is

$$L_{\mathrm{absorbed}} = A\sigma T_{\mathrm{air}}^4 = 2 \times 10^6 \times 5.7 \times 10^{-5} \times 263^4 = 5.45 \times 10^{11}\,\mathrm{erg/s}$$

The net rate of energy loss is

$$L_{\mathrm{emitted}} - L_{\mathrm{absorbed}} = A\sigma\left(T_{\mathrm{wall}}^4 - T_{\mathrm{air}}^4\right) = 2\times10^6\times5.7\times10^{-5}\times(283^4-263^4) = 1.86\times10^{11}\,\mathrm{erg/s}$$

2. (10 points) Describe two methods that can be used to determine the surface temperature of a star.

   (a) Color

   - color index
   - wavelength of peak emission

   (b) Spectrum

   - strength of spectral lines
   - spectral class

   (c) Flux + distance + radius
   (for eclipsing binaries)

$$F_* = F_{\rm obs}\left(\frac{D}{R_*}\right)^2 = \sigma T_{\rm eff}^4$$

3. (10 points) Derive the expression for the pressure of colliding particles,

$$P = \frac{1}{3} n (mv) v \ .$$

Use it to derive the relation between pressure density and temperature for an ideal gas.

(a) Pressure is the force per unit area due to colliding particles.
P = force exerted per collision × number of collisions per unit area per second
The force exerted per collision is the change in momentum

$$\Delta p = 2mv$$

The number of collisions per unit area per unit time is

$$\text{flux particles} = \frac{n}{6} v$$

Hence,

$$P = \frac{1}{3} n (mv) v$$

(b) Equation of State for an ideal gas

$$P = \frac{1}{3} n (mv) v$$

and

$$\frac{1}{2} m v^2 = \frac{3}{2} kT$$

so

$$P = nkT$$

4. (10 points) (a) What is the overall reaction for the fusion of hydrogen into helium, that is, all the initial particles $\rightarrow$ all the final particles?
(b) Calculate the rate at which the Sun is losing mass due to this fusion of hydrogen into helium.

(a) $4H \rightarrow 1He + 2\nu_e + 2e^+$

(b) The rate of mass loss is

$$\frac{dM}{dt} = \frac{L}{c^2}$$

since $E = mc^2$, so

$$\frac{dM}{dt} = \frac{4 \times 10^{33}}{9 \times 10^{20}} = 4.4 \times 10^{12}\ \mathrm{g/s} = 7 \times 10^{-27}\ \mathrm{M_\odot/yr}$$

5. (10 points) Draw a theoretical HR diagram.

- Include the axis scales in units of absolute magnitude M_V and $\log T_{\text{eff}}$. The absolute bolometric magnitude of the Sun is 4.75.
- Sketch the location of the Main Sequence in this diagram.
- Mark the location of the Sun in the diagram and label it "Sun".
- Mark the location of a star (labeled A) with the same spectral type of the Sun, but with three times larger radius.
- Outline the region occupied by White Dwarfs and label it.
- Mark the location of a main sequence star (labeled B) with luminosity 100 $L_\odot$.

6. (10 points) Star X has a parallax of 0.002 seconds of arc and an apparent magnitude m = 20.

   (a) What is the distance of star X?

   (b) What is the absolute magnitude of star X, assuming no interstellar absorption?

(a) Distance

$$d(\text{pc}) = \frac{1}{\pi} = \frac{1}{0.002} = 500\,\text{pc}$$

(b) Absolute Magnitude

$$m - M = 5log_{10}\left(\frac{d}{10\text{pc}}\right)$$

so

$$M = m - 5log_{10}\left(\frac{d}{10\text{pc}}\right) = 20 - 5log50 = 20 - 5 \times 1.7 = 20 - 8.49 = 11.5$$

7. (10 points) Estimate the time it would take the Sun to cool down if all thermonuclear fusion reactions were to cease.

There are two approaches

1. The thermal timescale

$$t = E_{\rm thermal}/L$$

2. The random walk timescale for the photons

$$t = \frac{N\ell}{c},\ N = \left(\frac{R}{\ell}\right)^2,\ \ell = 1/\rho\kappa\ .$$

Method 1:

$$E_{\rm thermal} = Nk < T >$$

Best way to get $E_{\rm thermal}$ is from the Virial Theorem

$$2E_{\rm thermal} + W_{\rm gravity} = 0$$

where

$$W_{\rm gravity} \approx -\frac{3}{5}\frac{GM^2}{R}$$

so

$$E_{\rm thermal} = \frac{3}{10}\frac{GM^2}{R} = \frac{3}{10}\frac{6.7\times10^{-8}\times(2\times10^{33})^2}{7\times10^{10}} = 1.15\times10^{48}{\rm erg}$$

and

$$l = 4\times10^{33}$$

so

$$t = \frac{1.15\times10^{48}}{4\times10^{33}} = 2.87\times10^{14}\,{\rm s} = 9\times10^6\ {\rm yrs}$$

Method 2:

$$t = \frac{R^2}{\ell c} = \frac{\rho\kappa R^2}{c}$$

The hard part is knowing what to take for the typical interior conditions for the Sun.

8. Below are spectra of 7 standard stars, types M-O and 4 stars of unknown spectral type for you to classify. Write down the spectral types of the unknown stars with as great an accuracy as you can.

Figure 1: Standard Stars

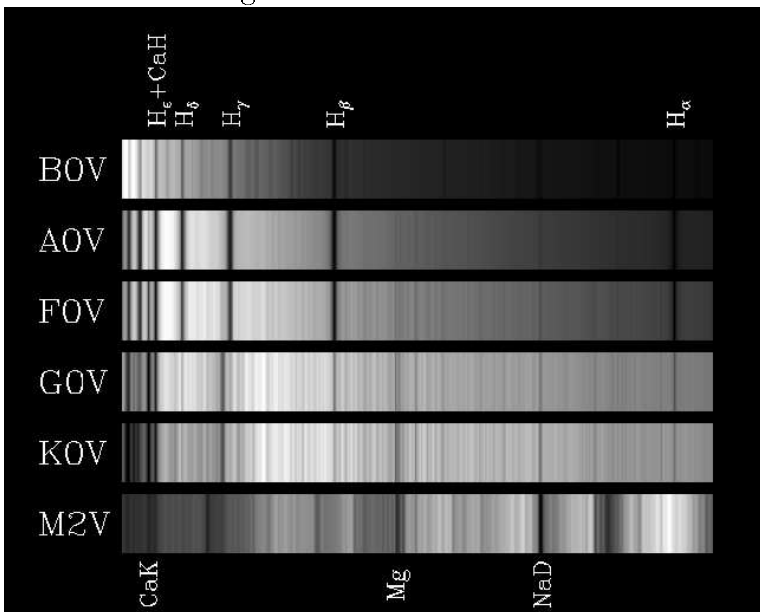
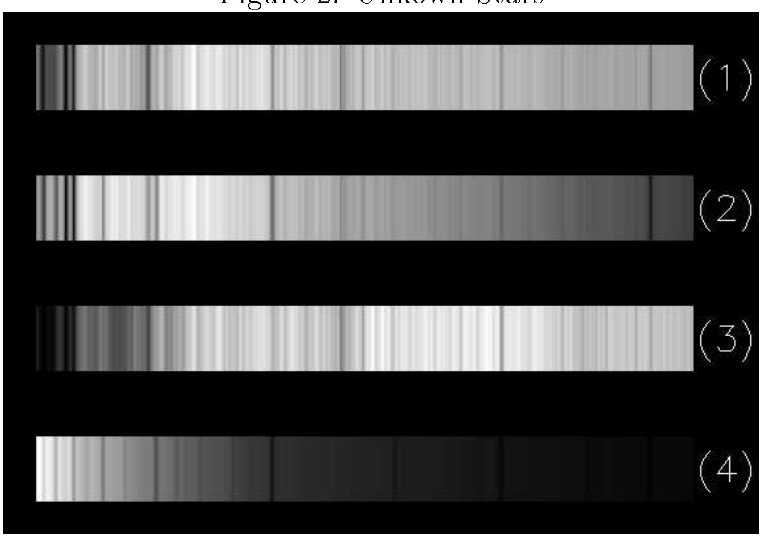


Figure 2: Unkown Stars



Star	Spectral Class
1	G6-8
2	F6-7
3	K5
4	O5

CONSTANTS

$D_{\rm earth-sun} = AU = 1.5 \times 10^{13}$ cm	$yr = 3.1 \times 10^{7}$ sec
$M_{\rm earth} = 6 \times 10^{27}$ g	$R_{\rm earth} = 6.4 \times 10^{8}$ cm
$M_{\rm sun} = 2 \times 10^{33}$ g	$R_{\rm sun} = 7 \times 10^{10}$ cm
$L_{\rm sun} = 4 \times 10^{33}$ erg s^{-1}	
$T_{c\odot} = 1.6 \times 10^{7}$ K	$\rho_{c\odot} = 162$ g cm^{-3}
$X_{c\odot} = 0.34$	$Y_{c\odot} = 0.64$
$M_{\rm moon} = 7 \times 10^{25}$ g	$R_{\rm moon} = 1.7 \times 10^{8}$ cm
$D_{\rm earth-moon} = 3.8 \times 10^{10}$ cm	$P_{\rm moon} = 27.3$ days
$c = 3 \times 10^{10}$ cm s^{-1}	$G = 6.7 \times 10^{-8}$ g^{-1} cm^{3} s^{-2}
$k = 1.4 \times 10^{-16}$ erg K^{-1}	$h = 6.6 \times 10^{-27}$ erg sec
$a = 7.6 \times 10^{-15}$ erg cm^{-3} K^{-4}	$\sigma = 5.7 \times 10^{-5}$ erg cm^{-2} s^{-1} K^{-4}
$m_{\rm H} = 1.7 \times 10^{-24}$ g	1 rad $= 2 \times 10^{5}$ arcsec
$m_{\rm e} = 9.1 \times 10^{-28}$ g	
$1pc = 2.06 \times 10^{5}$ AU $= 3.26$ LY	$1LY = 9.5 \times 10^{18}$ cm
$Q_{H} = 6 \times 10^{18}$ erg g^{-1}	$Q_{He} = 6 \times 10^{17}$ erg g^{-1}

FORMULAS

$\theta(\text{rad}) = \text{Arc}/\text{Radius}$

$R_\star = 2 \times 10^5 \text{AU}/\theta_{\text{arcsec}} = (\text{pc})/\theta_{\text{arcsec}}$

$m_1 - m_2 = -2.5\log_{10}(F_1/F_2)$

$m - M = 5\log_{10}(d/10pc)$

$a = F/m$

$F_{\text{gravity}} = GMm/R^2$

$V \sim L/t$

$a \sim L/t^2$

$t^2 \simeq L^3/GM$

$$M_1 + M_2 = (4\pi^2/G)(a^3/P^2) = Pv^3/2\pi G = v^2R/G$$

$KE + PE = E_{\text{total}}$

$\text{Work} = F\Delta x = -\Delta PE$

$3(\gamma - 1)U + PE_{\text{gravity}} = 0$

$KE = 1/2mV^2$

$PE_{\text{gravity}} = -GMm/R$

$P \simeq n(mV)V = nkT$

$P \times A = F_G$

$\Delta x \Delta(mv) \approx h$

$\lambda = h/mv$

$n^{j+1}n_e/n^j = (2Z^{j+1}/Z^j)\left(2\pi m_e kT/h^2\right)^{3/2} e^{-\chi/kt}$

$\left(2\pi m_e k/h^2\right)^{3/2} = 2.4 \times 10^{16}$

$\sigma(E) = S(E)/E exp\left[-2\pi Z_1 Z_2 e^2/\hbar\,(m/2E)^{1/2}\right]$

$S(E) \sim 657\,\text{keV barns}$

$\lambda = cP$

$P = 1/\nu$

$e_{\text{photon}} = h\nu = hc/\lambda$

$E \simeq kT$

$\lambda_{\text{peak}} \simeq hc/kT = 0.3/T(^\circ K)\ \text{cm}$

$n_\gamma \simeq \lambda^{-3}$

$B_\nu = \frac{2h\nu^3/c^2}{e^{h\nu/kT}-1}$

$B_\lambda = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT}-1}$

$U_R \simeq \left(\frac{k^4}{h^3c^3}\right)T^4 = aT^4$

$F = U_R c \simeq (kT/hc)^3 kTc = \frac{ac}{4}T^4 = \sigma T^4$

$P = dE/dt = FA = I\Omega A$

$dE_\nu = I_\nu(\mathbf{r}, \mathbf{n}, t)dA cos\theta d\Omega dt d\nu$

$dI_\nu/d\tau_\nu = \epsilon_\nu \ell_\nu - I_\nu$

$d\tau_\nu = ds/\ell_\nu$

$J_\nu = \frac{1}{4\pi}\int I_\nu d\Omega$

$F_\nu = \int I_\nu cos\theta d\Omega$

$L = F \times A$

$\theta_{\text{min}}(\text{radians}) = \lambda/d$

$F = -\frac{1}{3}n < v > \ell\frac{dQ}{dz}$

$L_{\text{rms}} = N^{1/2}\ell$

$\ell = 1/n\sigma$

$R = n_I n_T < \sigma v >$

$\kappa_{\text{es}} = 0.2(1 + X)\,\text{cm}^2\,\text{g}^{-1}$

$\kappa_{\text{H}^-} = 10^{-29}\rho^{1/2}T^{8.5}\,\text{cm}^2\,\text{g}^{-1}$

$\kappa_{\text{bf}} = 1.4 \times 10^{25}(1 + X)Z\rho T^{-3.5}\,\text{cm}^2\,\text{g}^{-1}$

$\kappa_{\text{ff}} = 3.7 \times 10^{22}(1 + X)(X + Y)\rho T^{-3.5}\,\text{cm}^2\,\text{g}^{-1}$