Physics 492 homework V, due Fri Feb 7

Reading: Chapters 4, 5

Problems:

1. In the semiclassical limit, the Fermi energy of an ideal gas of \( N \) identical spin-1/2 particles with mass \( m \) in a volume \( V \) is

\[
E_F(N) = \frac{\hbar^2}{2m} \left( \frac{3\pi^2 N}{V} \right)^{2/3}.
\]

Consider a nucleus with \( Z \) protons, \( N = A - Z \), and radius \( R = r_0 A^{1/3} \), where \( r_0 = 1.12 \) fm. In an ideal gas model the total kinetic energy is

\[
E = \frac{3}{5} Z E_F(Z) + \frac{3}{5} N E_F(N).
\]

(a) Determine \( E_F \) and \( E \) for \(^{16}\text{O}\).
(b) If \( N \approx Z \) then

\[
E \approx E_0 + a_A \frac{(N - Z)^2}{A},
\]

where \( E_0 = \frac{3}{5} A E_F(A/2) \). Determine the value of \( a_A \) in the ideal-gas limit.

Hint: Write

\[
N = \frac{A}{2} + \epsilon \quad \text{and} \quad Z = \frac{A}{2} - \epsilon,
\]

where

\[
N - Z = 2\epsilon,
\]

and expand the energy in \( \epsilon \).

(This is a modified Problem 4.3 in Williams.)

2. Williams, Problem 5.1. Use the coefficient values given in class, i.e. \( a_V = 15.85 \) MeV, \( a_S = 18.34 \) MeV, \( a_A = 23.22 \) MeV and \( a_C = 0.71 \) MeV. Note that to maintain the unit consistency, the mass formula in Williams should be actually written as

\[
M'(Z, A) c^2 = Z m_H c^2 + N m_n c^2 - a_V A + \ldots - \delta.
\]

3. Williams, Problem 5.4.

4. Williams, Problem 5.5. Hint: Calculate \( a_C \) from the \( Q \) value of the \( \beta^+ \) decay of \(^{35}\text{Ar}\). Estimate \( a_A \) using the fact that \(^{135}\text{Ba}\) is stable and thus the \( Q \) values of \( \beta^+ \) and \( \beta^- \) decays must be negative; \( Q(\beta^+) < 0 \) and \( Q(\beta^-) < 0 \) imply upper and lower bounds on \( a_A \), after substituting the value of \( a_C \).