Physics 492 homework VI, due Fri Feb 14

Reading: Chapters 6, 7.1-3

Problems:

1. Use the masses in the Table of Isotopes in the CRC Handbook of Chemistry and Physics (on reserve in Physics Library) or on the Web (http://www.nndc.bnl.gov/nndcscr/masses/MASS_RMD.MAS95) to show that $^{197}$Au is nominally unstable with respect to $\alpha$-decay. Calculate the kinetic energy of an $\alpha$ particle that would be emitted in the decay. (Note: Because of the recoil given to the daughter nucleus, the kinetic energy is slightly less than the $Q$-value for the decay.) Using the empirical Geiger-Nutall law, $\log_{10} t_{1/2} \simeq a + b Q^{-1/2}$, with $a \simeq -1.61 Z_D^{2/3} - 21.4$ and $b \simeq 1.61 Z_D$, estimate the half-life for the $\alpha$-decay of gold. The time in the empirical law is in seconds and the $Q$-value is in MeV. How does that half-life compare to the age of Universe? How does the $Q$-value compare to the one obtained in Problem 5.1 from Williams?

2. Use the tunneling formula derived in class, and given in the handout, to justify the empirical Geiger-Nutall law in Problem 2, including the values of the numerical coefficients there. The $\alpha$-particle velocity inside the parent nucleus may be assumed to take some representative value, such as outside the nucleus or larger.

3. Consider the strongly deformed nucleus $^{252}$Fm with the deformation parameter $\epsilon = 0.3$. That is, the nucleus is shaped like an ellipsoid of revolution with semimajor axis $a' = R(1 + \epsilon)$ and semiminor axis $a = R/(1 + \epsilon)^{-1/2}$, where $R \simeq r_0 A^{1/3}$ is the mean radius. Using a potential of the form suggested in the figure below, estimate the relative probabilities of polar and equatorial emission of $\alpha$ particles.

4. A typical induced fission reaction is

\[ n + ^{235}_{92}U \rightarrow ^{92}_{36}Kr + ^{142}_{56}Ba + 2n. \]

(a) Estimate the mass energy released, using the Weizsäcker semi-empirical mass formula.

(b) Calculate the mass energy released, using the exact atomic masses in the Table of Isotopes.

(c) Calculate the total mass energy, in joules, released when 1 kg of $^{235}$U undergoes fission.

Note: nuclear mass = atomic mass $- Z m_e$, 
1 u = 931.494 MeV/c$^2$, 
mass excess = atomic mass $- A \times 1$u.