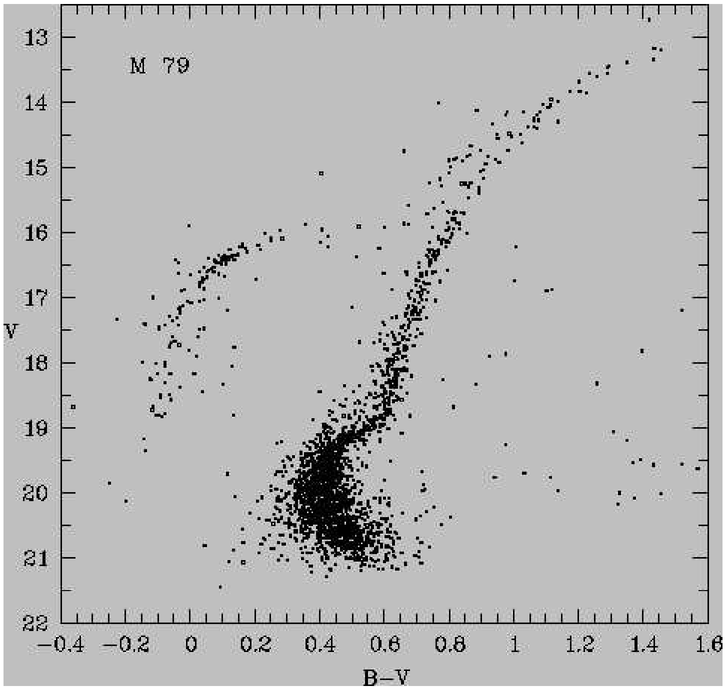
Astronomy 304, STARS

Homework # 12 Due Wednesday, April 23, 2003

Show All Your Work

1. Age of Star Cluster

Determine the age and distance of the cluster whose HR-diagram is shown below.



Carroll & Ostlie, figure 7.7, gives the mass-luminosity relation. Carroll & Ostlie, figure 13.1, gives evolutionary tracks for stars of several masses from 1-15 $M_\odot$. *Astrophysical Quantities* (Allen 1973) gives the following relation between B-V and $T_{\rm eff}$ for main sequence stars:

B - V	$\mathbf{T_{eff}}$
-0.35	40,000
-0.31	28,000
-0.16	15,500
0.00	9,900
+0.13	8,500
+0.27	7,400
+0.42	6,580
+0.58	6,030
+0.70	5,520
+0.89	4,900
+1.18	4,130
+1.45	3,480
+1.63	2,800
+1.8	2,400

The color at the turnoff is B-V = 0.3. From the table this corresponds to an effective temperature of about 7200 K, whose log is 3.86. From Fig 13.1 this corresponds to a mass of about 1.25 $M_\odot$. From table 13.1, the main-sequence lifetime of such a star is about 3×10^9 years. (The luminosity of such a star is, again from Fig 13.1, about 2 $L_\odot$.)

From Appendix E, a Main-Sequence star with B-V=0.3 has an absolute visual luminosity of M_V=2.7 and a Main-Sequence star with B-V=0.44 has an absolute visual luminosity of M_V=3.6. The stars at the turnoff in the cluster have an apparent magnitude $V \approx 20$ and the Main-Sequence stars with B-V=0.45 have an apparent magnitude $V \approx 20.5$. Hence the distance modulus is 16.9-17.3. The distance to the cluster is therefor,

$$d = 10^{(m-M+5)/5} = 24 - 29\text{kpc} \ .$$

2. Stellar Energy Budget

Consider a star which starts life as a 10 $M_\odot$ diffuse interstellar cloud of hydrogen (90%) and helium (10%) (by number). It ends its life as a 2 $M_\odot$ neutron star after having expelled 8 $M_\odot$ of its material in a supernova explosion. During its life it radiated energy at a rate of $L \sim 5 \times 10^4$ $L_\odot$ for about 2.6×10^7 yrs.

(a) Calculate the total (including rest mass) initial and final energy.

$$E_{\text{total}} = E_{\text{rest mass}} + E_{\text{gravitationalPE}} + E_{\text{thermal}}$$

Initially 10 $M_\odot$ is spread out in an interstellar cloud, where the gravitational potential energy is nearly zero because of the large size and the thermal energy is also nearly zero because it is cold.

$$E_{\text{initial}} = Mc^2 = 10 \times 1.989 \times 10^{33} \times (3 \times 10^{10})^2 = 1.79 \times 10^{55}\,\text{erg}$$

At the end of its life, its final energy is that of a 2 $M_\odot$ neutron star with a radius (eqn 15.22)

$$\begin{aligned} R_{\text{NS}} &= \frac{(18\pi)^{2/3}}{10} \frac{\hbar^2}{G M_{\text{NS}}^{1/3}} m_H^{-8/3} & (1) \\ &= \frac{(18\pi)^{2/3}}{10} \frac{(1.055 \times 10^{-27})^2}{6.67 \times 10^{-8} \times (2 \times 1.99 \times 10^{33})^{1/3}} (1.67 \times 10^{-24})^{-8/3} & (2) \\ &= 3.95 \times 10^5\,\text{cm} = 3.95\,\text{km} & (3) \end{aligned}$$

For a star in hydrostatic equilibrium the thermal and gravitational energies are related by the virial theorem

$$3(\gamma - 1)U_{\text{thermal}} + W_{\text{gravitational}} = 0$$

where, for a non-relativisitc perfect gas, $\gamma = 5/3$, so $U_{\text{thermal}} = -0.5W_{\text{gravitational}}$. Then the final energy is

$$\begin{aligned} E_{\text{final}} &= Mc^2 + \frac{1}{2}W_{\text{gravitational}} & (4) \\ &= Mc^2 - \frac{3}{10}\frac{GM^2}{R} & (5) \\ &= 2 \times 1.989 \times 10^{33} \times (3 \times 10^{10})^2 - \frac{3}{10}\frac{6.67 \times 10^{-8} \times (2 \times 1.99 \times 10^{33})^2}{3.95 \times 10^5} & (6) \\ &= 3.58 \times 10^{54} - 8.0 \times 10^{53} = 2.78 \times 10^{54}\,\text{erg} & (7) \end{aligned}$$

(b) Calculate the energy lost during its life and during the supernova explosion.

The energy lost during its life is

$$E_{\text{initial}} - E_{\text{final}} = 1.51 \times 10^{55}\,\text{erg}$$

(c) What are the sources of this energy – give quantitative results.

Most of the energy is just the rest mass energy of the 8 $M_\odot$ that is ejected to essentially ∞,

$$\Delta E = Mc^2 = 8 \times 1.989 \times 10^{33} \times (3 \times 10^{10})^2 = 1.43 \times 10^{55}\,\text{erg}$$

The remainder, 8×10^{53} erg, is the half of the gravitaional potential energy released when the star collapses from approximately a white dwarf configuration typical of the cores of old stars to a neutron star and produces a supernova, that is radiated away mostly as neutrinos. Some of this goes to accelerate the ejecta from the supernova. The other half remains as thermal energy in the neutron star.

The energy produced by thermonuclear fusion reactions is radiated away as luminosity during the stars life,

$$\begin{aligned}\Delta E &= (L/L_\odot)\, L_\odot \times \mathrm{age(yrs)} \times (\mathrm{sec/yr}) \\ &= 5 \times 10^4 \times 3.826 \times 10^{33} \times 2.6 \times 10^7 \times 3.156 \times 10^7 = 1.57 \times 10^{53}\, \mathrm{erg}\end{aligned}$$

Hence, the total energy released by thermonuclear fusion reactions during the stars life and the energy released as gravitational potential energy during the collapse from a white dwarf to a neutron star like core are comparable!

3. Carroll & Ostlie: Problem 13.10

Taking the distance to the Crab to be 2000 pc, and assuming that the absolute bolometric magnitude at maximum brightness was characteristic of Type II supernova, estimate its peak apparent magnitude. Compare this to the maximum brightness of the planet Venus (m $\approx$ -4), which is sometimes visible in the daytime.

The peak luminosity of a Type II supernova (p 514) is L=10^{43} erg/s. which corresponds to an absolute magnitude of

$$M_{\mathrm{crab}} = M_{odot} - 2.5 \log_{10}\left(\frac{L}{L_\odot}\right) = 4.76 - 2.5 \times 9.7 = -19.49$$

However, the book quotes a typical peak absolute magnitude of -18 (p 515). Then the apparent magnitude is

$$m = M + 5 \log_{10}(d/10\mathrm{pc}) = -18 + 5 \times 2.3 = -6.5\ ,$$

which is much brighter than Venus, m $\approx$ -4.

4. Carroll & Ostlie: Problem 13.16

An old version of stellar evolution, popular at the beginning of the twentieth century, maintained that stars begin their lives a large, cool spheres of gas, like giant stars on the H-R diagram. They then contract and heat up under th epull of their own gravity to become hot, bright blue O stars. For the remainder of their lives they lose energy, becomming dimmer and redder with age. As they slowly move down the main sequence, they eventually end up as cool, dim red M stars. Explain how observations of stellar cluster, plotted on an H-R diagram, contradict this idea.

Young clusters, as determined by their solar like heavy element abundance, have few red giants, and most of their stars are on the main sequence over its entire length. Since these stars were all formed about the same time, there does

not seem to be much evolution along the main sequence. Further, old clusters, with much smaller heavy element abundances than the Sun, have now bright blue main sequence stars, but do have many red giants, indicating there is evolution from the main sequence to the red giant brnach.