Astronomy 304, STARS

Homework # 2, Answers

Show All Your Work

1. Carroll & Ostlie: problem 5.14

A white dwarf is a very dense star, with it ions and electrons packed extremely closely together. Each electron may be considered to be located within a region of size $\Delta x \approx 1.5 \times 10^{-10}$ cm. Use Heisenberg's uncertainty principle, Eq. (5.18), to estimate the minimum speed of the electron. Do you think that the effects of relativity will be important for these stars?

$$\Delta v = \Delta p/m \approx \hbar/m_e \Delta x$$

$$\Delta v = \frac{6.62 \times 10^{-27}}{2\pi 9.1 \times 10^{-28} 1.5 \times 10^{-10}} = 7.7 \times 10^9 \, \mathrm{cm/s}$$

Relativistic corrections go as $(v/c)^2$, and here v/c=0.26, so relativistic corrections are about 6%. Note that the thermal speed of electrons at the center of the Sun is slightly less, $\approx 2 \times 10^9$ cm/s.

2. Carroll & Ostlie: problem 5.15

An electron spends roughly 10^{-8} s in the first excited stat of the hydrogen atom before making a spontaneous downward transition to the ground state.

(a) Use Heisenberg's uncertainty principle (Eq. 5.19) to determine the uncertainty ΔE in the energy of the first excited state.

$$\Delta E \approx h/(2\pi \Delta t)$$

The lifetime of the level is the uncertainty in time, so the uncertainty in energy of level 2 is

$$\Delta E = 1.05 \times 10^{-19} \, \mathrm{erg} = 6.58 \times 10^{-8} \, \mathrm{eV}$$

(b) Calculate the uncertainty $\Delta\lambda$ in the wavelength of the photon involved in a transition (either upward or downward) between the ground and first excited states of the hydrogen atom. Why can you assume that $\Delta E = 0$ for the ground state?

$\Delta\nu \approx \Delta E/h = c/\lambda^2 \Delta\lambda$.

The wavelength of $Ly_\alpha = 1.215 \times 10^{-5}$ cm, so the uncertainty in the wavelength is

$$\Delta\lambda \approx \frac{\lambda^2 \Delta E}{hc} = \frac{(1.215 \times 10^{-5})^2 (1.05 \times 10^{-19})}{(6.62 \times 10^{27})(3 \times 10^{10})}$$

$$\Delta\lambda \approx 7.8 \times 10^{-14} \text{cm} = 7.8 \times 10^{-7} \text{nm}$$

which is much less than the thermal Doppler width.

3. Carroll & Ostlie: problem 11.5a

(a) Using Eq. (9.58) and neglecting turbulence, estimate the full width at half-maximum of the hydrogen H_α absorption line due to random thermal motions in the Sun's photosphere. Assume that the temperature is the Sun's effective temperature.

Eq 9.58 gives the full width at half-maximum

$$(\Delta\lambda)_{1/2} = \frac{2\lambda}{c} \left[\left(\frac{2kT}{m} + v_{\text{turb}}^2 \right) \ln 2 \right]^{1/2}$$

Here,

$$\begin{aligned}
\lambda &= 656.2 nm = 6.562 \times 10^{-5} cm & (1)\\
c &= 2.998 \times 10^{10} cm/s & (2)\\
k &= 1.38 \times 10^{-16} erg/K & (3)\\
m &= 1.67 \times 10^{-24} g & (4)\\
T_{\text{eff.}} &= 5770 K & (5)\\
& & (6)
\end{aligned}$$

so

$$(\Delta\lambda)_{1/2} = 3.56 \times 10^{-9} cm = 0.0356 nm$$

4. Carroll & Ostlie: problem 9.11a

According to the "standard model" of the Sun, the central density is 162 g cm^{-3} and the Rosseland mean opacity at the center is 1.16 cm^2 g^{-1}.

(a) Calculate the mean free path of a photon at the center of the Sun.

$$\ell = 1/n\sigma = 1/\rho\kappa = 1/(162 \text{g cm}^{-3} \times 1.16 \text{cm}^2 \text{g}^{-1}) = 5.32 \times 10^{-3} \text{ cm}.$$

5. Carroll & Ostlie: problem 7.4

Sirius is a visual binary with a period of 49.94 yr. Its measured trigonometric paralax is 0.377" and, assuming that the plane of the orbit is in the plane of the sky, the true angular extent of the semimagjor axis of the reduced mass is $\alpha = 7.62$". The ratio of the distances of Sirus A and Sirius B from the center of mass is $a_A/a_B = 0.466$.

(a) Find the mass of each member of the system.

The distance to Sirius is

$$D = 1/\pi" = 2.65pc = 8.178 \times 10^{18} \text{ cm} .$$

The semimajor axis is

$$a = \alpha(\text{rad})D = 7.62" \frac{2\pi \text{ rad}}{1.296 \times 10^6 \text{ "}} \times 8.178 \times 10^{18} = 3.02 \times 10^{14} \text{ cm} .$$

Then the sum of the masses is

$$M_A + M_B = \frac{4\pi^2}{G}\frac{a^3}{P^2} = \frac{4\pi^2}{6.67 \times 10^{-8}} \frac{(3.02 \times 10^{14})^3}{(49.49 \times 3.156 \times 10^7)^2} = 6.68 \times 10^{33} \text{ g} = 3.36 \text{ M}_\odot .$$

The ratio of masses is

$$M_A a_A = M_B a_B$$

or

$$M_B = M_A \frac{a_A}{a_B}$$

so

$$M_A (1 + a_A/a_B) = 3.36 \text{ M}_\odot$$

or

$$M_A = 3.36/(1 + 0.466) = 2.29 \text{ M}_\odot ,$$

and

$$M_B = 3.36 - 2.29 = 1.07 \text{ M}_\odot .$$

(b) The absolute bolometric magnitude of Sirius A is 1.33 and Sirius B has a absolute bolometric magnitude of 8.57. Determine their luminosities, in units of the solar luminosity.

The magnitude is related to the flux by

$$m_1 - m_2 = -2.5 log_{10}\left(\frac{F_1}{F_2}\right) \quad (7)$$

$$\frac{F_1}{F_2} = 10^{-(m_1 - m_2)/2.5} . \quad (8)$$

The flux for a star at distance D and luminosity L is

$$F = L/4\pi D^2 .$$

For stars at the same distance the ratio of luminosities = the ratio of fluxes. For absolute magnitudes the distance is the nominal distance of 10 pc, and the absolute bolometric magnitude of the Sun is $M_{\mathrm{bol}\odot} = 4.76$, so

$$\frac{L_A}{L_\odot} = 10^{(4.76-1.33)/2.5} = 10^{1.37} = 23.6 \tag{9}$$

$$\frac{L_B}{L_\odot} = 10^{(4.76-8.57)/2.5} = 10^{-1.52} = 0.03 \; . \tag{10}$$

(c) The effective temperature of Sirius B is estimated to be approximately 27,000 K. Estimate its radius and compare it to the radii of the Sun and Earth.

The luminosity for given radius and effective temperature is

$$L = 4\pi R^2 \sigma T_{\mathrm{eff}}^4$$

so

$$R = \left(\frac{L}{4\pi\sigma T_{\mathrm{eff}}^4}\right)^{1/2} \; ,$$

or

$$R_B = \left(\frac{0.03 \times 3.83 \times 10^{33}}{4\pi \times 5.67 \times 10^{-5} \times (2.7 \times 10^4)^4}\right)^{1/2} = 5.5 \times 10^8 \,\mathrm{cm} = 8 \times 10^{-3} \, R_\odot = 0.86 \, R_\oplus \; .$$