Astronomy 304, STARS

Homework # 8, Due Wednesday, March. 19, 2003

Show All Your Work

1. Carroll & Ostlie: problem 11.8

   Suppose that you are attemping to make observations through an optically thick gas that has a constant density and temperature. Assume that the density and temperature of the gas are 2.5×10^{-7} gm cm^{-3} and 5770 K, respectively, typical of the values found at the base of the Sun's photosphere. If the opacity of the gas at one wavelength (λ_1) is $\kappa_{\lambda 1} = 0.26$ cm^2/g and the opacity at another wavelength (λ_2) is $\kappa_{\lambda 2} = 0.30$ cm^2/g, calculate the distance into the gas where the optical depth equals 2/3 for each wavelength. At which wavelength can you see farther in the gas? How much farther? This effect allows astronomers to probe the Sun's atmosphere at different depths (see Fig 11.17).

   Optical depth is

$$\begin{aligned} \tau &= \rho\kappa\Delta z \\ &= 2.5 \times 10^{-7} \times 0.26 \times \Delta z = 2/3 \\ &= 2.5 \times 10^{-7} \times 0.30 \times \Delta z = 2/3 \ . \end{aligned} \tag{1}$$

   Hence,

$$\begin{aligned} \Delta z_1 &= 103\,\mathrm{km} \\ \Delta z_2 &= 89\,\mathrm{km} \ . \end{aligned} \tag{2}$$

   Can see 14 km deeper at wavelength 1 which has the small opacity.

2. Carroll & Ostlie: problem 12.2

   Estimate the temperature of a dust grain that is located 100 AU from a newly formed F0 main-sequence star. *HINT:* Assume that the dust grain is in thermal equilibrium – meaing that the amount of energy absorbed by the grain in a given time interval must equal the amont of energy radiated away during the same interval of time. Assume also that the dust grain is spherically symmetric and emits and absorbs radiation as a perfect blackbody. You may want to refer to Appendix E for the effective temperature and radius of an F0 main-sequence star.

   The heating is

$$\begin{aligned} H &= F\pi R_g^2 \\ F &= 4\pi R_*^2 \sigma T_{\mathrm{eff}}^4 / 4\pi D^2 \\ &= \sigma T_{\mathrm{eff}}^4 \left(\frac{R_*}{D}\right)^2 \end{aligned}$$

The cooling is

$$C = 4\pi R_g^2 \sigma T_g^4$$

Thus in equilibrium

$$\begin{aligned} T_g &= \frac{1}{\sqrt{2}} T_{\text{eff}} \left(\frac{R_*}{D}\right)^{1/2} \\ &= \frac{7200}{1.414} \left(\frac{1.6 \times 7 \times 10^{10}}{10^2 \times 1.5 \times 10^{13}}\right)^{1/2} \\ &= 44\,\mathrm{K} \end{aligned}$$

3. Carroll & Ostlie: problem 12.7

Calculate the Jeans length for the giant molecular cloud in Example 12.2

$$R_J = \left(\frac{15kT}{4\pi G \mu m_H \rho_o}\right)^{1/2} .$$

From example 12.2 the giant molecular cloud has:

$$\begin{aligned} \rho_o &= 2 \times 10^{-16}\,\mathrm{g\,cm^{-3}} \\ T_O &= 150\,\mathrm{K} \\ \mu &= 1.3\,\mathrm{neutral\,H\,90\%,\ He\,10\%} \end{aligned}$$

Its Jeans length is then

$$R_J = \left(\frac{15 \times 1.38 \times 10^{-16} \times 150}{4 \times 3.14 \times 1.3 \times 6.67 \times 10^{-8} \times 1.67 \times 10^{-24} \times 2 \times 10^{-16}}\right)^{1/2} = 2.7 \times 10^{16}\,\mathrm{cm} .$$

4. Carroll & Ostlie: problem 12.8

(a) By using the ideal gas law, calculate $|dP/dr| \approx |\Delta P/\Delta r| \sim P_c/R_J$ at the beginning of the collapse of a giant molecular cloud, where P_c is an approximate value for the central pressure of the lcoud. Assume that $P = 0$ at the edge of the molecular cloud and take its mass and radius to be the Jeans values found in Example 12.2 and in Problem 12.7. You should also assume the cloud temperature and density given in Exampler 12.2.

Evaluate P_c/R_J. From example 12.2: n=10^8, T=150, so

$$P_c = nkT = 10^8 \times 1.38 \times 10^{-16} \times 1.5 \times 10^2 = 2.07 \times 10^{-6}\,\mathrm{dynes/cm^2}$$

From problem 12.7 above, the Jeans length is 2.7×10^{16} cm, so

$$|dP/dr| \approx P_c/R_J \approx 2.07 \times 10^{-6}/2.7 \times 10^{16} = 7.67 \times 10^{-23}\,\mathrm{dynes/cm^3}$$

(b) Show that, give the accuracy of our crude esitmates, $|dP/dr|$ found in part (a) is comparable to (i.e. within an order of magnitude of) $GM_r\rho/r^2$, as required for quasi-hydrostatic equilibrium.

$$
\begin{aligned}
GM(r)\rho/r^2 &\approx GM <\rho> /R^2 \approx G <\rho>^2 R_J \\
&\approx 6.67 \times 10^{-8}(2 \times 10^{-16})^2 \times 2.7 \times 10^{16} \\
&= 7.2 \times 10^{-23}\text{dynes/cm}^3 \qquad (3)
\end{aligned}
$$

[Aside: For a cloud in quasi-static equilibrium, the virial theorem is

$$3(\gamma - 1)U + W = 0$$

and

$$
\begin{aligned}
P &= (\gamma - 1)\frac{U}{V} \\
W &\approx \frac{GM^2}{R}
\end{aligned}
$$

so

$$P \approx \frac{GM^2}{R^4} = \frac{GM\rho}{R}]$$

(c) As long as the collapse remains isothermal, show that the contribution of dP/dr in Eq. (10.6) continues to decrease relative to $GM_r\rho/r^2$, supporting the assumption made in Eq. (12.9) that dP/dr can be neglected one free-fall collapse begins.

$$P_c = \frac{R_g}{\mu}\rho_c T_c$$

If T_c = const. because the cloud contracts isothermally, then

$$P_c \propto \rho_c \propto R^{-3}$$

and the pressure force increases as

$$\frac{dP}{dr} \sim \frac{P_c}{R} \propto R^{-4}$$

The force of gravity, on the other hand, increases as

$$GM(r)\rho/R^2 \propto R^{-5}$$

so the gravitational force increases faster than the pressure force.

5. Can you see photons from the interior of a star? Why or why not? From what level in a star do you see photons? Express your answer in terms of the photon mean free path?

   We can't see into the interior of stars because they are very many photon mfps deep. We can only see one photon mfp into a star, because photons can only escape, on average, if they are closer to space than one mfp, since the mfp is the average distance between collisions or in this case emission and absorption of a photon.