Astronomy 304, Spring 2003

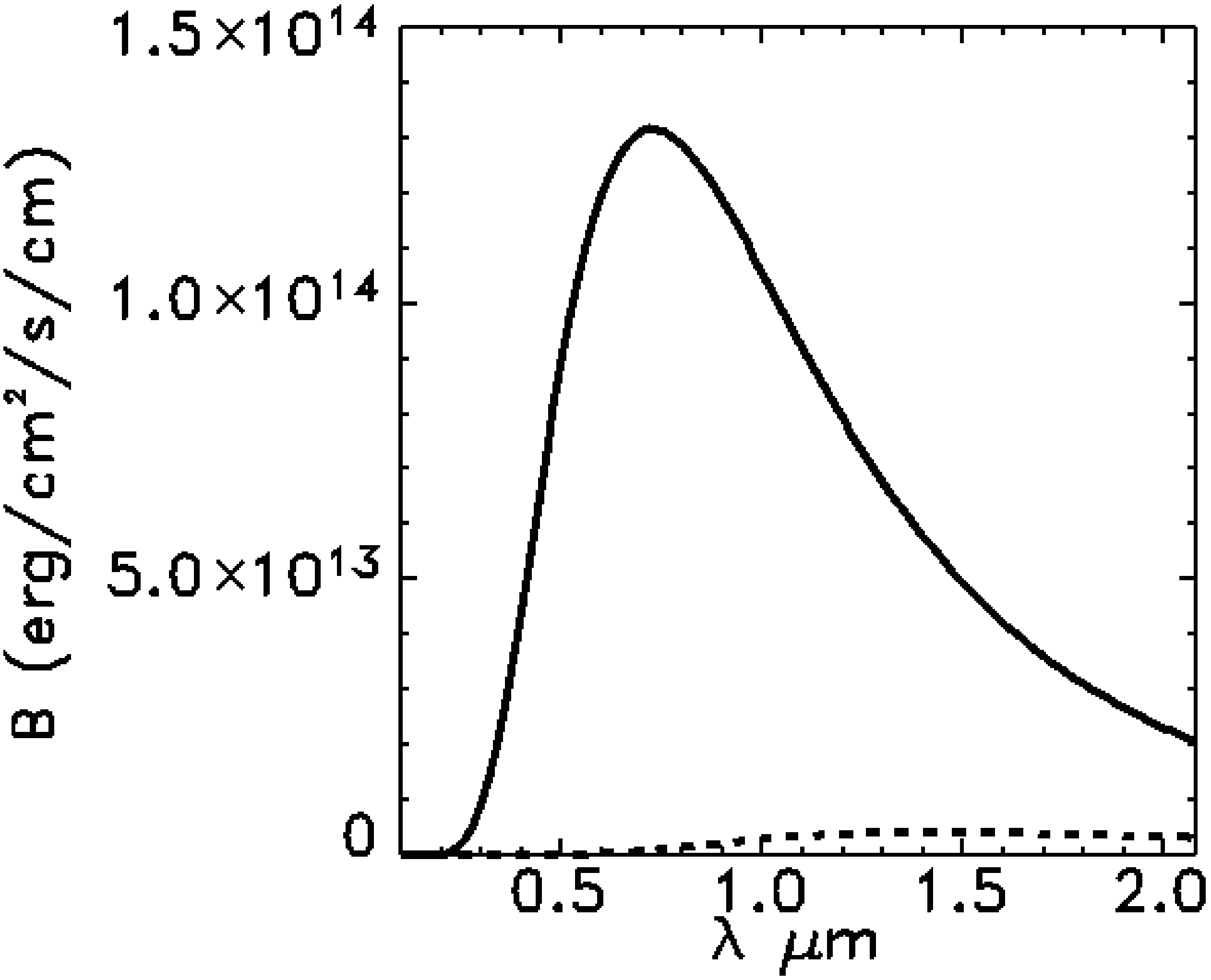
Final Exam

Name: ____________________

Student number: ________________

Show All Your Work! Remember units!

1. (5 points) Below is the black body spectrum for a temperature of 4000 K. On the same graph, draw the approximate black body spectrum for half the temperature.



The peak of the planck function shifts to twice the original wavelength

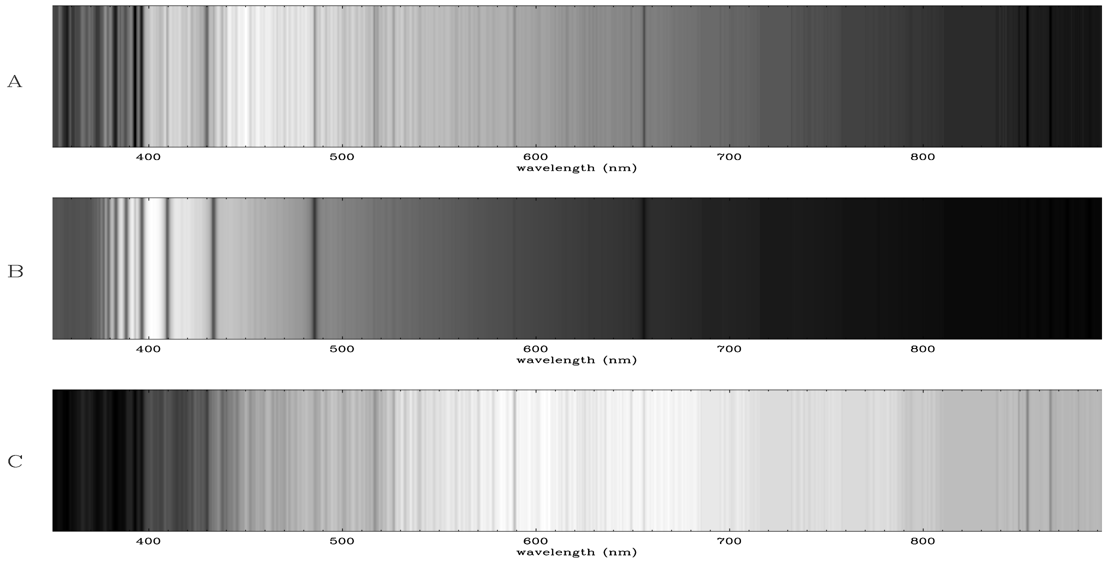
$$\lambda(\mathrm{cm}) \approx 0.3/T[K]$$

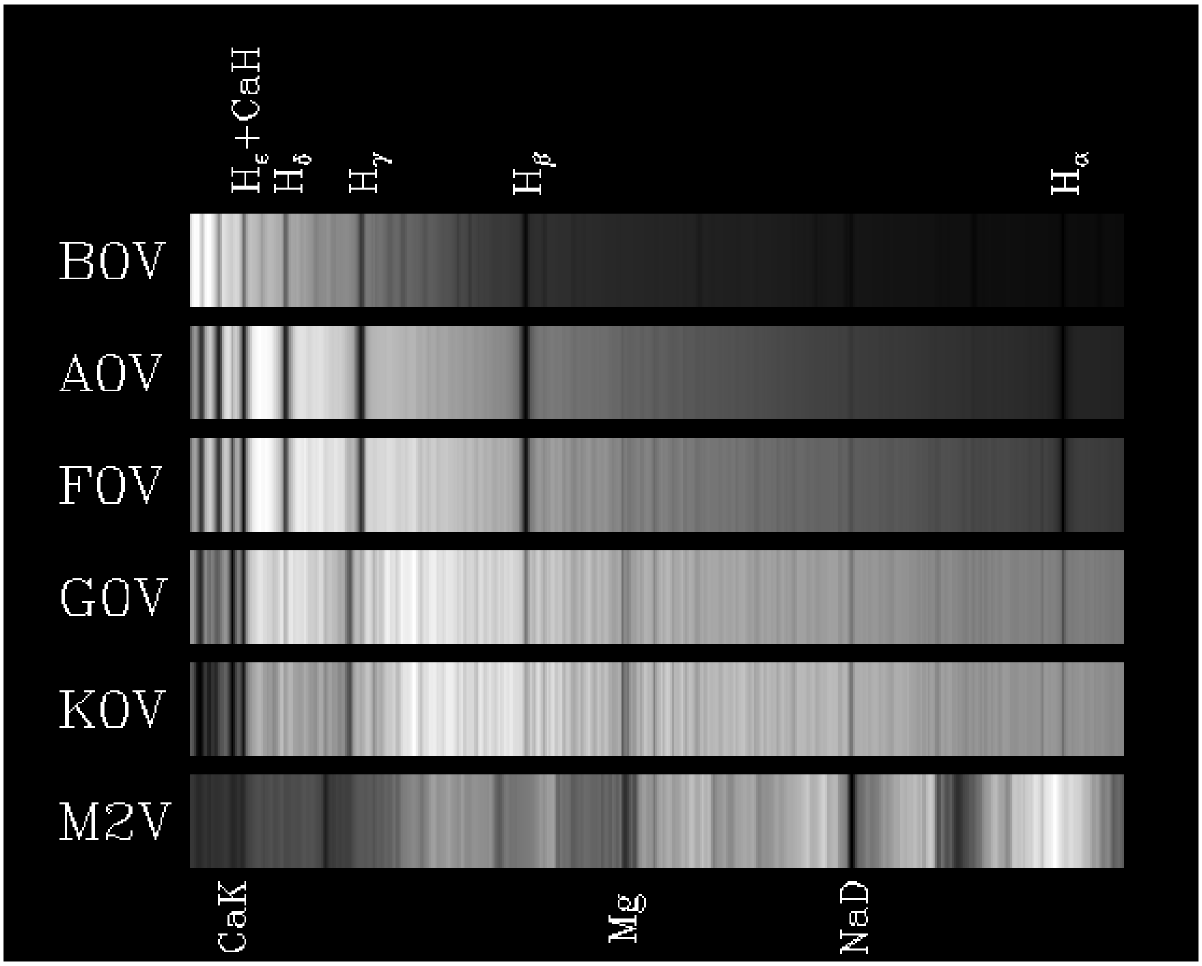
The area under the planck function is proportional to T^4, so it is reduced by a factor of 16.

2. (10 points) First arrange the following three stellar spectra in order of decreasing temperature, then classify their spectral type. Comparison spectra are on the next page.

   Temperature Sequence: B A C (in order of decreasing temperature)
   Spectral type:

   Star A: G2
   Star B: A2
   Star C: K5


A
B
C
400
500
600
700
800
wavelength (nm)


H_ϵ+CaH
H_δ
H_γ
H_β
H_α
B0V
A0V
F0V
G0V
K0V
M2V
CaK
Mg
NaD

3. (10 points) Derive the expression for the pressure of colliding particles,

$$P = \frac{1}{3} n(mv)v \ .$$

(1) Use it to derive the relation between pressure, density and energy for an extremely relativistic, non-degenerate gas.

Pressure is the force per unit area exerted by colliding particles
P = flux of particles × momentum change per collision
Particle flux = 1/6 n v
Momentum change per collision = 2 m v
So

$$P = \frac{1}{3} n(mv)v \ .$$

(2) For an extremely relativistic, ideal, non-degenerate gas
momentum = γmv, where $\gamma = (1 - v^2/c^2)^{-1/2}$
velocity = c
Hence,

$$P = \frac{1}{3} n\gamma mc^2$$

For an extremely relativistic gas the energy is

$$E = n\gamma mc^2$$

Thus,

$$P = \frac{1}{3} E$$

This is a general result for relativistic particles.

4. (10 points) Derive an approximate expression for the temperature at which the fusion of
$2^{12}C \rightarrow^{24} Mg$, $^{23}Mg + n$, $^{23}Na + p$, $^{20}Ne +^{4} He$, $^{16}O + 2^{4}He$ occurs.

Fusion occurs at approximately the temperature where the nuclei wavelength = the Coulomb barrier thickness

$$\lambda = h/p = h/\sqrt{2}mE = D = \frac{Z_1 Z_2 e^2}{E}$$

So

$$E = 2mZ_1^2 Z_2^2 e^4/h^2 = \frac{3}{2}kT$$

Then

$$\begin{aligned} T &= \frac{4}{3}\frac{m}{k}\frac{Z_1^2 Z_2^2 e^4}{h^2} \\ &= \frac{4}{3}\frac{1.67 \times 10^{-24}}{1.38 \times 10^{-16}} 6^4 \frac{(4.8 \times 10^{-10})^4}{(6.63 \times 10^{-27})^2} \\ &= 2.5 \times 10^{10} K \end{aligned} \tag{1}$$

5. (10 points) Below is the HR diagram for a cluster of stars. Estimate its DISTANCE and AGE. Table 1 is a table of absolute magnitude M_V and color index $B-V$ for the zero age main sequence. Following it is the mass luminosity relation. The lifetime of the Sun is approximately 10 billion years.

There are many ways of solving this problem, which give slightly different answers.

The color of the turnoff from the main-sequence, I read as

$$B - V = 0.2$$

and the apparent magnitude at this color is about

$$m_v \approx 11.5$$

The table then gives the absolute magnitude for the ZAMS at that color as

$$M_V = 2.6$$

The distance is

$$\log_{10}(d/10\,\mathrm{pc}) = (m_v - M_V)/5 = (11.5 - 2.6)/5 = 1.78$$

so

$$d = 603\,\mathrm{pc}$$

Alternatively, the cluster HR diagram has the absolute magnitude on the right hand side, and $M_V = 0$ corresponds to $m_v = 11.8$, so the distance is

$$\log_{10}(d/10\,\mathrm{pc}) = 11.8/5 = 2.36$$

so

$$d = 2291\,\mathrm{pc}$$

To determine the age, we need the mass of stars at the turnoff that are just leaving the main sequence.

From the mass-luminosity relation graph, $M_V = 2.6$ corresponds to

$$\log_{10}(M/M_\odot) \approx 0.25$$

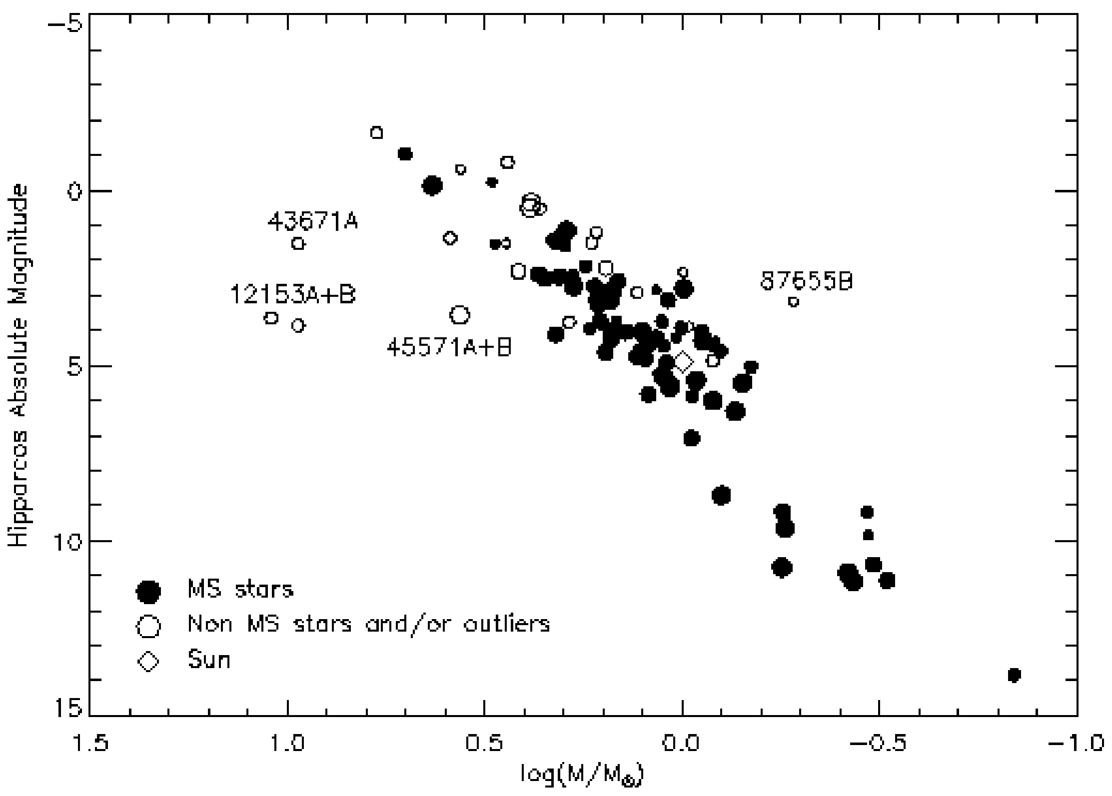
or

$$M = 10^{0.25} M_\odot = 1.78 M_\odot$$

Alternatively, Appendix E gives the mass of a main-sequence star with $B - V = 0.2$ as about 1.8 - 1.9 $M_\odot$.

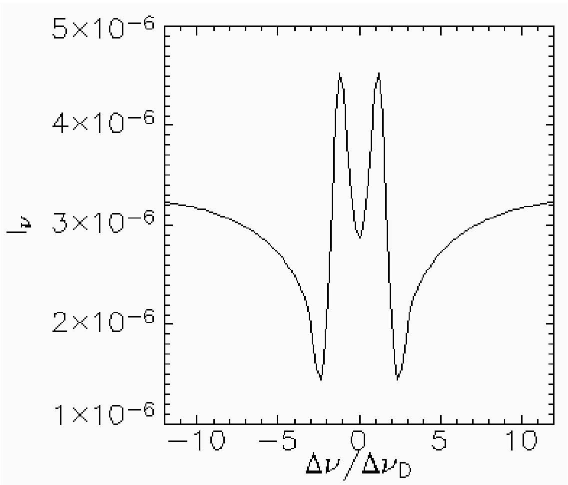
Then table 13.1 gives a main sequence lifetime of $\approx 8 \times 10^8$ years.

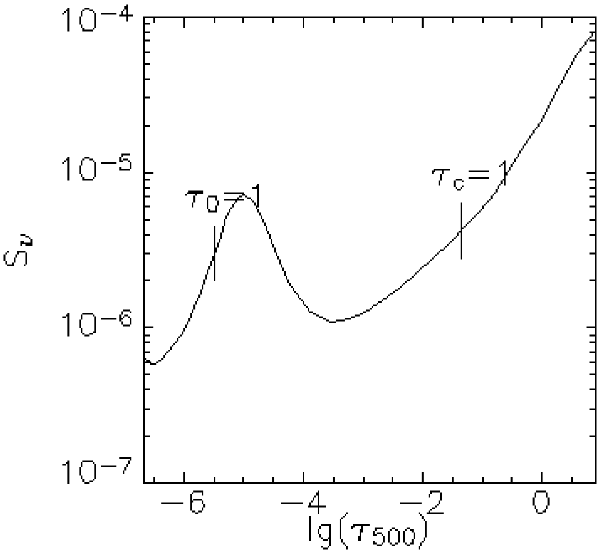
B - V	$\mathbf{M_V}$
-0.3	-3.3
-0.2	-1.0
-0.1	+0.5
0.00	+1.5
+0.1	+2.1
+0.2	+2.6
+0.3	+3.2
+0.4	+3.7
+0.5	+4.3
+0.6	+4.7
+0.7	+5.3
+0.8	+5.8
+0.9	+6.3
+1.0	+6.7
+1.1	+7.2
+1.2	+7.7
+1.3	+8.2
+1.4	+8.7
+1.5	+9.8
+1.6	+11.8
+1.7	+14
+1.8	+16

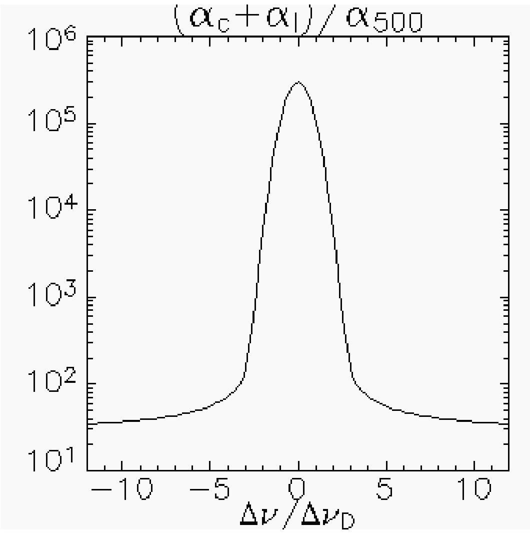


Hipparcos Absolute Magnitude
log(M/M$_\odot$)
43671A
12153A+B
45571A+B
87655B
MS stars
Non MS stars and/or outliers
Sun
−5
0
5
10
15
1.5
1.0
0.5
0.0
−0.5
−1.0

6. (10 points) Sketch the source function, S_ν, as a function of continuum optical depth, τ_{500}, in the blank right hand source function graph that produces the line profile, I_ν, shown in the left hand window. The profile of the opacity relative to the continuum opacity $(\alpha_c + \alpha_\ell)/\alpha_{500}$ is shown below these figures.

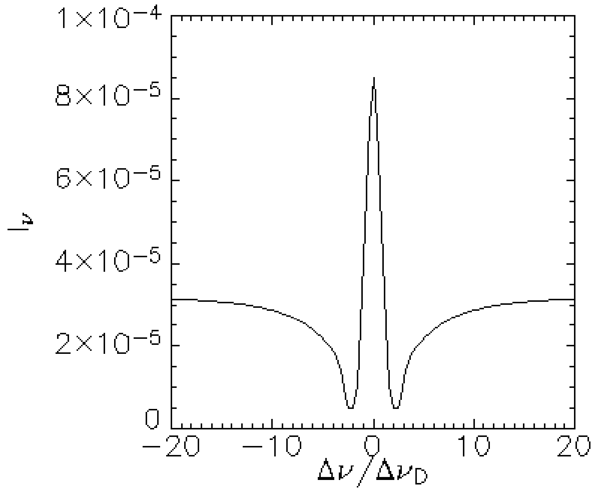


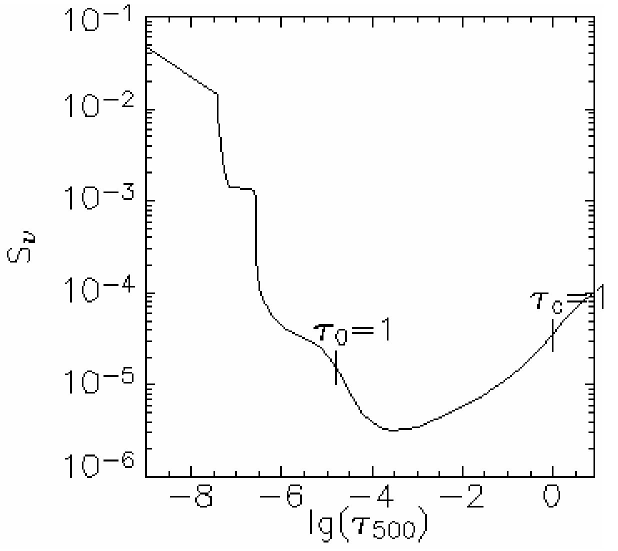




The source function should mimic the line profile. The source function value at line center $\tau_0 = 1$ corresponds to the intensity at $\Delta\nu = 0$. Since the intensity increases at first for increasing $\Delta\nu$, the source function must also increase with increasing optical depth. For larger $\Delta\nu$ the intensity again decreases, so the source function must also decrease for still larger τ. Finally, at large optical depth the source function always increases as the temperature increases going into the star. However, the intensity can not increase above the value of the source function for continuum optical depth = 1.

7. (10 points) Sketch the emergent intensity, I_ν, in the blank left hand graph that would be produced by the source function, S_ν, shown in the right hand graph. The profile of the opacity relative to the continuum opacity $(\alpha_c + \alpha_\ell)/\alpha_{500}$ is as shown in the previous question. On the source function the location of unit optical depth in the line center $\tau_0 = 1$ and in the continuum, $\tau_c = 1$ are shown.

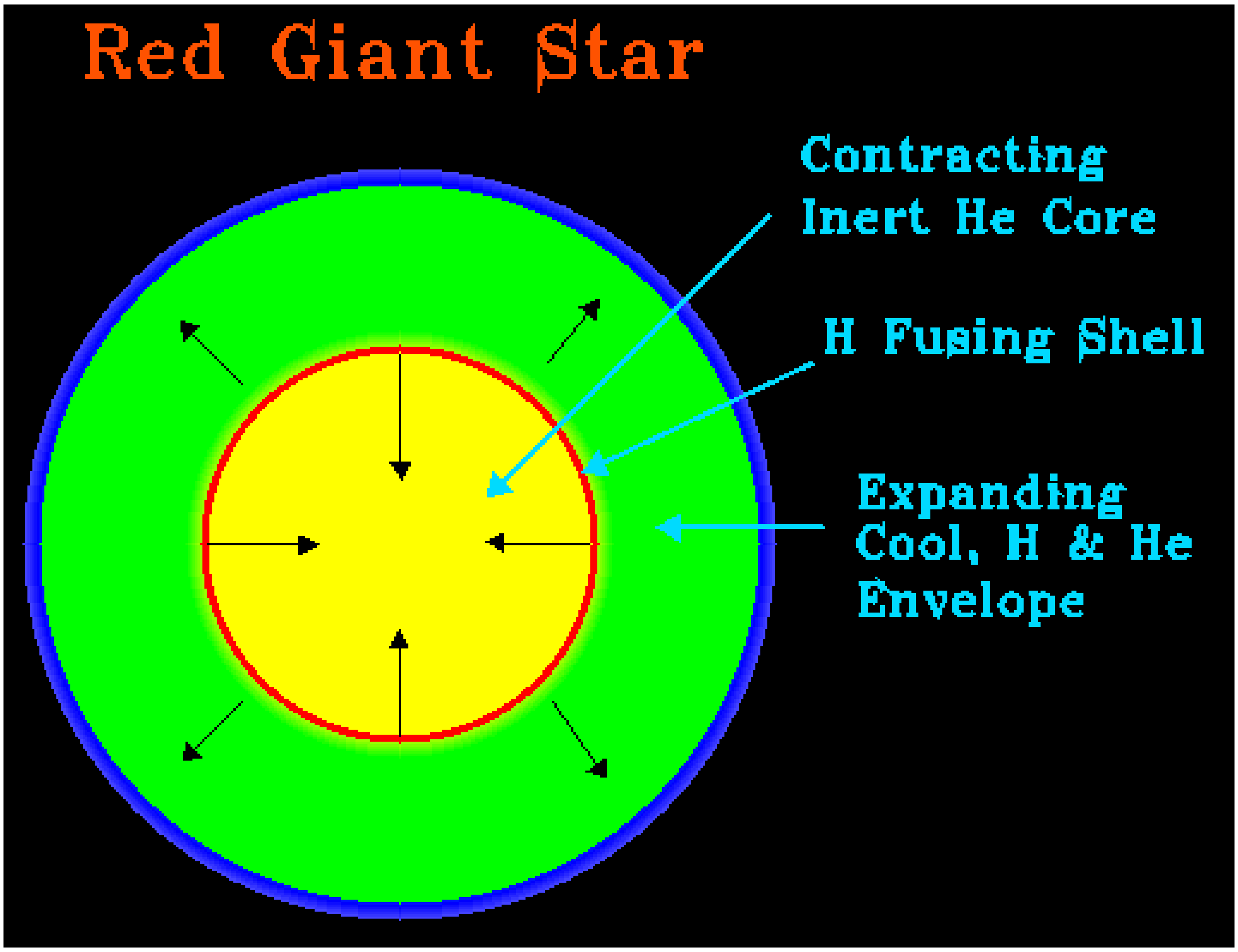




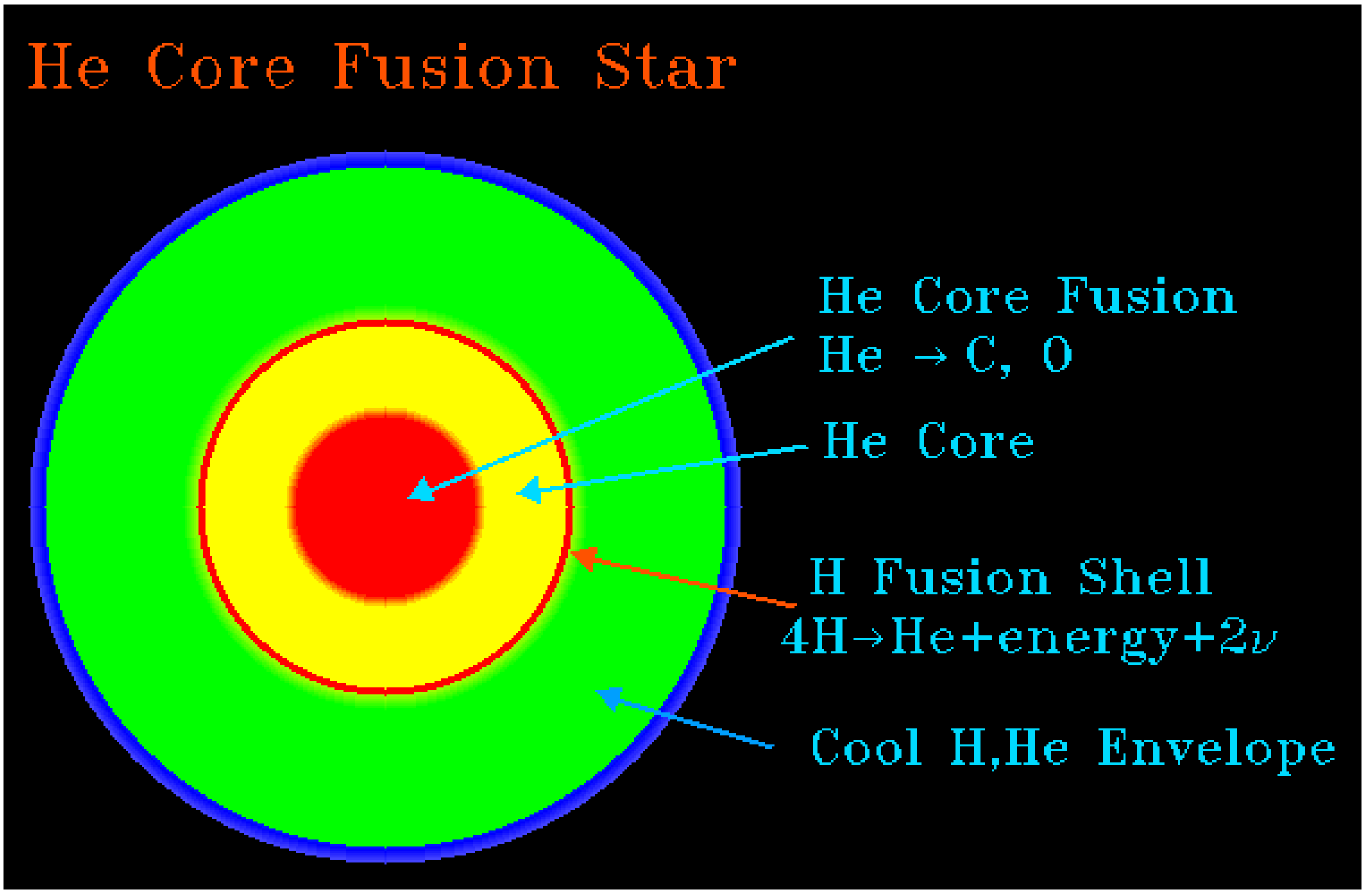
Note, the intensity profile must be symmetric about $\Delta\nu = 0$. Since the source function decreases from line center optical depth $\tau_0 = 1$, the line intensity must have a maximum at $\Delta\nu = 0$ and decrease with increasing $\Delta\nu$. Since the source function then increases toward continuum optical depth $\tau_{500} = 1$, the line intensity must increase again in the wings as it approaches the continuum value.

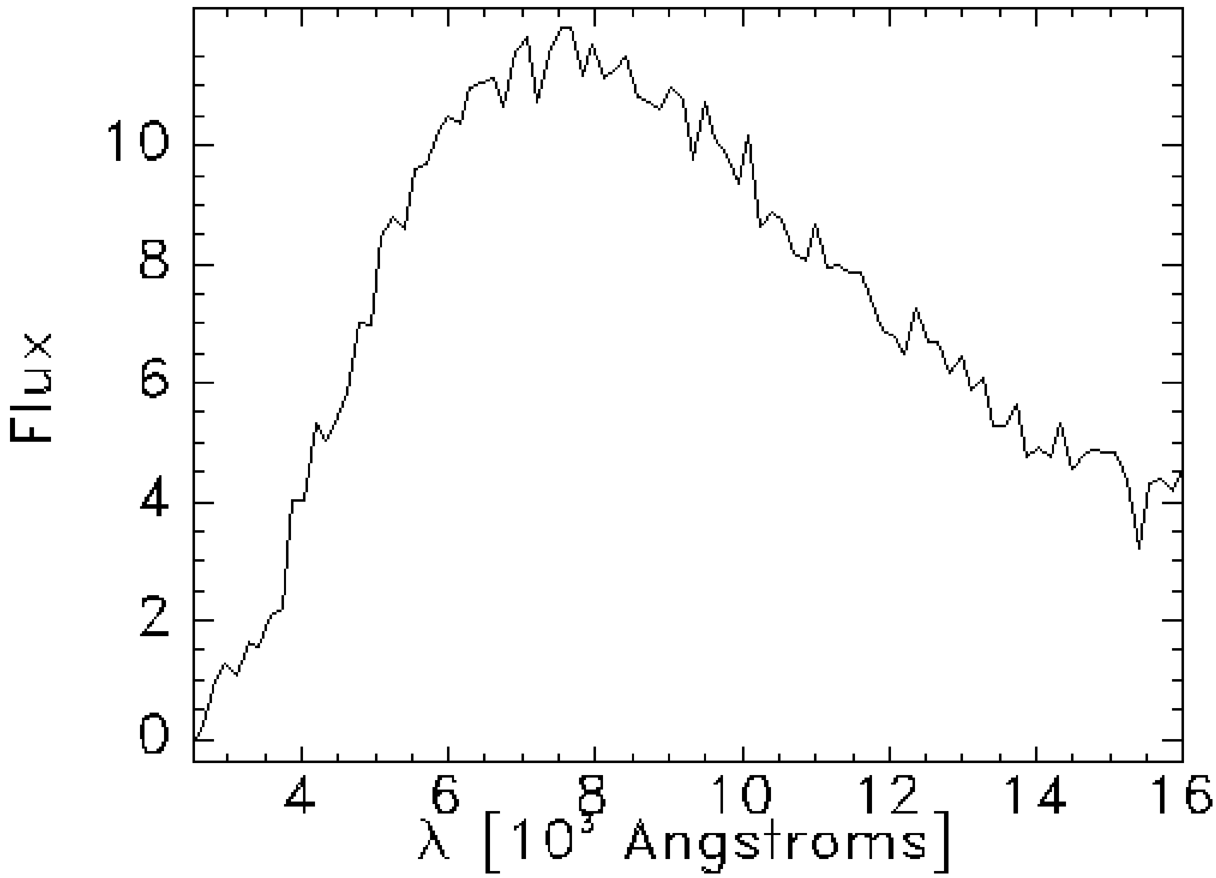
8. (10 points) Sketch the internal structure of the star Aldebaran. [See the HR diagram for bright stars near the end of the exam.] Show the regions of where thermonuclear fusion reactions, if any, are occurring and write each reaction down. Show the regions of different composition and label the dominant species in each region. Indicate if a region is static, expanding or contracting.

Aldebaran is a Red Giant, so it has one of the two following structures:



or


He Core Fusion Star
He Core Fusion
He → C, O
He Core
H Fusion Shell
4H→He+energy+2ν
Cool H,He Envelope



9. (10 points) Calculate the temperature of the surface of Arcturus. The observed radiation from Arcturus is shown on the graph above of Flux vs. Wavelength.

$$\lambda_{\text{peak}} \approx 7500\mathring{A} = 7.5 \times 10^{-5}\text{cm}$$

and

$$T = 0.3/\lambda[\text{cm}] \approx 4000K$$

10. (10 points) Explain why there is a maximum possible mass for white dwarfs.

Increasing mass increases the gravity which needs an increased pressure to balance it and support the star. The pressure in a WD is supplied by degenerate electrons, whose speed depends on their separation according to the uncertainty principle or their density according to the Pauli exclusion principle. Electrons squeezed closer together move faster and exert more pressure. Hence pressure is increased by squeezing the electrons closer together by decreasing the size of the WD and increasing its density. There is a maximum possible pressure because when the electrons are squeezed their energy increases, and when their energy becomes larger than the n-p mass difference the electrons are squeezed onto the protons to make neutrons. This decreases the number of electrons supplying the pressure. More simply, electrons become relativistic and can't increase their speed beyond the velocity of light c.

11. (10 points) As the Sun lives out its main sequence lifetime it slowly gets brighter. Explain this change in the Sun as it ages.

    As the Sun ages, it fuses 4 H $\rightarrow$ He, which reduces the number of particles in the core. As a result the core must contract and heat up to maintain the pressure. Higher temperature make the fusion reactions proceed faster and so the luminosity increases and the outer envelope expands to let the photons out.

12. (10 points) Star X has a parallax of 0.002 seconds of arc and an apparent magnitude m = 20.

    (a) What is the distance of star X?

    (b) What is the absolute magnitude of star X, assuming no interstellar absorption?

(a)

$$d = 1/\pi = 1/(2 \times 10^{-3}) = 500\,\mathrm{pc}$$

(b)

$$m - M = 5\log_{10}\left(\frac{d}{10\mathrm{pc}}\right)$$

so

$$\begin{aligned} M &= m - 5\log_{10}\left(\frac{d}{10\mathrm{pc}}\right) \\ &= 20 - 5\log_{10}(50) \\ &= 11.5 \end{aligned} \tag{2}$$

13. (3 points) A star fusing 4H $\rightarrow$ He + energy in its <u>CORE</u> is
(a) Betelgeuse
(b) Capella
(c) Pollux
(d) <u>Spica</u>
(e) none of these

14. (3 points) When we look at the Sun, we see radiation emitted how many photon mean free paths into the interior?

<u>one mfp</u>

15. (3 points) On the attached HR diagram, DRAW the evolutionary track of the Sun to the start of He fusion.

16. (3 points) In what astronomical body were the atoms you are composed of, other than hydrogen and helium, created?

<u>stars, supernova</u>

17. (3 points) You observe a photon escaping from the surface of the Sun. How long ago was the energy of this photon produced by a fusion reaction in the core of the Sun?
(a) 8 minutes ago
(b) several days ago
(c) when your parents were born.
(d) <u>hundreds of thousands of years ago</u>
(e) 4.5 billion years ago

18. (3 points) Vega and Rigel emit the same flux of radiation from each square meter of their surface, but Rigel is 1000 times more luminous than Vega. This is because Rigel is
(a) cooler than Vega
(b) <u>larger than Vega</u>
(c) closer than Vega
(d) hotter than Vega
(e) none of these

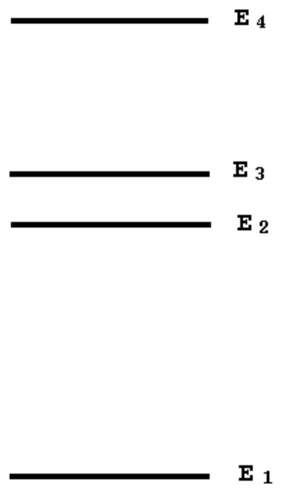
19. (3 points) For the toy atom whose electronic energy states are sketched below, the electron jump that corresponds to ABSORBING the SMALLEST possible energy photon (for this atom) is

From 2 to 3

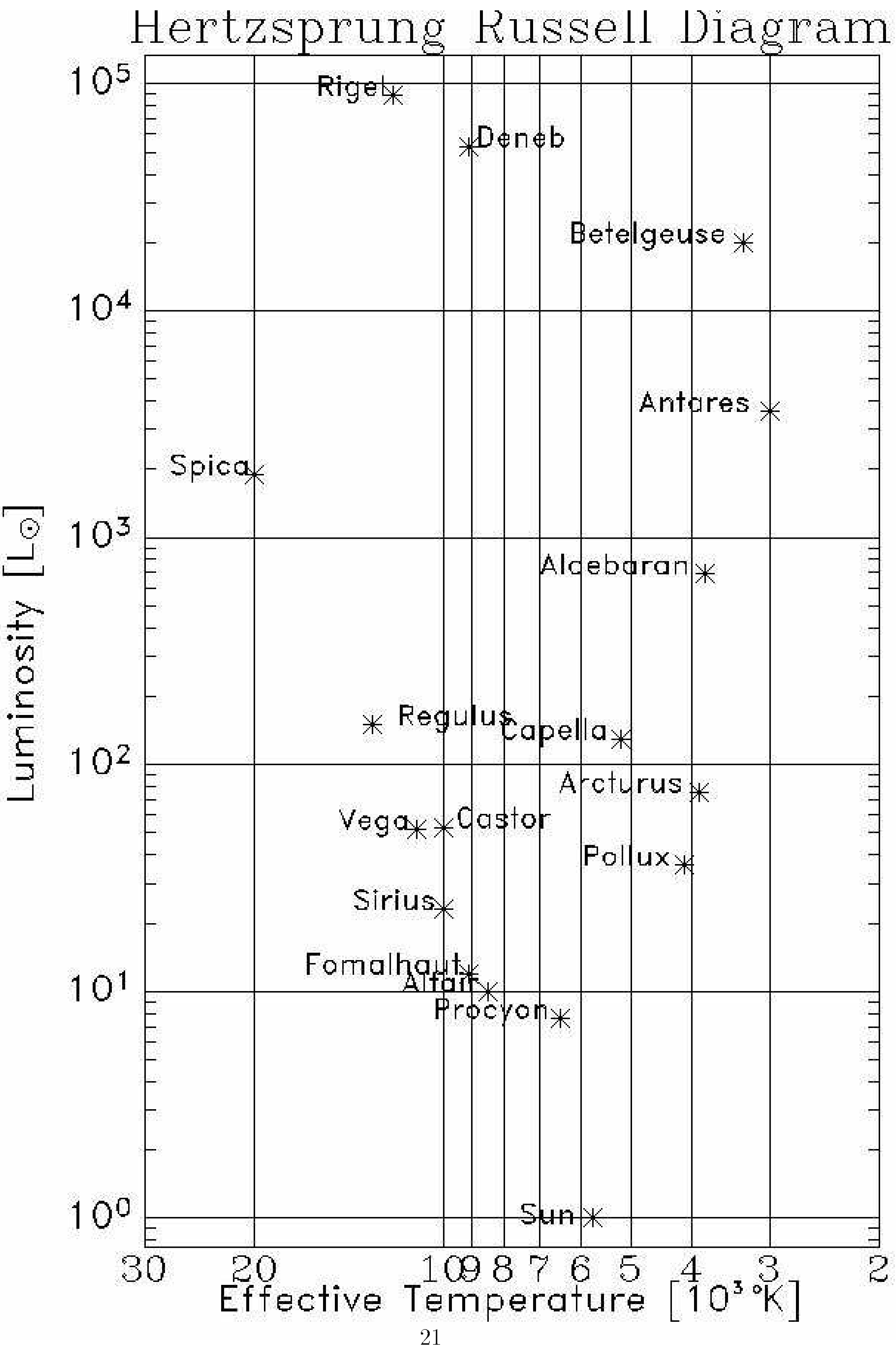
The electron jump that corresponds to EMITTING the LARGEST energy photon possible (for this atom) is

From 4 to 1

Energy level diagram of toy atom:



20. (3 points) Temperature measures how
(a) close together particles are
(b) often parcels collide
(c) fast particles are moving
(d) massive particles are
(e) none of these


Hertzsprung Russell Diagram
Luminosity [L⊙]
10^5
10^4
10^3
10^2
10^1
10^0
Rigel
Deneb
Betelgeuse
Antares
Spica
Alcebaran
Regulus
Capella
Arcturus
Vega
Castor
Pollux
Sirius
Fomalhaut
Altair
Procyon
Sun
30 20 10 9 8 7 6 5 4 3 2
Effective Temperature [10^3 °K]

CONSTANTS

$D_{\rm earth-sun} = AU = 1.5 \times 10^{13}$ cm	$yr = 3.1 \times 10^7$ sec
$M_{\rm earth} = 6 \times 10^{27}$ g	$R_{\rm earth} = 6.4 \times 10^8$ cm
$M_{\rm sun} = 2 \times 10^{33}$ g	$R_{\rm sun} = 7 \times 10^{10}$ cm
$L_{\rm sun} = 4 \times 10^{33}$ erg s^{-1}	
$T_{\rm c\odot} = 1.6 \times 10^7$ K	$\rho_{\rm c\odot} = 162$ g cm^{-3}
$X_{\rm c\odot} = 0.34$	$Y_{\rm c\odot} = 0.64$
$M_{\rm moon} = 7 \times 10^{25}$ g	$R_{\rm moon} = 1.7 \times 10^8$ cm
$D_{\rm earth-moon} = 3.8 \times 10^{10}$ cm	$P_{\rm moon} = 27.3$ days
$c = 3 \times 10^{10}$ cm s^{-1}	$G = 6.7 \times 10^{-8}\,{\rm g^{-1}\,cm^3\,s^{-2}}$
$k = 1.4 \times 10^{-16}\,{\rm erg\,K^{-1}}$	$h = 6.6 \times 10^{-27}\,{\rm erg\,sec}$
$a = 7.6 \times 10^{-15}\,{\rm erg\,cm^{-3}\,K^{-4}}$	$\sigma = 5.7 \times 10^{-5}\,{\rm erg\,cm^{-2}\,s^{-1}\,K^{-4}}$
$m_{\rm H} = 1.7 \times 10^{-24}$ g	$1\,{\rm rad} = 2 \times 10^5$ arcsec
$m_{\rm e} = 9.1 \times 10^{-28}$ g	
$1pc = 2.06 \times 10^5\,{\rm AU} = 3.26\,{\rm LY}$	$1LY = 9.5 \times 10^{18}$ cm
$Q_H = 6 \times 10^{18}\,{\rm erg\,g^{-1}}$	$Q_{He} = 6 \times 10^{17}\,{\rm erg\,g^{-1}}$

FORMULAS

$\theta(\mathrm{rad}) = \mathrm{Arc}/\mathrm{Radius}$

$m_1 - m_2 = -2.5\log_{10}(F_1/F_2)$

$a = F/m$

$V \sim L/t$

$t^2 \simeq L^3/GM$

$M_1 + M_2 = (4\pi^2/G)(a^3/P^2)$

$\quad = Pv^3/2\pi G = v^2R/G$

$KE + PE = E_{\mathrm{total}}$

$3(\gamma - 1)U + PE_{\mathrm{gravity}} = 0$

$KE = 1/2mV^2$

$P \simeq n(mV)V = nkT$

$\Delta x \Delta(mv) \approx h$

$n^{j+1}n_e/n^j = (2Z^{j+1}/Z^j)\left(2\pi m_e kT/h^2\right)^{3/2} e^{-\chi/kt}$

$\sigma(E) = S(E)/E exp\left[-2\pi Z_1 Z_2 e^2/\hbar\,(m/2E)^{1/2}\right]$

$\lambda = cP$

$e_{\mathrm{photon}} = h\nu = hc/\lambda$

$\lambda_{\mathrm{peak}} \simeq hc/kT = 0.3/T(^oK)$ cm

$B_\nu = \frac{2h\nu^3/c^2}{e^{h\nu/kT}-1}$

$U_R \simeq \left(\frac{k^4}{h^3c^3}\right)T^4 = aT^4$

$P = dE/dt = FA = I\Omega A$

$dI_\nu/d\tau_\nu = \epsilon_\nu \ell_\nu - I_\nu$

$J_\nu = \frac{1}{4\pi}\int I_\nu d\Omega$

$L = F \times A$

$F = -\frac{1}{3}n < v > \ell\frac{dQ}{dz}$

$\ell = 1/n\sigma$

$\kappa_{\mathrm{es}} = 0.2(1+X)\,\mathrm{cm^2\,g^{-1}}$

$\kappa_{\mathrm{bf}} = 1.4 \times 10^{25}(1+X)Z\rho T^{-3.5}\,\mathrm{cm^2\,g^{-1}}$

$R_\star = 2 \times 10^5\,\mathrm{AU}/\theta_{\mathrm{arcsec}} = (\mathrm{pc})/\theta_{\mathrm{arcsec}}$

$m - M = 5\log_{10}(d/10pc)$

$F_{\mathrm{gravity}} = GMm/R^2$

$a \sim L/t^2$

$\mathrm{Work} = F\Delta x = -\Delta PE$

$PE_{\mathrm{gravity}} = -GMm/R$

$P \times A = F_G$

$\lambda = h/mv$

$(2\pi m_e k/h^2)^{3/2} = 2.4 \times 10^{16}$

$S(E) \sim 657\,\mathrm{keV\,barns}$

$P = 1/\nu$

$E \simeq kT$

$n_\gamma \simeq \lambda^{-3}$

$B_\lambda = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT}-1}$

$F = U_R c \simeq (kT/hc)^3 kTc = \frac{ac}{4}T^4 = \sigma T^4$

$dE_\nu = I_\nu(\mathbf{r},\mathbf{n},t)dA cos\theta d\Omega dt d\nu$

$d\tau_\nu = ds/\ell_\nu$

$F_\nu = \int I_\nu cos\theta d\Omega$

$\theta_{\mathrm{min}}(\mathrm{radians}) = \lambda/d$

$L_{\mathrm{rms}} = N^{1/2}\ell$

$R = n_I n_T < \sigma v >$

$\kappa_{\mathrm{H^-}} = 10^{-29}\rho^{1/2}T^{8.5}\,\mathrm{cm^2\,g^{-1}}$

$\kappa_{\mathrm{ff}} = 3.7 \times 10^{22}(1+X)(X+Y)\rho T^{-3.5}\,\mathrm{cm^2\,g^{-1}}$