Physics 410 - 2003

Thermal Physics

Problem Set 1

1. Consider a particle of mass m confined in a two-dimensional square box of size L. The potential energy has the form

$$U(x,y) = \begin{cases} 0, & 0 < x, y < L \\ \infty, & \text{otherwise} \end{cases}$$

   Find the 8 lowest energy levels and their multiplicities. (5 pt)

2. Derive the expression for the binomial coefficient $\begin{pmatrix} n \\ m \end{pmatrix}$ in the expansion of $(x+y)^n$ (4 pt) [*Hint*: you can use mathematical induction]. Consider a system of N spins. Show that the number of combinations in which there are $N_\uparrow$ spins pointing upward and $N_\downarrow = N - N_\uparrow$ spins pointing downward is given by $\begin{pmatrix} N \\ N_\uparrow \end{pmatrix}$ (4 pt)

3. Consider a system of N spins, each of which has equal probability to point upward or downward. Find the probability that $N_\uparrow$ of the spins are pointing upward. (4 pt).

4. The probability density for a classical molecule of mass m to have a given velocity v_x along the axis x is

$$p(v_x) = C \exp[-mv_x^2/2k_BT],$$

   where T is temperature and k_B is the Boltzmann constant. Find the constant C (3 pt). Find the probability for a molecule to have the velocity v_x lying between $-(k_BT/m)^{1/2}$ and $4(k_BT/m)^{1/2}$ (5 pt).

You need to have 20 points out of 25 (5 points are extra credit).