Digital Synthesis and Analysis of Periodic Complex Waves

Introduction

In this lab you will synthesize periodic waveforms by addition of sine waves. A waveform is generated by using a computer to add up the Fourier series using the LabView program "Arbitrary Waveform Generator.vi". This program has the capability of generating two separate outputs but, you will use only one. The other will be needed to help you assemble a complex waveform. Once the waveform is assembled to your satisfaction on Channel 0, it will be output via the DAC component on the LabView board. You will observe analog waveforms on the oscilloscope and you will also hear the waveforms through headphones.

The frequency of the output wave is determined by setting the number of waveforms in the construction window which contains 1000 points and by specifying the number of points to read every second.

Wave Generation

The Sine Wave

Use the library feature of the program to set up a sine wave with amplitude 6 V, 2 waveforms in the window and an acquisition rate of 100,000 points/second. What is the frequency of the resulting wave? Verify your conclusion by making a measurement with the oscilloscope. Also listen to the tone on the earphones.

The Triangle

Make an approximate 200 Hz triangle by adding a third harmonic to the sine above with amplitude chosen from the Fourier series expansion for a triangle:

$$x(t) = \frac{8}{\pi^2} \sum_{n=1,3,5...}^{\infty} \frac{1}{n^2} \cos(2\pi nt/T)$$

Add five more harmonics and observe the improvement of the triangle shape.

The Square Wave

Make an approximation to a 200 Hz square wave by using two components, the first and third harmonics, as given in the series below:

$$x(t) = \frac{4}{\pi} \sum_{n=1,3,5...}^{\infty} \frac{1}{n} \sin(2\pi nt/T)$$

Observe the oscillations. Now add eight more terms one at a time. Observe that the oscillations do not go away but tend to be concentrated in the region of the discontinuity. In fact, with a finite number of terms, the oscillations at the discontinuity never go away. This effect is known as the "Gibbs phenomenon".

Other waves

Add up the first 4 terms of the series for the half-wave rectified sine:

$$x(t) = \frac{1}{\pi} + \frac{1}{2}\cos(2\pi t / T) - \frac{2}{\pi}\sum_{n=2,4,6...}^{\infty} (-1)^{\frac{n}{2}} \frac{1}{n^2 - 1}\cos(2\pi nt / T)$$

Waveforms and pitch

Using the built-in library generate a sawtooth wave with frequency 200 Hz. Compare the pitch with the sine oscillator. You should find that the pitch of the saw is close to the pitch of the 200 Hz sine.

The case of the missing fundamental

The Fourier expansion of a sawtooth wave with unity amplitude, starting at x = -1 when t = 0, is given by:

$$x(t) = -\frac{2}{\pi}\sum_{n=1}^{\infty} \frac{1}{n}\sin(2\pi nt / T)$$

· Create a sawtooth wave with a missing fundamental frequency. The simplest way to do this is to construct a sawtooth wave of amplitude A and of 0° phase in one channel, the fundamental with amplitude A*2/π and 0° phase in the other and then add them. The choice of sawtooth expansion phase ensures that we subtract the fundamental.

· Observe that the periodicity of the waveform is still 1/200 Hz i.e. 1/200 s. Explain how this can be.

· Study the pitch of the waveform with the missing fundamental. You should find that it is still 200 Hz. And yet, there is no spectral energy at 200 Hz. This observation played an important role in developing theories of human pitch perception.

Crest factor

The shape of the waveform of a tone depends on the amplitude and the phases of the components. The power spectrum depends only on the amplitudes. Therefore, one can change the waveform shape while leaving the power spectrum the same by changing the phases of the components.

The crest factor is the maximum value of the waveform, divided by the RMS value. In communications practice, there is an advantage to keeping the crest factor low.

Below we consider a waveform having the first three harmonics, all with the same amplitude.

· Show that the largest possible crest factor is obtained by choosing phases so as to add up cosine waves.

- Show that the crest factor for three cosines of equal amplitude is $\sqrt{6}$.
- Generate this wave. Observe it and listen to it.
- The smallest crest factor can be obtained by reversing the sign of the third harmonic. (It is not obvious why should be so, but it is so.) Generate this wave. Compare its shape and its sound with the wave from part (c)

Components above the Nyquist frequency and fold-over distortion

Digital Synthesis creates components with high frequencies that are not desired in the output. The purpose of a reconstruction filter is to remove them. The generation of a sine tone is the simplest illustration. Suppose we want to generate a 5,000-Hz sine, using the 20,000 sample rate. In fact we generate quite a complex spectrum. Not only do we get 5,000 Hz. We get 20,000 ± 5,000. We also get (2 * 20,000) ± 5,000, and so on. Explain why this means that the reconstruction filter ought to cutoff below a frequency, which is half the sample rate. Half the sample rate is a frequency known as the "Nyquist frequency".