Experiment 1

Simple Measurements and Error Estimation

Reading and problems (1 point for each problem):

Homework 2: turn in as part of your preparation for this the first week of this experiment.

Read sections 3.1-3.10 of Taylor (you can skip 3.2). Read again the handout on the important things in Taylor. Look carefully at the definition of independence in the handout.

> Do problems 3.12, 3.16 (see below), 3.18(see below), 3.22, 3.23, 3.28; they (and the analysis and discussion requested below) will help prepare you for the uncertainty calculations needed for this lab.

For 3.16: calculate only for the sums a+c and a+e. Summarize your calculations in a table and comment on the comparative sizes of the uncertainties in each problem. You can use either a calculator or a spreadsheet for your calculations.

For 3.18, only do the calculations for a+b+c and m b/ t. Before you start, predict which errors will be most important in each case. Show the formulas you will use, and arrange your results in a table. Explain whether the uncertainty of a+b-c will be the same, or different from, the uncertainty of a+b+c. From this answer, explain whether the fractional uncertainty of a+b+c will be larger or smaller than the fractional uncertainty of a+b-c, and why. Explain which uncertainty was most important in each case. Why did the importance of δb change from the a+b+c case to the m b/ t case?

For 3.22, start by evaluating the fractional uncertainty of I, and V in percent, then calculate δP/P in percent following the example on p 62. Finally, derive δP from δP/P. For 3.22b, see if you can avoid repeating the entire calculation. For 3.23 (and any calculations in the lab involving products!) proceed in the same way. A very useful step is to pause and explain if any of the terms are obviously negligible before performing the final calculation of the fractional uncertainty. Also, explain why the fractional uncertainty of R^2 is twice that of R.

Homework 3: Turn in at start of 2nd week of experiment.

Read chapter 4: introduction, sections 4.1, 4.6 can be read together; then read the rest of chapter 4; then read chapter 5 through section 5.2. Do problems 4.2, 4.10, 4.16, and 4.23 .

For problem 4.2, do the calculation laid out in table style initially so you see exactly how it works; the entries in the table you can calculate either by a spreadsheet or with your calculator. But if you use a spreadsheet, you should spot-check results with your calculator! For 4.2 and 4.10, the *checking* calculations requested can be done with either your calculator, or (easier) Excel—but you should really do them (the purpose is to be sure you know how, and the check is that you get the right answer). You can use either explicit formulas or the built-in functions.

Experiment 1 Goals

1. Perform simple measurements as accurately as possible and to estimate uncertainties in these measurements.
2. Understand the strengths and limitations of different length measuring instruments.
3. Practice computing errors for quantities derived from several measurements.
4. Distinguish between systematic and random errors.
5. Learn one method for estimating the random errors.
6. Learn how known random errors make it possible to estimate systematic errors.

Theoretical introduction

The main purpose of this experiment is to introduce you to methods of dealing with the uncertainties of the experiment (for more background, see the Appendix). The basic procedures to correctly estimate the uncertainty in the knowledge of the measured value (*the error of the measurement*) include:

- Estimation of the uncertainty in the values directly measured by, or read from, the measurement device (*directly measured quantities, Taylor, Chapter 1*);
- correct treatment of the random errors and systematic errors of the experiment (*Chapters 4 and 5*);
- calculation of the errors of the quantities which are not measured directly (*the propagation of errors, Chapter 3*);
- rounding off the insignificant digits in the directly measured and calculated quantities (Chapter 2 and the Appendix to this lab).

You will be dealing with all these topics in more detail throughout the semester, but to understand this introduction to the topics, you will have to read much of the first 5 chapters of Taylor to see what is happening.

1. Preliminary discussion (15-30 minutes).

Before the lab, you are asked to read and understand the theoretical material for this lab (Exp1 and *Taylor*). Before the experiment starts, your group needs to decide which information will be relevant to your experiment. Discuss what you will do in the lab and what preliminary knowledge is required for successful completion of each step.

Think hard about organizing your work in an efficient way. What measurements will you need to make? Go through your lab manual with a highlighter, then make checklist of the needed measurements. What tables or spreadsheets will you need to make to organize the calculations data? How should you use *Kgraph* to expedite your calculations and unit conversions (when necessary)? What tables will you need to summarize your analysis and conclusions from the data? This lab will have more explicit reminders about tables than future labs, but you should be thinking about this organization of data taking, data reduction, and summarization in every lab.

Questions for the preliminary discussion

You should write your own answer to each question in your lab book, but leave space to change it after discussion. If you do change your answer, say why.

1.1 Suppose during each of several measurements we find a value, which lies in the same interval of the scale of the measuring device. For example, each time we measure the length to be between 176 and 177 mm, with the length between the ticks on the ruler equal to 1 mm. How do we estimate an uncertainty in the measured length in this case?

1.2 Now suppose we use a much more precise device (say a laser micrometer). Due to a higher precision, this device can resolve the miniscule changes in the length due to the random mechanical deformations of the object, and in each measurement we will see the slight unsystematic changes in the observed length. From these data, how can we find the most likely value of the length? How can we characterize numerically the typical variation in the measurements? Which statistical measures and formulas will we use for these two points? (Hint: see Chapter 4 of Taylor).

1.3 Next, we are going to find out if the two independent measurements from 1 and 2 are consistent with each other. Which procedure will we use? Is there a quantitative method to find out if two measured lengths are in agreement? Is there a quantitative method to estimate how certain our conclusion about the agreement or the disagreement of these measured values is?

1.4 If we are going to use the results of our measurements to calculate some other quantities (e.g., calculate the density of the rod using the measurements of its dimensions and the mass), which formulas will we use to calculate the mean values and the uncertainties of these quantities?

1.5 In our calculation, the calculator (computer) will typically return the results with as many digits as possible, including digits well beyond our measurement uncertainty. What procedure will you follow to systematically get rid of these insignificant digits?

1.6 From the homework problems, it was evident that much labor can be saved by judicious simplification of the uncertainty calculations. If you choose to do so, how will you justify approximations to the uncertainty calculations?

1.7 The measurement of the density of the pipes poses special problems, which only begin with obtaining a mathematically correct formula for the volume: one should also consider which form to put the formula to best minimize the fractional uncertainty in the volume. Discuss which instrument(s) would be best for this measurement. Record your choices and your reasons. As part of the discussion, consider how your answer would change if the diameter of the pipe were much larger, or much smaller; and if the wall was much thicker or thinner.

Part I: Density Measurements

2. Introduction

Your text (Sec. 1.3, p. 5) describes how Archimedes was able to determine the composition of a king's crown by measuring its density. We will attempt to perform a similar exercise, but we shall use copper instead of gold. Copper has a density of 8.91 g/cm^{3}at 20 degrees Celsius (C). We will consider later what to do if the temperature is not exactly 20 degrees C. Your task today is to measure the density, calculate the appropriate uncertainties and decide whether your measurement agrees with the given value.

Please report both % and absolute uncertainties for your final values; using % uncertainty in your uncertainty calculation tables will usually make things much clearer, both for you and the grader.

In our lab, we will use rulers, vernier calipers and micrometers. Discuss in your group the following questions:

2.1 Which of these instruments is the most precise; the least precise? How do you know? In particular, is the caliper more precise than the ruler? Hint: If you don't see how to use the caliper, refer to the Appendix.
2.2 What tables will you need for the measurements below? For the uncertainty calculations?

Now begin your measurements.

Write down in your lab notebook the sample code for the unknowns you are measuring.

Write down the uncertainties of the length measurements with each of these instruments. Use the ruler to measure the three dimensions of the block. Repeat these measurements with the vernier caliper and the micrometer. Assign uncertainties to your measurements. For each instrument, measure the length with the highest precision possible.

Measure the mass of your block and estimate its uncertainty.

For the ruler measurement, compute the volume of the block and its uncertainty. Make two estimates of your uncertainty using alternatively Eqs. 3.18 and 3.19 on p. 61 of the text.

2.3 Which estimate is more appropriate for this calculation? Why?

Pay attention to the units. In the calculation, follow the rules for rounding off the insignificant figures. If the calculation is done correctly, the smallest significant figure in the final mean value will be of the same order as the final uncertainty.

Compute the density and its uncertainty. Use Eq. 3.18 on p. 61 of Taylor.

Calculate the density and its uncertainty using the data obtained with the help of the caliper and micrometer. Compare the three measurements (and their inputs and uncertainties) in a table!

Answer the following questions:

2.4 Is your value consistent with the density of pure copper? Could it, instead, be a copper alloy? (Copper alloys (e.g. bronze and brass) have densities which can range from 7.5 g/cm^3 to 10 g/cm^3.). For more information, see the appendix on Alloy Densities. Justify your conclusion quantitatively. (Hint: compare the discrepancy between the theoretical and experimental values with some other number; see the Taylor handout).

2.5 In the computation of density, what were the greatest sources of uncertainty? Which were the smallest?

2.6 Based on only your measurements made with the ruler, would you arrive to the same answer for the Question 1? Why?

2.7 What systematic errors might we be overlooking? Are any of these big enough to affect your estimate of uncertainty? Consider, one at a time, the temperature dependence of the density of metal (see below), irregularities in the shape of the block, and any other errors you can think of. Try to give an estimate of the size of each effect. Based on this estimate, could it be an important source of error in your density calculation?

Thermal Expansion

If a metal is heated, its length increases by an amount ΔL given by:

$$\Delta L = \alpha \cdot L \cdot \Delta T$$

where L is the original length, ΔT is the increase in temperature, and α is the thermal coefficient of linear expansion. For aluminum, $\alpha = 23 \times 10^{-6}$ (per degree C). For steel, $\alpha = 11 \times 10^{-6}$ (per degree C). For copper, $\alpha = 17 \times 10^{-6}$ (per degree C).

3. Density measurements of other objects

Perform a density measurement to determine the material of 3 other unknown objects. Be sure to record what dimension you measured, the instrument used, and the uncertainty. Again, organize in a table, and use uncertainties (and propagation of errors) to present the numeric arguments supporting your conclusions.

In your report, include 3 measured densities of the rectangular block and the densities of 3 unknown objects, as well as the relevant uncertainties, and the material or materials you deduce them to be made of.

II. Random Uncertainties

In this part of the experiment we ask you to perform one of the simplest of repetitive measurements in order to investigate random and systematic errors. The idea is that we will compare the time interval from the large digital clock at the front of the room, with a more precise instrument (the hand timers). We want to perform a test to see if the time scale of the digital clock is correct or not—that is, whether using the large digital clock would cause systematic errors were we to use it to measure time.

The problem we have is that although the timers have very accurate time scales, we need to use rather imprecise hand-eye coordination to operate them. The systematic error of the timers is small, and guaranteed by the manufacturer. But you will have to measure the random error from your hand-eye coordination, since at the start you don't know it. We will do it by repeating measurements of the same time interval on the large digital clock, and using the variation of the measurements to calculate the random error in a single time measurement. We will then use the fact that by repeatedly measuring time intervals, we can decrease the uncertainty of our estimate of the wall clock counting rate.

4. Time Measurements

At the front of the room is a large digital clock. Assume that it counts at a constant rate, but do not assume that the rate is one count per second. The clock will count from 1 to 20, blank out for some unspecified length of time, and then begin counting again. To measure the clock's count rate, you will be given a timer whose systematic error is less than 0.001 seconds for time intervals of ten seconds or less. However, you will be relying on hand-eye coordination, which means your measurements will have random uncertainties. Your reaction time is unavoidably variable. You may also be systematically underestimating or overestimating the total time.

Observe the clock. Write down whether you believe the clock is counting reasonably close to one count per second. Also, before doing any measurements, guess how much your reaction time would vary from one measurement to the next (this is your initial guess for your random error).

Now choose a counting interval at least 10 counts long, and time 25 of them. Write down how many clock counts you are using as the interval.

One person should time while the other records the data on the data sheet belonging to the person timing. <u>To avoid an unconscious skewing of data, the person timing should not look at the data sheet until all 25 measurements have been recorded</u>. This is essential; otherwise, you will introduce a bias into your measuring procedure! Make a few practice runs before taking data. Avoid starting a timing interval on the first count. Use the first of three counts to develop a tempo with which to synchronize your start. Exchange places with your partner, and time 25 more counting intervals. Thus, each person will have a data sheet with 25 timings recorded on it. Assuming you have prepared well for the lab, you should both be able to analyze your own data. In any case, you should provide your own answers to the questions at the end.

5. Data Analysis

From your data set of 25 measurements compute the average or "mean" time per interval $\overline{T} = \frac{1}{N}\sum_{i=1}^{N} T_i$, where the Σ stands for a summation, T_i represent the I-th single measurement of the time per interval, and N = 25, the number of measurements. Thus, this formula directs you to add up the 25 values of T, and divide by N. (If these formulas don't make sense to you, check Taylor chapter 4 again for the definition of the notation). This gives the average time per interval. Now use this value of $\overline{T}$ (pronounced "T bar") to compute the standard deviation, σ, of the values of T, defined as:

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(T_i - \overline{T}\right)^2}$$

Compare your N=25 value of σ with your previous guess of the variability of your reaction time.

The standard deviation of the mean value you arrived at is related to the standard deviation of the individual values:

$$\sigma_m = \frac{\sigma}{\sqrt{N}}$$

To demonstrate that you understand them, write out the calculations of these three quantities explicitly for the first 3 measurements (N=3). You may then use a calculator or the computers for the full 25. Find which item from Kgraph Functions | Statistics does this calculation for you (how could you check?). You will need the values of σ and σ_m in what follows.

Use Kgraph to calculate σ and σ_m for the first N=3, 5, 10, and 25 (all) of your measurements. First predict how you think that each will vary with N. Then comment whether on your prediction for the change as N gets larger is approximately correct. Why do σ and σ_m behave differently?

The standard deviation of the mean σ_m, is the best estimate of the uncertainty in the measurement of the mean. Note that, unlike the standard deviation, this uncertainty can be made arbitrarily small by taking a sufficiently large number of measurements.

Use Kgraph to make a bin histogram from your twenty-five measurements. The x axis should represent the time T, and the y axis should be the number of measurements which fall in the kth time bin. Adjust settings so the bin width is about w

= 0.4σ (Show the calculation in your notebook. How can you find the bin width Kgraph is using?).

Clearly mark the points of $\bar{T}$ and $\bar{T} \pm \sigma$ for your measurements. These quantities can be shown to be the best estimate of your measurement. The region included in the range $\pm\sigma$ should contain about 68% of your data points if your errors are random and consequently the distribution of your measurements is normal or Gaussian (Taylor, chapter 5). What fraction of your data lies within this range?

5.1 Extra Credit: (Reference: Chapter 5.2 – 5.3)

Draw on your stack histogram an appropriate Gaussian distribution given by a curve (function) of the form

$$g(T) = Ae^{-\frac{1}{2}\left(\frac{T-\bar{T}}{\sigma}\right)^2}$$

To do this, you need values for the constants in g. For $\bar{T}$ and σ, use your best estimates for the mean and standard deviation of your time measurements. Choose A to match your histogram . Hint: what is the value of the function g(T) at $T=\bar{T}$?

Calculate g(T) at five points (Hint: Kgraph and Excel both use *exp* for the exponential function.). Plot them by hand on your histogram plot. Then connect these points with a smooth curve, which should resemble the Gaussian curves in Taylor. With a finite number of measurements such as 25, your histogram may not resemble the expected "bell" shape curve to a great degree.

6. Drawing Conclusions from the timing measurements

6.1 Calculate the time interval per clock tick (How is this related to $\bar{T}$?). Does it appear that the clock tick is 1.0 seconds, as we assumed at the beginning? What is the uncertainty in your estimate?

In other words, is there a significant discrepancy in the time measured by the large clock? Note that we have already made the hypothesis that the large digital clock is running correctly, and now we want to check whether this hypothesis is in agreement with our statistical analysis.

6.2 Now we will make this test quantitative by calculating the number size of discrepancy from expectations in units of the uncertainty of that difference. The discrepancy we want is that between the average time counted, $\bar{T}$, with its expected value T_{exp} ,assuming 1 count per second. So, from the Taylor handout, we will use $D = \bar{T} - T_{exp}$. And we can use σ_m as our estimate of δD, since that gives our uncertainty in how well we know $\bar{T}$, and there is no uncertainty in our prediction, T_{exp} . Then

$$t = \frac{\left|\overline{T} - T_{\text{exp}}\right|}{\sigma_m}$$

This expression is also the same as Eq. 5.67 on p. 150 of your text. Here, in accord with standard statistical notation, *t* has the meaning of the number of the standard deviations of the mean needed to cover the difference between the mean and expected times. That is, *t is not a time*: it has no units since both the numerator and denominator are in seconds.

6.3 Based on this calculation, is $\overline{T}$ compatible with T_{exp} ? That is, is the discrepancy (statistically) significant?

6.4 Why is σ_m used instead of σ in the formula above?

6.5 Suppose you were going to use the timer at the front of the room for timing measurements. Would doing so cause a systematic error in measurements of time intervals?

6.6 Suppose the difference *was* statistically significant and you needed to use the large digital clock as a timer. Based on your data, can you correct for its systematic error? Explain how you would give the best estimate (in seconds) of a time interval of 120 counts? What uncertainty would you report for that time interval?

6.7 Suppose you had recorded only your first 5 measurements. What would you have concluded about the existence of a significant discrepancy? Were the remaining 20 measurements necessary in your opinion? Explain.

Appendices

Appendix 1. Theory of Uncertainties

Contrary to the naïve expectation, the experiments in physics typically involve not only the measurements of various quantitative parameters of nature. In almost all the situations the experimentalist has also to present an argument showing how confident she is about the numeric values obtained. Among other things, this confidence in the validity of the presented numeric data strongly depends on the accuracy of the measurement procedure. As a simple example, it is impractical to measure a mass of a feather using the scale from the truck weigh station, which is hardly sensitive to the weight less than a few pounds.

Another challenge has to be met when the scientist tries to compare the results of her experiment with the data from another experiments, or with the theoretical predictions. Since the conditions of the measurement almost always vary from an experiment to an experiment, and since they are also different from the idealized situation of the theoretical model, the compared values most likely will not match each other exactly. The task is then to figure out how important the factors creating this discrepancy are. If these factors are stable (do not change from measurement to measurement) and well noticeable, they are called systematic errors. If, on the contrary, these factors are more or less random and on average compensate each other, they are called random, or statistic errors. An important fact is that the uncertainty due to the random errors can be reduced by increasing the number of measurements.

Appendix 2. Significant figures

In the calculations, it is always important to distinguish significant figures in the presented numbers from insignificant. The following simple rules will help you in this task.

1. Since the error of the measurement is only an approximate estimate of the uncertainty of the measurement, we do not need to keep more than one or two largest digits in it. The smallest digit in the mean value should be of the same order as the smallest significant digit of the uncertainty. Examples:

Incorrect	Correct
517.436 ± 0.1234	517.4 ± 0.1 or 517.44 ± 0.12
24.3441364 ± 0.002	24.344 ± 0.002
12385 ± 341	12400 ± 300 or 12390 ± 340

2. When adding or subtracting two numbers, the result should have the same number of the significant digits after the decimal point as the least precise summand. Example:

$$517.45 + 34.7824 = 552.23$$

3. When multiplying or dividing, the result should have the same <u>total</u> number of the significant digits as the least precise multiplier. Example:

$$1234.3 \times 23.45 \approx 28940$$

4. For other operations (raising to power, square root, exponent) the rule is similar to the one for the multiplication and division: you should keep as many significant digits in the final result as you had in the input. Example:

$$\sqrt{3.567} \approx 1.889$$

Appendix 3. Commercial Metal and Alloy Densities

Table of density (specific gravity) of alloys.

SG = Specific Gravity; the units are either g cm^{-3} or kg m^{-3}
CE = Coefficient of linear Expansion (μ inch/ inch-°F)

Common name and classification		SG	CE
Aluminum alloy 380	ASTM SC84B	2.7	11.6
Aluminum alloy 3003, rolled	ASTM B221	2.73	12.9
Aluminum alloy 2017, annealed	ASTM B22	2.8	12.7
Hastelloy C		3.94	6.3
Cast gray iron	ASTM A48-48. Class 25	7.2	6.7
Ductile cast iron	ASTM A339, A395	7.2	7.5
Ni-resist cast iron	type 2	7.3	9.6
Malleable iron	ASTM A47	7.32	6.6
Cast 28-7 alloy (IID)	ASTM A297-63T	7.6	9.2
Aluminum bronze			
	ASTM B169, alloy A;ASTM B124, B150	7.8	9.2
Ingot iron	(included for comparison)	7.86	6.8
Plain carbon sheet	AISI-SAE 1020	7.86	6.7
Stainless steel	type 304	8.02	9.6
Beryllium copper 25	ASTM B194	8.25	9.3
Inconel X, annealed		8.25	6.7
Yellow brass (high brass)	ASTM B36, B134, B135	8.47	10.5
Copper	ASTM B152, B124, B133, B1, B2, B3	8.91	9.3
Haynes Stellite alloy 25 (L605)		9.15	7.61

Appendix 4. THE VERNIER CALIPER

A vernier caliper consists of a high quality metal ruler with a special vernier scale attached which allows the ruler to be read with greater precision than would otherwise be possible. The vernier scale provides a means of making measurements of distance (or length) to an accuracy of a tenth of a millimeter or better. Although this section will be devoted to the use of the vernier caliper, it should be noted that vernier scales can be used to make accurate measurements of many different quantities. In the future, you will also use an instrument with a vernier scale to make precise readings of angular displacements.

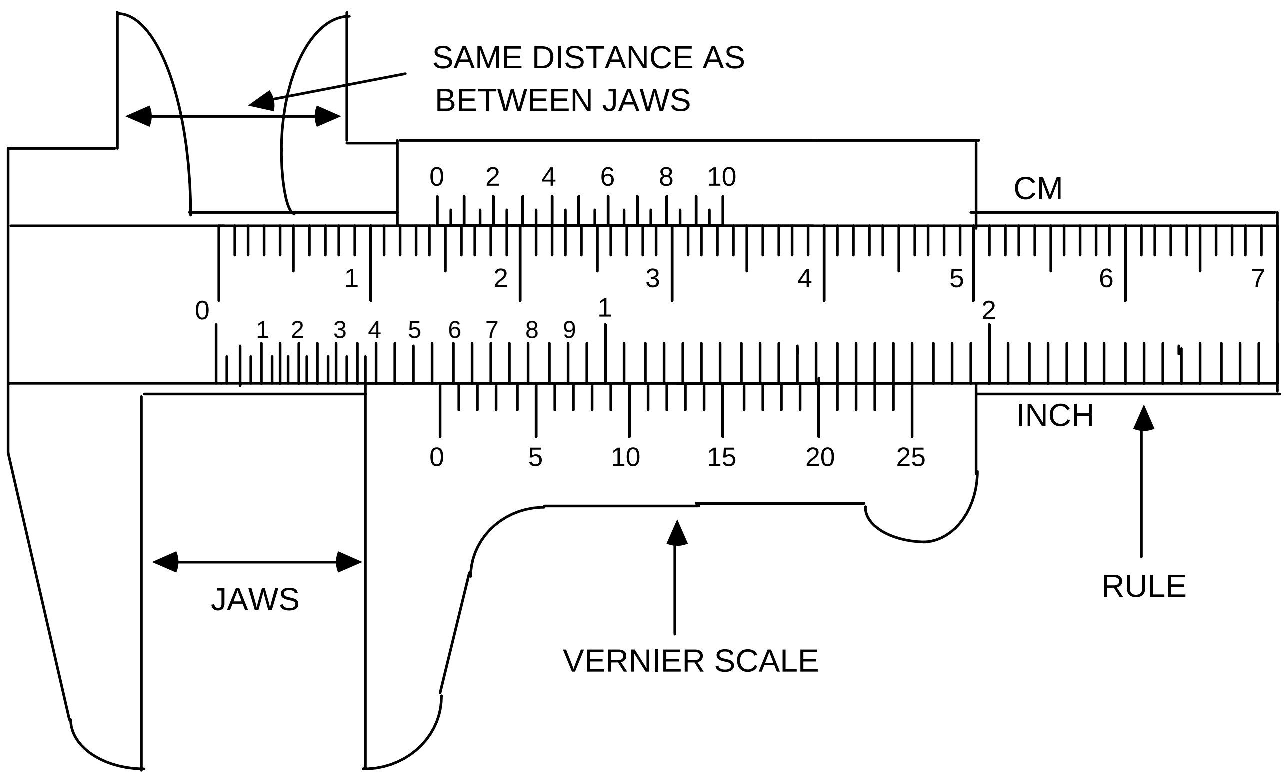


Figure 1: Vernier Caliper

Looking at the vernier caliper in Fig. 1, notice that while the units on the rule portion are similar to those on an ordinary metric ruler, the gradations on the vernier scale are slightly different. The number of vernier gradations is always one more than the number on rule for the same distance. The line on the vernier which is aligned with one on the rule tells us the fraction of the units on the rule. For example, in Fig. 1 the vernier reads 1.440 cm or 0.567 in.

To use the vernier caliper:

(1) Roll the thumb wheel until the jaws are completely closed (touching each other). Now check whether the caliper is reading exactly zero. If not, record the caliper reading, and subtract this number from each measurement you make with the caliper.

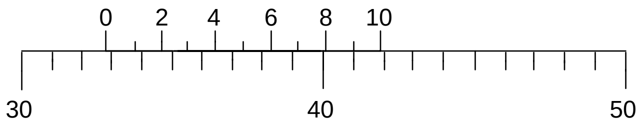
(2) Use either the inside edges of the jaws, or the outside edges of the two prongs at the top of the caliper to make your measurement. Do <u>not</u> use the tips of the prongs. Roll the thumb wheel until these surfaces line up with the end points of the distance you are measuring.

(3) To read the caliper:

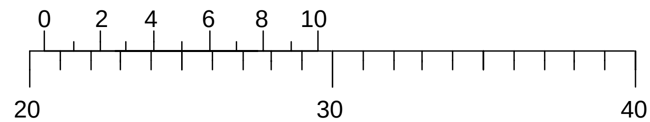
(a) record the numbers which correspond to the last line on the rule which falls before the index line on the vernier scale. On the following page, this would be 32 since the index line falls just after the 32 cm line.

(b) count to the right on the vernier scale until you reach a vernier line which lines up with a line on the rule and record the number of this vernier line as your last digit. on the following page it is the ninth vernier line which is aligned with one on the rule, so the whole distance is 32.9 cm.

The following pages show six vernier scales, similar to that on the vernier caliper, which will allow you to test your ability to read a vernier caliper.



Ans: 32.9



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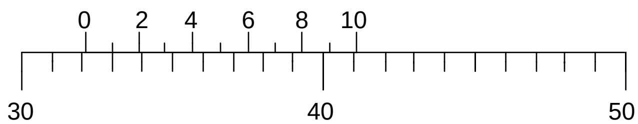
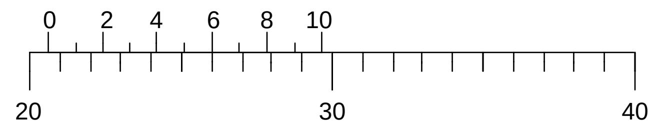


Ans: 32.1



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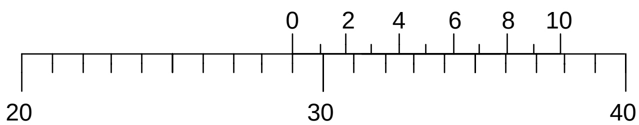

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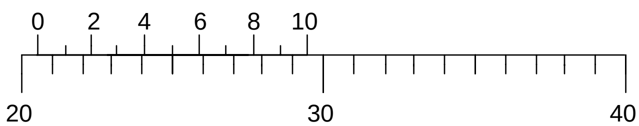

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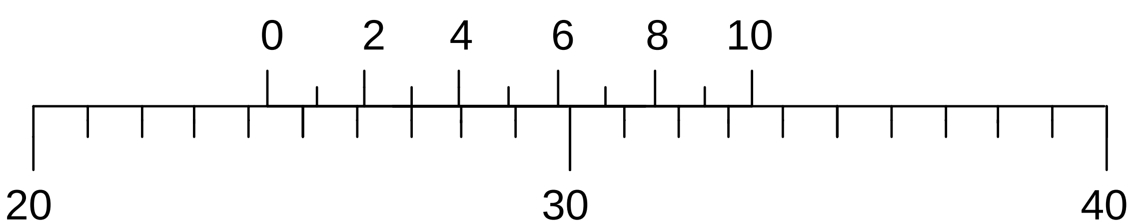

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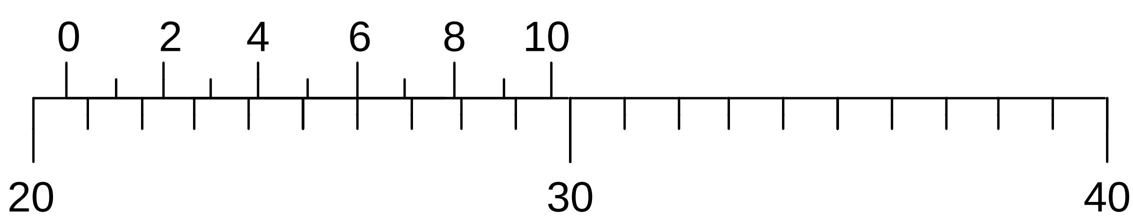

Ans: