# Experiment 2

# F r e e  F a l l

## Suggested Reading for this Lab

Taylor, Section 2.6, and 2 standard deviation rule in Taylor handout. Review Chapters 3 & 4, Read Sections 8.1 - 8.6. You will also need some procedures from Exp0 and the Kgraph Quick Tour, and possibly do a first look at Chapter 12.
Halliday, Resnick, Walker Ch 2 (as needed)

**Homework 4: turn in as part of your preparation for this the first week of this experiment.**

1. Taylor 3.36 (a, b) Review Chapter 3.8; show your work!
2. Taylor 3.37 (for (b), calculate the t value)
3. Taylor 3.38
4. Taylor 8.8 (a)
5. Using Eq. (3) below show that $v_i$, as defined in Eq. (4) is the instantaneous velocity at the middle of the time interval.

**Homework 5: Turn in at start of 2nd week of experiment.** Based on Taylor 8.2:

1) First draw the data by hand on squared paper, draw by hand a best line through the points with a ruler, and measure the slope by finding the rise / run, using a large interval. Why does using a large interval help?
2) Use a spreadsheet and table layout like Table 8.1 and calculate the slope by formulas 8.10-8.12.
3) Now use your calculator or spreadsheet special functions to calculate slope and intercept. Do you think the three values obtained are sufficiently close?
4) Finally, extend your spreadsheet to calculate the residuals from the linear fit, and evaluate Eqs 8.15-8.17. Eq 8.15 is what you use to estimate $\sigma_y$ (the uncertainty in y measurements, assumed to be the same for each individual measurement), when you don't have any other way to estimate it. Eq 8.16-8.17 use the estimate of the $\sigma_y$ to find the uncertainty in the slope and intercept (independent of the method you used to find $\sigma_y$).
5) Now calculate t to test to see whether your fit slope is consistent with your hand-measured slope.

## Goals

1. To study the time dependence of the velocity and position of a body falling freely under the influence of gravity.

2. To understand and apply the “two standard deviations” definition of statistical compatibility.
3. To measure the value of the gravitational constant in East Lansing and compare it to the accepted value g = 9.804 $m/s^2$.
4. To use least squares fitting methods to obtain best values for unknown parameters and their uncertainties.

## Theoretical Introduction

An object falling freely near the surface of the Earth experiences a constant downward acceleration caused by the pull of the Earth’s gravity, g. If we choose the upward direction as positive, the sign of the body’s acceleration is negative, a = -g. We now ask the question: “If the acceleration a(t) is given, how do we find the velocity v(t) and the distance y(t) that the body has traveled in a time t?” To derive the equations of motion we apply integral calculus. Thus, choosing the direction of motion along the y-axis only, we can write

$$a(t) = \frac{dv_y}{dt} = \frac{d^2 y}{dt^2} = -g \; . \tag{1}$$

We integrate this equation with respect to time to get the instantaneous downward velocity v(t):

$$\int_{v_0}^{v} dv = -\int_{0}^{t} g dt$$

$$v(t) = v_0 - gt \tag{2}$$

where $v_0$ is the velocity at time t = 0. Since *v(t) = dy / dt*, we can integrate Eq. (2) once more to find the distance that the object has fallen in a time t:

$$y(t) = y_0 + v_0 t - \frac{1}{2} g t^2 \tag{3}$$

where $y_0$ is the object’s position at time t = 0.

You may recall from your study of linear motion in kinematics, that we could have arrived at the same expressions, if we just substituted a = -g (and y = x) in the equations of linear motion:

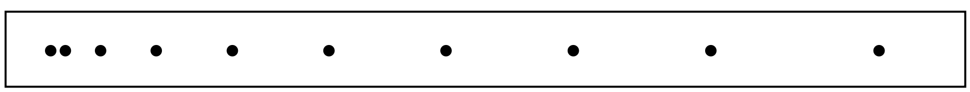
$$v(t) = v_0 + at \; ; \quad x(t) = x_0 + v_0 t + \frac{1}{2} a t^2 \, .$$

## Questions for Preliminary Discussion

1. How should v depend on t? Draw a sketch
2. How should y depend on t? Draw a sketch
3. What kind of systematic errors might influence your experiment?
4. What is the reason for using 1/60 s time intervals?
5. How will you calculate parameters and uncertainties in least squares fits?
6. What tables will you need to summarize and compare your data at the end?

## Experimental Procedure

In Exp2, you will perform measurements of g with the Behr free fall apparatus. A cylinder is dropped and a record of its fall is made. Before measurement, the cylinder is suspended at the top of the stand with an electromagnet. When the electromagnet is turned off, the cylinder begins to fall. Simultaneously, the spark timer starts to send high-voltage pulses between two wires. Your instructor will demonstrate how to operate the Behr free fall apparatus for you. As the cylinder falls, it closes the gap between the two wires and a spark will jump from one wire to the other at the point where the cylinder passes. At the time of each pulse a spark goes through the wires and the cylinder, leaving a mark on the paper tape. The time interval ($\Delta t$) between the two adjacent sparks is about 1/60th of a second. You can measure the actual interval using the period counter. The appearance of the beginning portion of such a tape is indicated below, with time increasing to the right.



You will use two methods for determining the kinematical trajectory of the cylinder. On your paper tape, you should have about 30 burn marks. (1) Take the points in order and measure the differences, $\Delta y_i$, between adjacent points, using the most precise measuring instrument available. (2) Measure the position, $y_i(t_i)$, starting with the first point and making your measurements using a metric tape measure or ruler.

## More Questions for Discussion

7. How should you determine which point to start with and which part to end with? (If you choose a graphical method, you may need to remove some data, or learn Kgraph's data selection tools).
8. Are the uncertainties of successive values of $\Delta y_i$ independent? Why or why not?
9. [Extra Credit] Can you think of another way to plot or analyze the data which would give you more direct access to the acceleration?

## Data and Graphical Analysis

Assign the first usable point as $y = 0$, $t = 0$. Justify your choice! **Assign uncertainties to the measurements, stating in your report how you arrived at these values**. The time associated with the start of each time interval is given by $t_i = i \times \Delta t$, where i is the number of the interval, and $\Delta t$ is the time between measurements.
From the intervals $\Delta y_i$, the average velocity for each interval is calculated as:

$$v_i = \frac{\Delta y_i}{\Delta t}. \tag{4}$$

As pointed out in Section 2.6 of your text, a graph of instantaneous velocity versus time can be used to test the linear dependence of v(t) and the quadratic dependence of y(t) on $\Delta t$. The data that you have taken give you the average velocity in each interval.

Compute $v_i$ for each i and, using your estimated uncertainties in $\Delta y_i$, compute an uncertainty for each velocity $v_i$. Prepare a data sheet in *Kgraph* which includes $\Delta y_i$, error in $\Delta y_i$, $y_i(t)$, error in $y_i(t)$, $t_i = i^* \Delta t$, $v_i$, error in $v_i$, etc. Be sure to label each column correctly with appropriate units. You may also find it useful to be able to create a series; refer to Kgraph Help | Function | Create Series.

Show your method of calculating the uncertainty for $v_i$ in your notebook; this should also be included in your report. Be sure to label the axes and include units for each variable; all plotted points should have error bars representing the uncertainties.

(1) Make a graph of v(t) vs. t by plotting $v_i$ vs. $t_i$ .
(2) Make a second graph of y(t) vs. t by plotting $y_i$ vs. $t_i$.

On both types of plots, label the axes using SI units. Your notebook should have graphs of both experiments with preliminary calculations shown.

Refer to the *Kgraph* quick tour, or Kgraph Help | Search | Error Bars to find out how to put error bars on your graph.
For *Kgraph* to calculate the errors in the curve fit parameters, you *cannot* use the polynomial curve fit routines from the menu. You need instead to create a user-defined function as show in page 14 of the Quick Tour. If you want to try out the example, use the file Sinc.QPC instead of Inhibition.

# Questions to be discussed

These questions should be addressed in your report; please refer to them by number. Answers such as "no" or "yes" are not useful. You should summarize your various results in a table or two, allowing easy comparison among various methods of measurement.

1. Does v depend linearly on t? Does y depend quadratically on t? Which is a better fit? Use Kgraph least squares fit results to address these questions quantitatively. Hints:

(R, the standard deviation of the residuals, eq 8.15, and Chi-squared (Taylor Ch 12) are all related to the quality of the fit).

2. Determine the slope (by hand, using rise over run) from your graph of v(t) vs. t and the value of g in SI units. Show the calculation in your notebook as well as in your report. Is it compatible with the value Kgraph reported?
3. Does your straight line pass within all error bars? (You'd expect 68% if the uncertainties are Gaussian, and the theory is correct.) If not, suggest reasons why this may be. For instance, your estimates of error bars could be faulty or there may be systematic errors.
4. What is the y-axis intercept value as determined from your *v* vs. *t* graph, and what does it mean?
5. Now analyze the results from y vs. t. Calculate $y_0$, $v_0$, g and their uncertainties using *Kgraph*. Do the values for g and its uncertainty agree with your determinations in v vs. t?
6. Are the values of $v_0$ obtained in the two measurements compatible?
7. Which method do you believe is best for measuring g? Why?
8. Are your results reproducible? That is, when you repeat your measurements do you find g values that differ by less than two standard deviations from one another? (If your group obtained only one set of data, compare your data with that of another group.)
9. Using your most reliable results for g, compute the percentage deviation of your result from the accepted value and discuss whether the deviation is statistically significant.
10. If you replaced the cylinder by one with different mass and then performed the experiment again, how would your results differ?
11. What does "terminal velocity" of a falling object mean? (Look it up if you don't know.) What are the implications for your experiment?
12. What kind of systematic errors, if any, might be affecting your experiment?

# Appendix

### Significant Figures in Your Report

When reporting results, for example g, you must always include value and uncertainty in the form $g \pm \delta g$. Refer to the text or the Appendix in Exp1 for guidance on assigning a valid number of significant figures.