

Experiment 4 Elastic and Inelastic Collisions

Reading :Halliday, Resnick, Walker: Chapter 10 as needed

Homework 8: turn in as part of your preparation for this the first week of this experiment.

1. Work through the introduction to *Excel* in the Appendix to this lab, and print the resulting spreadsheet twice: once with the numbers, and once with the formulas.

Assuming random and independent errors, and using

$$v = L/t \quad p = m v \quad K = \frac{p^2}{2m}$$

$$\text{measured: } L = 2 \pm .06 \quad t = 1 \pm .04 \quad m = 2 \pm .0001 \text{ (negligible uncertainty)}$$

In each case: give the algebraic formula, then evaluate the numerical uncertainty.

2. Calculate the uncertainty in v in terms of the uncertainty in L and in t .
3. Calculate the uncertainty of p in terms of the uncertainty of v .
4. Calculate the uncertainty of K in terms of the uncertainty of p .

Homework 9: turn at start of 2nd week of experiment. No homework 9! However, you should work out the uncertainties for the fractional momentum and energy changes for each kind of collision. There is also a calculation (extra credit) in the Questions for Discussion to find an equation to compare with your inelastic collision data.

1. Goals

1. Study momentum and energy conservation in inelastic and elastic collisions
2. Understand use of Excel in analyzing data
3. Carry out uncertainty calculations of moderate complexity

2. Theoretical Introduction

The following experiment explores the conservation of momentum and energy in a closed physical system. As you probably know from the accompanying theoretical course, the conservation of energy and momentum plays an important role in physics and is a consequence of fundamental symmetries of nature.

2.1 Momentum

For a single particle (or a very small physical object), momentum is defined as the product of the mass of the particle and its velocity:

$$\vec{p} = m \vec{v} \tag{1}$$

Momentum is a vector quantity, making its direction a necessary part of the data. To define the momentum in our three-dimensional space completely, one needs to specify its three components in x , y and z direction. The momentum of a system of more than one particle is the vector sum of the individual momenta:

$$\vec{p} = \vec{p}_1 + \vec{p}_2 + \cdots = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \cdots \tag{2}$$

The 2nd Newton's law of mechanics can be written in a form which states that the rate of the change of the system's momentum with time is equal to the sum of the external forces acting on this system:

$$\frac{d\vec{p}}{dt} = \Sigma \vec{F} \quad (3)$$

From here we can immediately see that when the system is closed (which means that the net external force acting on the system is zero), the total momentum of the system is conserved (constant).

2.2 Energy

Another important quantity describing the evolution of the system is its energy. The total energy of a given system is generally the sum of several different forms of energy. Kinetic energy is the form associated with motion, and for a single particle:

$$KE = \frac{mv^2}{2} \quad (4)$$

Here v without the vector symbol stands for the absolute value of the velocity,

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}.$$

In contrast to momentum, kinetic energy is NOT a vector; for a system of more than one particle the total kinetic energy is the algebraic sum of the individual kinetic energies of each particle:

$$KE = KE_1 + KE_2 + \dots \quad (5)$$

Another fundamental law of physics is that the total energy of a system is always conserved. However within a given system, one form of energy may be converted to another (such as potential energy converted to kinetic in the Pendulum experiment). Therefore, kinetic energy alone is often not conserved.

2.3 Collisions

An important area of application of the conservation laws is the study of the collisions of various physical bodies. In many cases, it is hard to assess how exactly the colliding bodies interact with each other. However, in a closed system, the conservation laws often allow one to obtain the information about many important properties of the collision without going into the complicated details of the collision dynamics. In this lab, we will see in practice how the conservation of momentum and total energy relate various parameters (masses, velocities) of the system independently of the nature of the interaction between the colliding bodies.

Assume we have two particles with masses m_1, m_2 and speeds \vec{v}_{1i} and \vec{v}_{2i} which collide, without any external force, resulting in speeds of \vec{v}_{1f} and \vec{v}_{2f} after the collision (i and f stand for *initial* and *final*). Conservation of momentum then states that the total momentum before the collision \vec{P}_i is equal to the total momentum after the collision \vec{P}_f :

$$\vec{P}_i = m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i}, \quad \vec{P}_f = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \quad \text{and} \quad \vec{P}_i = \vec{P}_f \quad (6)$$

2.4 Elastic and inelastic collisions

There are two basic kinds of collisions, elastic and inelastic:

2.4.1 In an elastic collision, two or more bodies come together, collide, and then move apart again with no loss in total kinetic energy. An example would be two identical "superballs", colliding and then rebounding off each other with the same speeds they had before the collision. Given the above example conservation of kinetic energy then implies:

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad \text{or} \quad KE_i = KE_f \quad (7)$$

2.4.2 In an inelastic collision, the bodies collide and (possibly) come apart again, but now some kinetic energy is lost (converted to some other form of energy). An example would be the collision between a baseball and a bat. If the bodies collide and stick together, the collision is called completely inelastic. In this case, all of the kinetic energy relative to the center of mass of the whole system is lost in the collision (converted to other forms).

- In this experiment you will be dealing with
- a completely inelastic collision in which all kinetic energy relative to the center of mass of the system is lost, but momentum is still conserved, and
 - a nearly elastic collision in which both momentum and kinetic energy are conserved to within a few percent.

2.5 Conservation laws for macroscopic bodies

So far we were talking about the system of point-like particles. However, the conservation of the momentum is also valid for macroscopic objects. This is because the motion of any macroscopic object can be decomposed into the motion of its center of mass (which is a point in the space) with a given linear momentum, and a rotation of the object around this center of mass. Then, the conservation of the linear momentum is again valid for the motion of the centers of the mass of the objects. However, some of the linear kinetic energy can be transformed into the rotational energy of the objects, which should be accounted for in a real experiment.

2.6 Kinetic Energy in Inelastic Collisions.

It is possible to calculate the percentage of the kinetic energy lost in a completely inelastic collision; you will find that this percentage depends only on the masses of the carts used in the collision, if one of the carts starts from rest.

After the completely inelastic collision, the carts move together, so that

$$v_{1f} = v_{2f} = v_3$$

The initial KE is given by:

$$KE_i = \frac{m_1 v_{1i}^2}{2} + \frac{m_1 v_{2i}^2}{2}. \quad \text{But, since } v_{2i} = 0$$

$$KE_i = \frac{m_1 v_{1i}^2}{2} \quad (8)$$

The final KE is given by:

$$KE_f = v_3^2 \frac{m_1 + m_2}{2} \quad (9)$$

From conservation of momentum:

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_3 \quad \text{or, since } v_{2i} = 0$$
$$m_1 v_{1i} = (m_1 + m_2) v_3 \quad (10)$$

Since the collision is inelastic, the initial KE is not equal to the final KE . Use equations (8), (9), and (10) to obtain an expression for $D_K (\%) = \frac{(KE_f - KE_i)}{KE_i}$. Hint: define $x = m_1/(m_1+m_2)$ and use it to eliminate v_3 .

3 Experimental setup

We will study the momentum and energy conservation in the following simplified situation:

- we will look on the collision of only 2 objects;
- the motion of these objects will be linear and one-dimensional, so that we can choose the reference frame in such a way that only x -components of the objects' momenta are non-zero; the sign of these components depends on the direction of the motion;
- the experimental apparatus can be set up in a way to almost completely eliminate the net external force on the system.

Our objects will be two carts of different masses, with one initially at rest. The carts move on an air track, which ensures that the motion is one-dimensional and reduces the friction between the carts and the surface. The velocities of the carts can be measured with the help of the photogates, which are described in more details below. Also, it is possible to attach various bumpers (rubber bands, etc) to the carts, which will change the nature of the interaction forces between the carts.

Before the beginning of the measurements, spend at least 15 minutes to figure out which external factors can disturb the motion of the carts on the track, and what you should do to reduce or eliminate these factors. Remember, the successful completion of this lab strongly depends on your ability to create an almost closed system. Make a few practice trials to see if you can achieve an unperturbed one-dimensional collision of the carts. Adjust the level of the air track and the power of the air supply if necessary.

Questions for preliminary discussion

- Draw a diagram of all forces acting on each cart when they collide. Which forces will influence the total P and KE most?
- In our experiment, can we achieve a completely elastic collision? a completely inelastic collision?
- In a collision in a closed system, can some of the total momentum be lost? Some of the kinetic energy?
- In an elastic collision in a closed system, can the total momentum be lost? Can kinetic energy be lost? What about momentum or kinetic energy loss in inelastic collisions?
- What tables should you make in your lab book for writing down your original data?
- How will you calculate the uncertainty for the fractional changes in total momentum and kinetic energy? (Be sure to show formulas).
- Extra Credit: Find a formula for $D_K(\%)$ as suggested in the end of section 2.6 .

Measurements

4. Inelastic collision

4.1 In the first part of the lab we make sure that after the collision the carts stick together and move with some velocity common to both masses. Thus, we have to measure the velocity of cart 1 before the collision and the common velocity of the carts 1 and 2 after the collision. For this purpose, we use two photogates (see Figure 1). Each of them allows measuring the time it takes the cart to go through it. The velocities are calculated by dividing the length of the plate on the cart 1 by the measured time (speed = length/time); for velocity you have to pick a + direction.

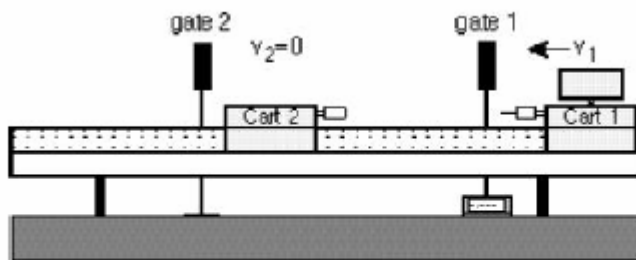


Figure 1: The initial state of the carts before inelastic collision (note that the fin on cart 2 can be removed).

Position cart 2 close to the gate 2 and set the photogate timer to "GATE" mode and the memory switch in "ON" position. In this mode the photogate will display the first time interval measured. Subsequent measurements will not be displayed (only the first one is), but the times are added in the memory. By pushing the "READ" switch you can display the memory contents, which is the sum of all measurements. Example: the initial reading for cart 1 (the time that it took to pass through the gate 1) is 0.300 seconds. Cart 1 collides with cart 2 and they go together through the photogate 2 (Figure 2). Suppose it now takes 0.513 seconds. The display will remain at 0.300, but the memory will contain $.300 + 0.513 = 0.813$ seconds. To find the second time, you have to subtract the first time from the contents of the memory. Try this out by moving the cart through the gate by hand a few times.

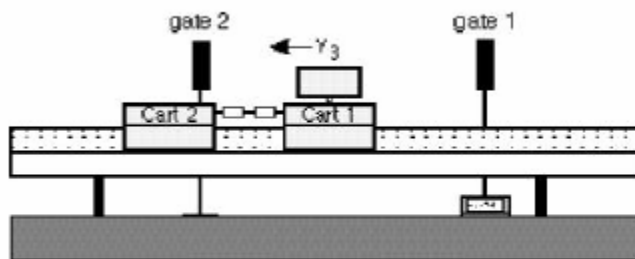


Figure 2: The final state of the carts after inelastic collision.

4.2 Do 3 sets of inelastic collisions consisting of 2 trials each. Vary the masses of the carts by adding the masses (small metal disks) to them. In these measurements, use the needle and putty bumpers and measure the initial and final velocities for the following sets of masses:

Trial 1+2: no mass disks on cart 1, 4 mass disks on cart 2;

Trial 3+4: 2 mass disks on cart 1, 2 mass disks on cart 2;

Trial 5+6: 2 mass disks on cart 1, no mass disks on cart 2.

Extra Credit: In the 2nd set of collisions, use the magnets as bumpers and do the measurements for the same sets of masses.

4.3 In each measurement, you need to find all the initial and final masses and velocities, and use them to calculate the initial and final total momentum and kinetic energy. **Make tables in your lab book to organize your recordings.**

In your directory, you will find 2 preset datasheets, *Coll1* and *Coll2*. You will use *Coll1* for inelastic collisions, and *Coll2* for elastic collisions. Open them and **save into your section's folder**. When you have completed the spreadsheets, print them showing the numbers, and **print again, showing the formulas used!**

5. Elastic collisions

In an almost elastic collision, the main difference from the previous part of the lab is that after the collision the carts move separately. The elastic bumper allows them to bounce off with almost no conversion of the kinetic energy into the other forms of energy.

As before, cart 2 initially stays at rest, and before the collision we have to measure only the velocity of the cart 1 v_{1i} (Figure 3). However, after the collision we have to measure the velocities of both carts, v_{1f} and v_{2f} (Figure 4). Thus, all in all we have to measure three times (t_{1i}, t_{1f}, t_{2f}), while the photogate system allows to simultaneously measure only two of them.

We can get out of this situation if, after the measurement of the initial time t_{1i} , but before the collision, we reset the timer. You have to make several practice trials to quickly remember and reset the contents of the timer before the carts collide. Then, we can again see the contents of the timer display and the memory to find t_{1f} and t_{2f} .

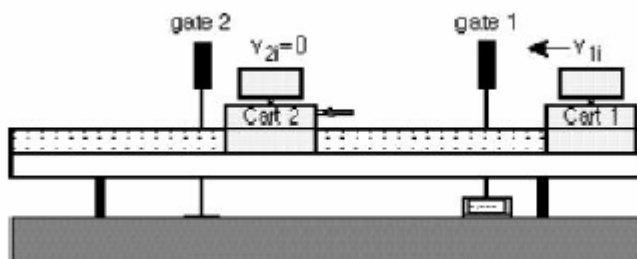


Figure 3: The initial state of the carts before elastic collision.

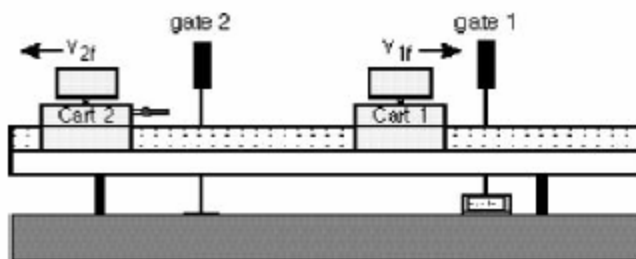


Figure 4: The final state of the carts after elastic collision.

The plates on two carts have the same length (check this!). The experiment will be done with cart 2 initially at rest. You will do 6 trials with the following choices of m_1 and m_2 :

Trial 1+2: no mass disks on cart 1, 4 mass disks on cart 2

Trial 3+4: 2 mass disk on cart 1, 2 mass disks on cart 2

Trail 5+6: 4 mass disks on cart 1 no mass disk on cart 2

Extra Credit: In the 2nd set of collisions, use the magnets as bumpers and do the measurements for the same sets of masses.

Pay attention to the **sign of the velocities**, which depends on the direction of motion of the cart. If the percentage change in momentum or kinetic energy before and after the collision is greater than 10%, repeat the measurement more carefully (collide slower/faster, etc.). Since the datasheet is set up it is easy to see whether momentum/energy is better conserved with every trial you do.

If one of the times you measure is too long for the timer to measure, substitute a large number for the time in your spreadsheet.

6 Questions to be discussed

Conservation of momentum

- 6.1 Create a graph (one for inelastic and one for elastic collisions) of the relative change of the total momentum $D_p(\%) = (P_f - P_i) / P_i \times 100\%$ versus the number (1-6) of the measurement. Show the uncertainty of $D_p(\%)$ with the help of error bars. Also, show the theoretical prediction for $D_p(\%)$. Extra Credit: Use different symbols to show the data obtained with different types of bumpers.
- 6.2 According to the graphs, does the change in the momentum depend on the type of interaction? In other words, does $D_p(\%)$ show a systematic dependence on the type of the bumper that you have used?
- 6.3 According to the graphs, was the total momentum conserved in the collisions? Use the “two standard deviations” rule to justify your answer. If the momentum was not conserved, discuss the reason why. Also, explain what you have done to improve the situation and whether you succeeded.

Conservation of energy

- 6.4 For the inelastic collision, plot $D_K(\%)$ vs. $x = \frac{m_1}{m_1 + m_2}$. Show the error bars for $D_K(\%)$ and the line you predicted for the theoretical prediction. What are the slope and intercept of a straight line fit to these data? Does a straight line fit these data reasonably? Extra Credit: Discuss the consistency of your data for $D_K(\%)$ with the theory.
- 6.5 Make and analyze a $D_K(\%)$ graph for the nearly elastic collision (as you analyzed $D_p(\%)$ above in steps 1-3). What was the average loss of the kinetic energy in this part of the experiment? Did you achieve the goal of 10% loss of KE in the nearly elastic collisions? Which factors is this loss most sensitive to?

Appendix: INTRODUCTION TO EXCEL

This introduction will show you the basics you need for this lab. Open Excel by double-clicking the icon with the mouse. Then open a new spreadsheet by File | New (and possibly Blank Workbook). Files that end in “.xls,” are Excel spreadsheets. You can copy and paste data from the *Excel* spreadsheet to the *Kgraph* spreadsheet. *Kgraph* can also open spreadsheets directly, and if it’s just data and single-cell column headers, it will even label the columns.

- The *Excel* spreadsheet is made up of rectangles called “cells.”
- Text is entered into a “cell” by clicking on the cell with the mouse, typing the desired text, and pressing Return or Enter.
- To execute a mathematical formula in a cell, you must **always first type “=”** (an equals sign), followed by the desired formula. For example, to compute $5+6$, type “**=5+6**” into an empty cell and press “return.” The desired answer, “11,” appears in the cell where you entered the formula. The actual formula you entered in the cell appears at the top of the screen underneath the menu bar. This works for any cell. If you want to see the formula for a any cell, just “click” on the cell using the mouse, and the formula for that cell appears at the top of the spreadsheet.

Now we will fill a **column A** labeled X with numbers ranging from 0 to 1.4 in steps of 0.2. Do not take the time to do this all by hand! You can have *Excel* do it for you. In cell **A1** enter X and type Enter. In the next cell of the column, **A2**, enter 0, our first value. Then in **A3**, we can give *Excel* a formula we want it to follow in order to fill in the rest of the column. In each cell, we want it to add 0.2 to the value in the cell just above. That will fill in the column in steps of 0.2. Therefore in cell **A3** enter the formula “**=A2+0.2**” and press return. Highlight 7 cells of the column, starting with the cell that has the formula in it, **A3**. Then go to Edit | Fill | Down and then let go of the mouse button. Your column should be filled in increments of 0.2.

In the next **column B** (Label it Sin(x) in **B1**) you are going to compute the sine of X using a formula. In **B2** type “**=Sin(A2)**” and hit Return. By typing **A2** in the parentheses we are telling *Excel* to take the sine of the value (in radians) which it finds in **A2**. Then you can use the same process as above to “fill-down” the entire column. If you do this just as described above, *Excel* knows to start with **A2**, take the sine of each successive value in the A-column, and place the new value in each successive cell of the B-column. Now repeat this same procedure, only this time for cosine X in the **C-column**.

More hints:

If in doubt, use parentheses to make sure things get calculated in the right order. For example, $3 + 5/2$ results in 5.5, but $(3 + 5)/2$ results in 4. It would have been better to use $3 + (5/2)$ in Excel for the first case.

To **check your spreadsheet formulas**, type Ctrl-` (that is, hold down the Ctrl key and type the ` key (which has a ~ above it). If you now print your spreadsheet, you can see all the formulas. Typing Ctrl-` again toggles back to showing the calculated numbers.

Instead of typing a cell number (say O27) you can click on the cell while entering the formula.

You can change formatting with the % and .00 → .0 and .0 → .00 buttons.

Copying formulas:

Edit | Fill | Down is equivalent to:

clicking the cell with the formula	cell will be highlighted, with dark borders
hovering the mouse over the lower right corner	mouse pointer turns from a hollow to a solid cross
dragging the mouse down over more cells	the formula is "copied"

The formula =A17 **copies the contents** of cell A17 into the cell in which you typed the formula.

When the formula is copied, the cell references change automatically! So if your formula in A3 was A2+.2, when it is filled or dragged down to cell A4, it becomes A3 + .2 ; if you are doing the same set of calculations on a whole column of numbers, you can set up the formulas in the first row, then highlight and drag down the formulas in several columns at once.

A similar process allows you to copy formulas sideways (across columns), or copy and pasted into another column. Put 100 in D2. Next, enter =D2 in D3 to copy D2's contents. Now highlight A3:A8, hit copy (Ctrl-c) and then move the mouse to D4 and paste (Ctrl-v). This fills the **D column** with numbers spaced by .2, but starting from 100.

Sometimes this automatic renumbering isn't what you want to do, so you need to be able to stop Excel from adjusting cell numbers. One way to always refer to single cell is to give it a name. Type 10 into cell E1. Highlight E1, then click your mouse in the name box (it will say E1) left of where the formula appears. Type fred there and when you use the name fred in your formulas and it will always refer to that specific cell. Enter in E3 the formula =fred^A3 and copy it down through E4:E9. You should see numbers ranging from 1.58 through 25.12 in the **E column**.

When your spreadsheet is complete, make sure you save! Print your spreadsheet **both** as calculated numbers and as formulas.

Common operations in Excel

you can substitute cells for numbers with these operations

What?	Mathematics	Excel Equation	Value
Addition	$11 + 12$	= 11 + 12	23
Subtraction	$29 - 21$	= 29 - 21	8
Multiplication	30×15	= 30 * 15	450
Division	$44 / 12$	= 44/12	3.66666667
Example	$3 + \frac{4}{5 \times 2} - 3 \times 7$	= 3 + (4/(5*2) - (3*7))	17.6
Power	6^3	= 6^(.3)	1.7117
Exponential	$e^{6.1}$	=exp(6.1)	445.86
Square Root	$\sqrt{3^2 + 4^2}$	= sqrt (3^2 + 4^2)	5
Sine or Cosine	Sin(x) or Cos(x)	=sin(x) or =cos(x)	x = .5 (radians): sin = .479
Pi	Cos (2 π x)	=cos(2*pi()*x)	.809 for x = .1

Operations on groups of cells: use as an example cells A3...A9 containing .2, .4, 1.4

The group can be denoted by A3:A9, or you can highlight a group of cells while entering the formula

Sum	$\sum a_i$	=sum(A3:A9)	5.6
Mean		=average(A3:A9)	0.8
Standard Deviation		=stdev(A3:A9)	.316