Uncertainty for h in Pendulum experiment

$h = L(1-\cos\theta)$

From this we need the uncertainty in h. We can do this in steps, as in Taylor section 3.8.

Start by taking $y = (1-\cos\theta)$ so that $h = L\, y$.

So we need (from Eq 3.18)

$$\delta h/h = \sqrt{ (\delta L/L)^2 + (\delta y/y)^2 }$$

What is $\delta y/y$? From Taylor section 3.6, Eq 3.23, we notice

$$\delta y = |dy/d\theta|\, \delta\theta = \delta\theta\, |\, d(1-\cos\theta)/d\theta\, | = \delta\theta \sin\theta \text{ and}$$

$$\delta y/y = \delta\theta \{ \sin\theta/(1-\cos\theta) \}$$

We can also simply further by consider only small angles (that might not be appropriate, so we'll check). The small angle approximation can be found by keeping only the first terms involving θ in the Taylor series for each of the trig functions:

$$\delta y/y \approx \delta\theta \{ \theta/[1-(1-\theta^2/2)] \} = \delta\theta\, 2/\theta$$

Let's check for our biggest angle, θ = 30 degrees, or .524 radians:

$$\sin\theta/(1-\cos\theta) = 3.73$$
$$2/\theta = 3.82$$

So for one significant figure uncertainty calculations, the simpler form is fine:

$$\boldsymbol{\delta h/h = \sqrt{ (\delta L/L)^2 + (2\,\delta\theta/\theta)^2 }}$$

If you prefer partial derivatives (which are explained in Taylor section 3.11 as a mild variation on regular differentiation), the derivation looks like the following. This method is more general—it can handle cases where the step by step method falters—but is more laborious, in that you may effectively re-derive Eq 3.16, 3.18, 3.9, or 3.26 as part of your calculation . We start from the general equation 3.47:

$$\delta h = \sqrt{ (\delta L\, \partial h/\partial L)^2 + (\delta\theta\, \partial h/\partial\theta)^2 } \quad \text{or}$$
$$\delta h/h = \sqrt{ [\delta L\,(1/h)\,\partial h/\partial L]^2 + [\delta\theta\,(1/h)\,\partial h/\partial\theta]^2 }$$

But $\partial h/\partial L = dh/dL$ (assuming θ is a constant) $= 1-\cos\theta$, and $(1/h)\partial h/\partial L = 1/L$ (of course you knew that from Eq 3.18 since L appears in h only as a simple factor). Similarly

$$\partial h/\partial\theta = \sin\theta, \text{ and } (1/h)\partial h/\partial\theta = \sin\theta/(1-\cos\theta)$$

So that

$$\delta h/h = \sqrt{ [\delta L/L]^2 + [\delta\theta \sin\theta/(1-\cos\theta)]^2 } \text{ as before.}$$