



Figure 1

In the above diagram, the first red input lead has voltage V_a , the second has V_b , and the black lead defines a reference voltage V_r . The input signals can be thought of as the voltage differences between the input lead and the reference voltage:

$$A = V_a - V_r \quad B = V_b - V_r$$

The subtractor part of the differential amplifier forms the difference between the two input leads: $C = A - B = V_a - V_b$. The voltages here refer to voltages at any particular instant of time.

This signal $(A - B)$ is then sent through an amplifier and its amplitude gets increased “k” times. The signal $k \times (A - B)$ becomes the output from the differential amp. The value “k” is the GAIN of the amp.

$$\text{GAIN} = \frac{\text{output}}{\text{input}}$$

The amplifier part of the differential amplifier makes voltages bigger at each moment. It cannot make the input voltage vary more or less quickly. Thus, an ideal amplifier has no effect on the frequency of its input signal $C = A - B$ or on the shape as a function of time. It only changes its size. Of course, the signal $A - B$ can be quite different from A or B by themselves.

It might appear as if the differential amplifier takes the hard way by amplifying $(V_a - V_r) - (V_b - V_r)$ instead of amplifying $(V_a - V_b)$ directly. Any real amplifier actually produces (as you will measure) an output related to not just the difference of its inputs, but also to their sum. So without the reference signal, we would have:

$$\text{Output} = k \times (V_a - V_b) + g \times \left(\frac{V_a + V_b}{2} \right)$$

where g is known as the “common mode” gain, the gain for an input presented in “common” to both inputs of the amplifier: if $V_a = V_b = V_{\text{comm}}$, $V_{\text{comm}} = (V_a + V_b) / 2$. An ideal differential amplifier would amplify *only* the difference, with $g = 0$ and $k=100$ or so. How close it comes to this is measured by the common mode rejection ratio,

$$\text{CMRR} = (1 - g/k) \times 100\%$$

Stray electrical signals from outside sources, called noise, pervades the room where voltages V_a and V_b are measured. The amplitude of this noise is often much greater than the amplitude of the biological signals to be studied. Since this noise is common to any signals measured in the same area, we can make a third measurement, V_r , of just the noise:

$$V_a = A + \text{noise} \quad V_b = B + \text{noise} \quad V_r = \text{noise}$$

Without using the reference signal, the noise cancels in the difference term, but not in the sum term. Our imperfect amplifier would produce:

$$\text{Output} = k \times (A - B) + g \times \left(\frac{A+B}{2} + \text{noise} \right)$$

The differential arrangement uses as inputs $A = (V_a + \text{noise}) - (V_r + \text{noise})$ and $B = (V_b + \text{noise}) - (V_r + \text{noise})$. Now, the noise also cancels in the sum term and we get:

$$\text{Output} = k \times (A - B) + g \times \left(\frac{A+B}{2} \right)$$

Note: if all inputs are equal, $V_a = V_b = V_r$, then $A = B = 0$, and we expect zero output.

Since the noise is much larger than the desired signals A and B , the arrangement which subtracts the reference voltage produces much less contamination of the output signal. The biological signals would be completely obscured if not for this property of the amplifier, which is known as Common-Mode Rejection, because it rejects signals sent in common to both of the input leads.

We will measure the characteristics of the amplifier by arranging input signals of $A=0$, then $B=0$, and finally $A=B$. From the equations above, the output for these conditions should be

$$\text{Out}(A=0) = -B(k - g/2) \approx -kB \text{ if } g \ll k$$

$$\text{Out}(B=0) = A(k + g/2) \approx kA \text{ if } g \ll k$$

$$\text{Out}(A=B) = Ag$$

A second feature of the differential amplifier is that it can be "AC coupled". This means that there is an electronic circuit that passes only input potentials varying fairly rapidly in time. The AC coupling circuitry will not pass constant voltage DC or slowly varying voltage at frequencies below 1/2 cycle per second. AC coupling also removes any DC component from an AC signal. For example, a signal that varies from 5 mV to 15 mV at, say, 10 Hz, is an AC signal with a DC component of 10 mV. (See Figure 2.) The AC coupler will remove the 10mV DC component and pass an AC signal varying from -5 mV to +5 mV to the amplifiers. AC coupling is accomplished by capacitors in the input circuit that act as a large resistance to DC signals. The differential amplifier may also be "DC coupled" with no restriction on the input. It amplifies whatever it sees at

- Plot gain (vertical axis) vs. frequency (horizontal axis) on a semi-logarithmic graph.

Questions

- Is the gain you measured at 200 Hz consistent with your measurement in part 1?
- What happens to the gain of the differential amp at high frequencies?
- Estimate the highest and lowest frequency signal one would encounter when looking at cardiac signals. Is this amplifier adequate? (Hint: To see good detail in the shape of the heartbeat, following all the wiggles in the signal, you would need a frequency range of roughly $100f$, where f = the basic heart frequency.)

PART III. COMMON-MODE REJECTION

To measure the percentage of common-mode rejection, we connect both input leads (red) to the OUTPUT of the signal generator. Attach the black lead to GROUND.

- Set the input to 0.2 volt and 200 Hz. Measure the peak-to-peak voltage of the input by using the voltage cursors (channel A).

INPUT = _____ volts

- Measure the peak-to-peak voltage of the output by using the voltage cursors (channel B).

OUTPUT = _____ mV = _____ volts

- Calculate the common-mode gain.

Common-mode gain = $g = \text{OUTPUT}/\text{INPUT} = \underline{\hspace{2cm}}$

- Calculate the % common-mode rejection ratio (CMRR).

$$\begin{aligned} \% \text{ CMMR} &= (1 - g/k) \times 100 = \left(1 - \frac{\text{common_mode_gain}}{\text{gain_of_one_side}} \right) \times 100 \\ &= \underline{\hspace{2cm}} \% \end{aligned}$$

(Use the gain of either side that you determined from Part I.)