Physics 820 homework XIII, due Mon Nov 29

Reading: Chapter 8.1-3, 8.5, 6

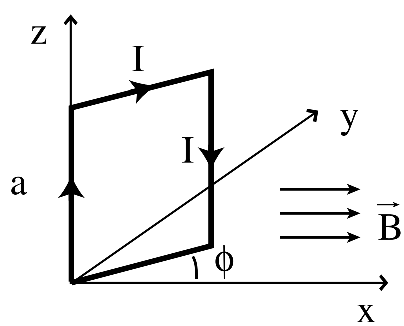
Problems:

1. Goldstein, Problem 8-14.

2. The cartesian coordinate system $\mathcal{O}'$ rotates about an inertial system $\mathcal{O}$ at a constant angular velocity $\vec{\omega}$. (a) Consider a particle of mass m moving in a potential $V(\vec{r})$. How is the particle velocity $\vec{v}$ in $\mathcal{O}$ related to the velocity $\vec{v}'$ perceived in $\mathcal{O}'$? Obtain a Lagrangian for the particle in terms of the coordinates $\vec{r}'$ in $\mathcal{O}'$. (b) Obtain the Hamilton's function and the canonical equations of motion for the particle in $\mathcal{O}'$, relying on the coordinates $\vec{r}'$.

3. Goldstein, Problem 8-23.

4. From the August '03 subject exam:
An ideally conductive square loop can rotate around its side placed on the z-axis, as shown, within a constant uniform magnetic field $\vec{B}$ along the x-axis.



The loop's side length is a, moment of inertia is J and self-inductance is $\mathcal{L}$. As generalized coordinates describing the loop, one can use the angle ϕ of the loop relative to the x-axis and the net charge q that passed around the loop in the clockwise direction. (The current is $I = \dot{q}$.) In terms of those coordinates, the Lagrangian for the loop can be written as

$$L(\phi, \dot{\phi}, q, \dot{q}) = \frac{1}{2} J \dot{\phi}^2 + \frac{1}{2} \mathcal{L} \dot{q}^2 - \dot{q} a^2 B \sin\phi .$$

Here, one can recognize the rotational and inductive energies of the loop and an interaction term of the loop's magnetic moment with the field. (a) From the Lagrangian, find the conserved quantities for the motion of the loop. Can you interpret those quantities? (b) Obtain a Hamiltonian for the loop in terms of the specified coordinates and generalized momenta. (c) Exploit the conservation laws from (a) to obtain an effective potential $U_{eff}(\phi)$ for the motion of the loop in ϕ. Sketch the potential and discuss qualitatively the possible motions in ϕ depending on initial conditions.