Physics 820 homework IV, due Mon Sep 27

Reading: Chapter 2, 3.1-6

Problems:

1. Goldstein, Problem 2-2.

2. A particle is free to move on the surface of a sphere of unit radius under the influence of no forces other than those that constrain the particle to the sphere. It starts at a point q_1 and ends at another point q_2 (without loss of generality, both points may be taken to lie on a meridian of longitude).

(a) Show that there are many physical paths (i.e. $q(t)$ functions) the particle can take in going from q_1 to q_2 in a given time τ. How many? Under what conditions are there uncountable many?

(b) Calculate the action for each of two possible paths the particle can take and show that they are not in general equal. Now construct two new, nonphysical paths close to the original ones, going from q_1 to q_2 in the same time τ. This can be done by adding to each physical path a small distortion of the form $\eta_k(t)$ such that $\eta_k(0) = \eta_k(\tau) = 0$. Show that for each of the nonphysical paths the action is greater than it is on the neighboring physical one, thus demonstrating that each physical path minimizes the action locally.

3. Goldstein, Problem 2-14.

4. A ladder of length L and mass M rests against a smooth wall and slides without friction on the wall and the floor. Assume that the ladder is initially at rest at an angle α_0 with respect to the floor. Use the method of Lagrange undetermined multipliers to find the angle α_1 at which the ladder leaves the wall.

5. By considering the respective Lagrangian, determine the integrals of motion for a particle moving in a uniform field $V = -\vec{F} \cdot \vec{r}$.