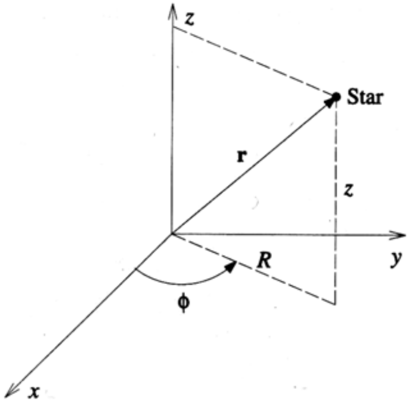


Epicycles… the short form.

For lurid details, see [CO pp.1018-1030]

Ptolemy
Alexandria, 140AD

U = potential energy
$\Phi = U/M$

$$M\frac{d^2\mathbf{r}}{dt^2} = -\nabla U(R,z) = -\frac{\partial U}{\partial R}\hat{\mathbf{e}}_R - \frac{1}{R}\frac{\partial U}{\partial \phi}\hat{\mathbf{e}}_\phi - \frac{\partial U}{\partial z}\hat{\mathbf{e}}_z$$

Circular symmetry ➔ independent of ϕ

$$\frac{d^2\mathbf{r}}{dt^2} = -\frac{\partial \Phi}{\partial R}\hat{\mathbf{e}}_R - \frac{\partial \Phi}{\partial z}\hat{\mathbf{e}}_z \qquad (23.11)$$

$$\frac{d^2\mathbf{r}}{dt^2} = \left(\ddot{R} - R\dot{\phi}^2\right)\hat{\mathbf{e}}_R + \frac{1}{R}\frac{\partial\left(R^2\dot{\phi}\right)}{\partial t}\hat{\mathbf{e}}_\phi + \ddot{z}\hat{\mathbf{e}}_z$$

Separate d^2r/dt^2 into R, ϕ, z components ➔ 3 equations. (23.13-23.15)

$$\ddot{R} - R\dot{\phi}^2 = -\frac{\partial \Phi}{\partial R},$$

$$\frac{1}{R}\frac{\partial\left(R^2\dot{\phi}\right)}{\partial t} = 0,$$

$$\ddot{z} = -\frac{\partial \Phi}{\partial z}.$$

Conservation of specific angular momentum $J_Z = R^2\, d\phi/dt$

Define an effective potential:

$$\Phi_{\text{eff}}(R,z) \equiv \Phi(R,z) + \frac{J_z^2}{2R^2}$$

$$\ddot{R} = -\frac{\partial \Phi_{\text{eff}}}{\partial R}$$

$$\ddot{z} = -\frac{\partial \Phi_{\text{eff}}}{\partial z}$$

Conservation of J_Z ➔ acceleration in ϕ direction when r changes.

Taylor series expansion around position of minimum Φ_{eff} (circular orbit):

$$\Phi_{\text{eff}}(R,z) = \Phi_{\text{eff},m} + \left.\frac{\partial \Phi_{\text{eff}}}{\partial R}\right|_m \rho + \left.\frac{\partial \Phi_{\text{eff}}}{\partial z}\right|_m z + \frac{1}{2}\left.\frac{\partial^2 \Phi_{\text{eff}}}{\partial R\partial z}\right|_m \rho z + \frac{1}{2}\left.\frac{\partial^2 \Phi_{\text{eff}}}{\partial R^2}\right|_m \rho^2 + \frac{1}{2}\left.\frac{\partial^2 \Phi_{\text{eff}}}{\partial z^2}\right|_m z^2 + \cdots$$

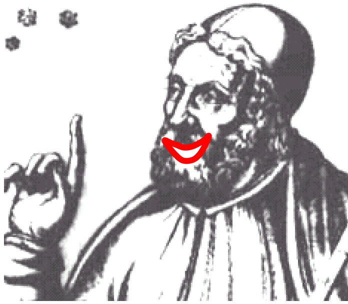
$$\Phi_{\text{eff}}(R,z) \simeq \Phi_{\text{eff},m} + \frac{1}{2}\kappa^2\rho^2 + \frac{1}{2}\nu^2 z^2. \qquad (23.24)$$

$$\ddot{\rho} \simeq -\kappa^2\rho \qquad \kappa^2 \equiv \left.\frac{\partial^2 \Phi_{\text{eff}}}{\partial R^2}\right|_m$$

$$\ddot{z} \simeq -\nu^2 z \qquad \nu^2 \equiv \left.\frac{\partial^2 \Phi_{\text{eff}}}{\partial z^2}\right|_m$$

(23.25-23.26)

Harmonic oscillation in R, ϕ, z about circular orbit (Epicycles)



$$\ddot{\rho} \simeq -\kappa^2 \rho$$

$$\ddot{z} \simeq -\nu^2 z$$

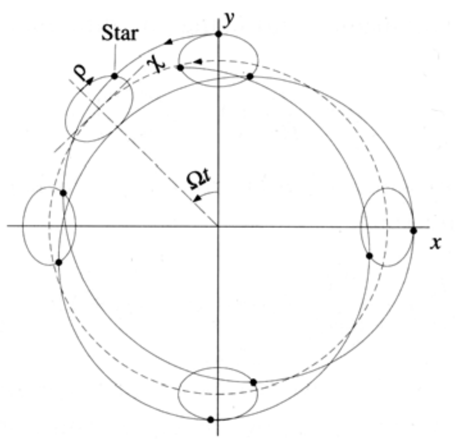
$$\rho(t) = R(t) - R_m = A_R \sin \kappa t$$

$$z(t) = A_z \sin(\nu t + \zeta)$$

$$\phi(t) = \phi_0 + \frac{J_z}{R_m^2} t + \frac{2J_z}{\kappa R_m^3} A_R \cos \kappa t = \phi_0 + \Omega t + \frac{2\Omega}{\kappa R_m} A_R \cos \kappa t$$

R_m = R at min. $\Phi_{\mathbf{eff}}$
Ω = circular ang. vel.

In inertial frame:



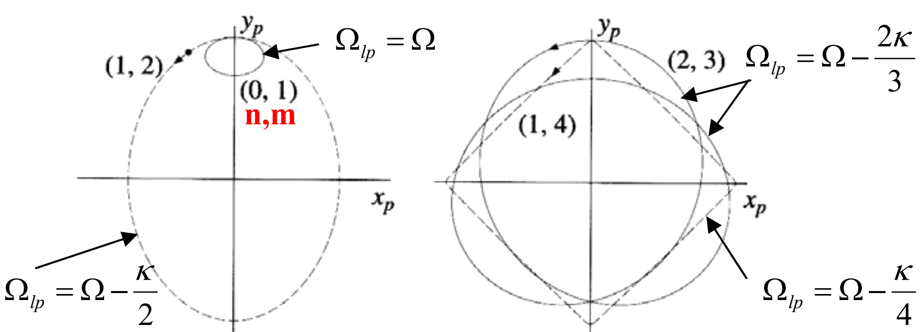
"local pattern speed"

Orbits closed if:

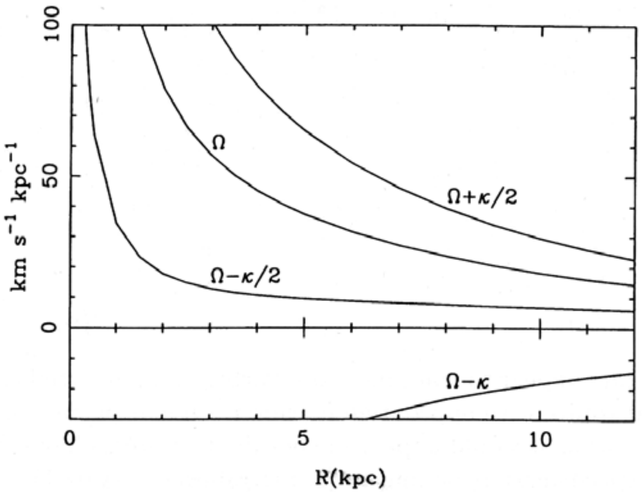
$$m\,(\Omega - \Omega_{lp}) = n\kappa$$

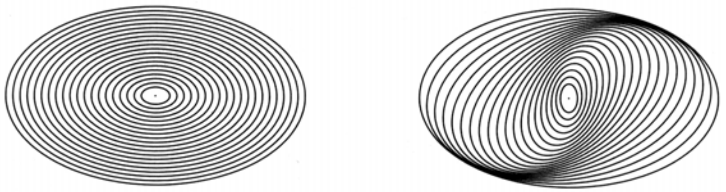
$$\Omega_{\ell p}(R) = \Omega(R) - \frac{n}{m}\kappa(R)$$

Viewed from frame rotating with Ω_{lp}:



Ω_{lp} calculated from rotation curve for Milky Way.





Two ways to line up closed elliptical orbits (as seen from frame rotating with Ω_{lp})

Basic nature of a density wave

From: Toomre, Annual Review of Astronomy & Astrophysics, 1977 Vol. 15, 437.

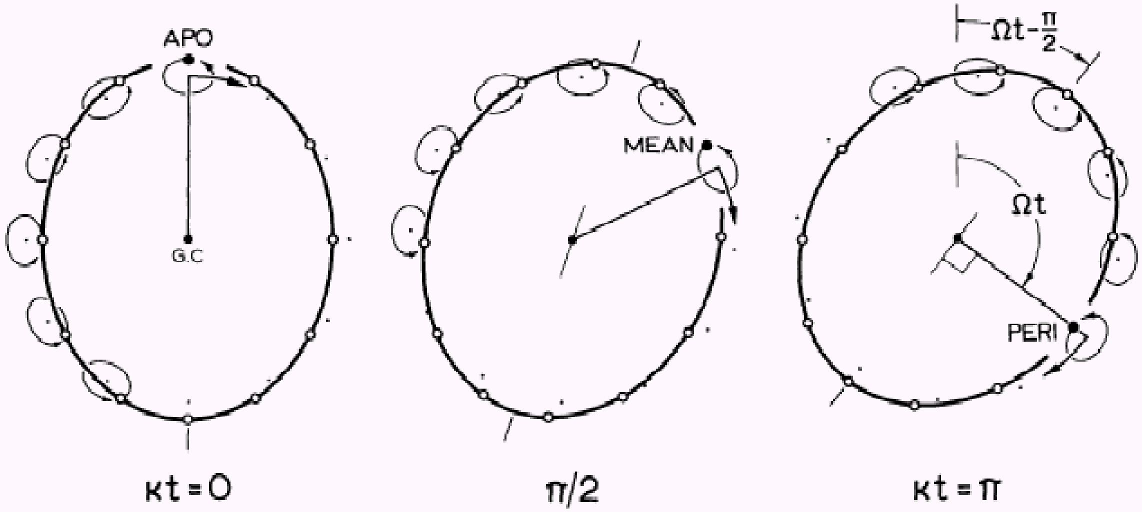
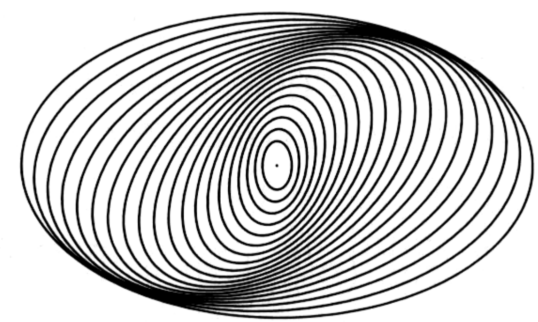


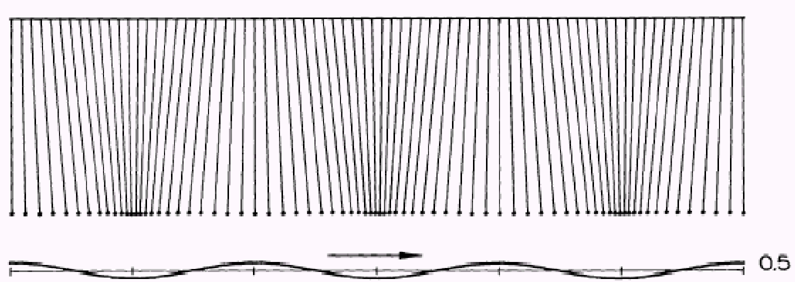
Figure 2 Slow $m = 2$ kinematic wave on a ring of test particles, all revolving clockwise (like the 12 shown) with mean angular speed Ω in strictly similar and nearly circular orbits. The small elliptical "epicycles," traversed counterclockwise in the above sequence of snapshots separated in time by exactly one-quarter of the period $2\pi/\kappa$ of radial travel along each orbit, depict the apparent motions of these particles relative to their mean orbital positions or "guiding centers." Drawn for the case $\kappa = \sqrt{2}\Omega$—or one where the rotation speed $V(r) = r\Omega(r) =$ const at neighboring radii—the diagram emphasizes that the oval *locus* of such independent orbiters advances in longitude considerably more slowly than the particles themselves. That precession rate equals $\Omega - \kappa/2$, as one can verify at once by comparing the last frame with the first.

- At each R_m, stars' positions in epicycles are forced into a specific pattern by gravitational potential of spiral arm.
- Sum of positions of stars at this R_m forms an ellipse rotating at pattern speed.



- Spiral density pattern is sum of many ellipses, all rotating at same pattern speed.

Pendulum example



Lin & Shu's theory

Perturbed form of collisionless Boltzmann equation.
"quite complicated"

Fluid hydrodynamics

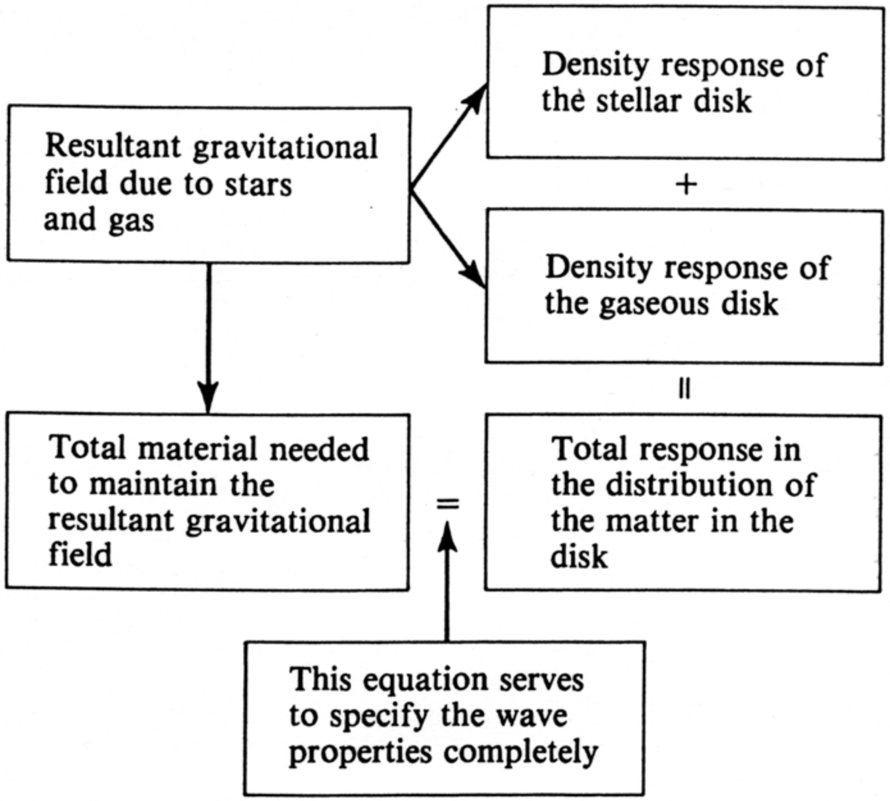


Figure 12.26. The calculational scheme used to calculate the normal modes of oscillation of a disk galaxy. (After C. C. Lin.)

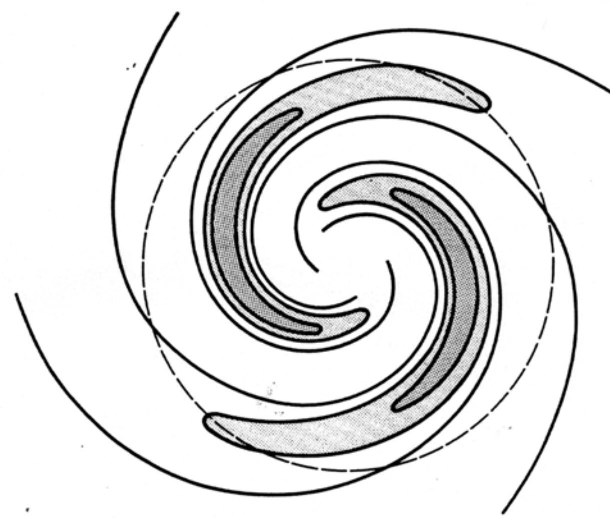
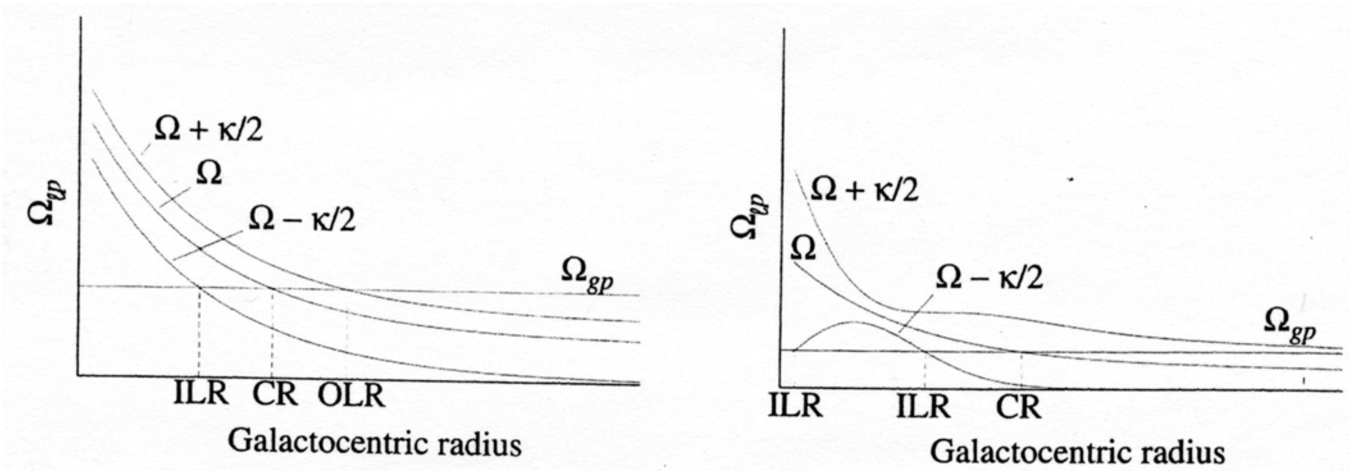


Figure 12.27. Contours of equal density excesses above the average value around a circle in a typical spiral mode. The dashed circle gives the radius where the material rotates at the same speed as the wave pattern.

Inner Lindblad Resonance (ILR)
Co-rotation Radius
Outer Lindblad Resonance (OLR)

} Important in all disk galaxies

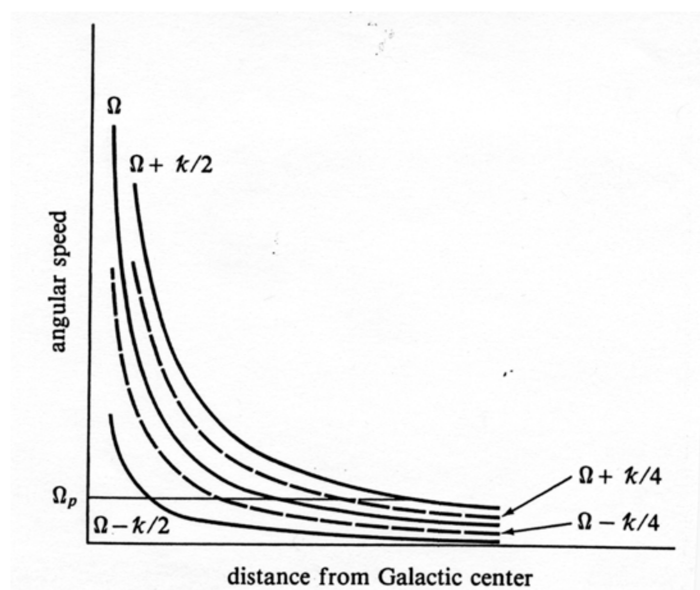


Density waves cannot propagate across ILR or OLR

Density wave theory interprets most spirals as 2-armed

- 4-armed pattern is n / m = 1 / 4
- exists over a narrow range of radius.
  - ➔ less likely to be seen.

•Actual 4-armed spirals are superposition of two 2-armed patterns

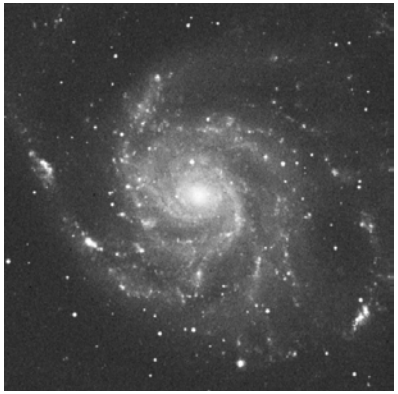


Spiral Structure

[CO 23.3]

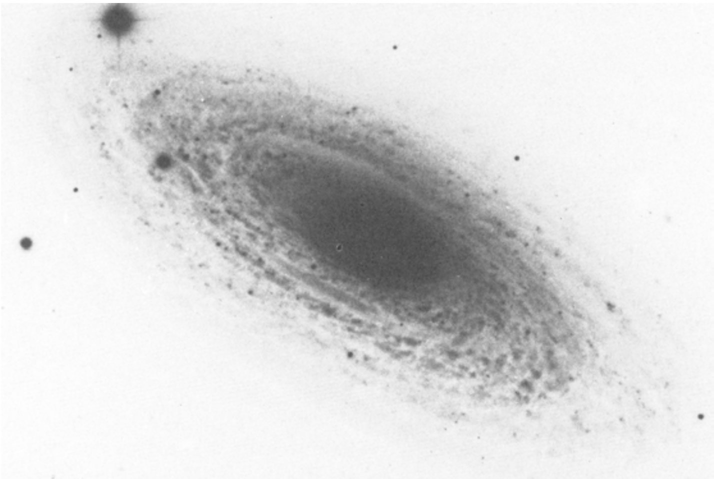
Grand design (10%)

M51



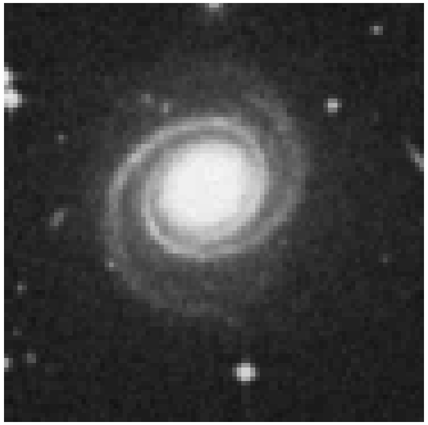
Multi-arm (60%)

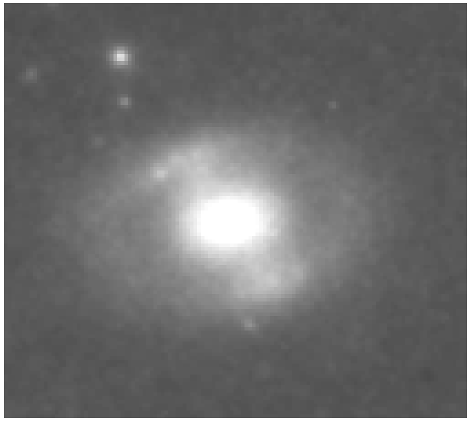
M101



Flocculent (30%)

NGC 2841





Inner rings

NGC 7096

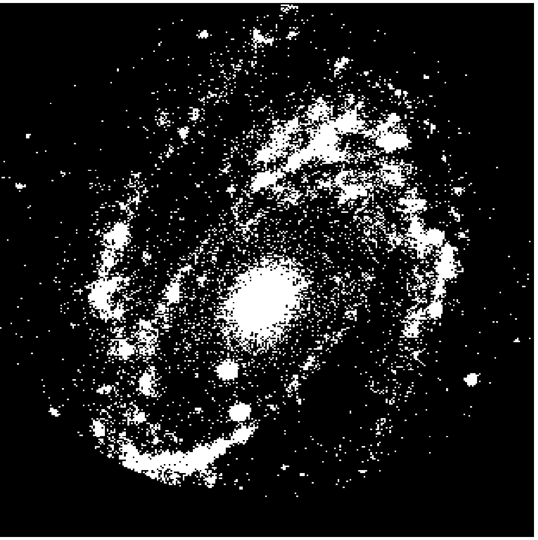
M81

Outer Ring

NGC 4340

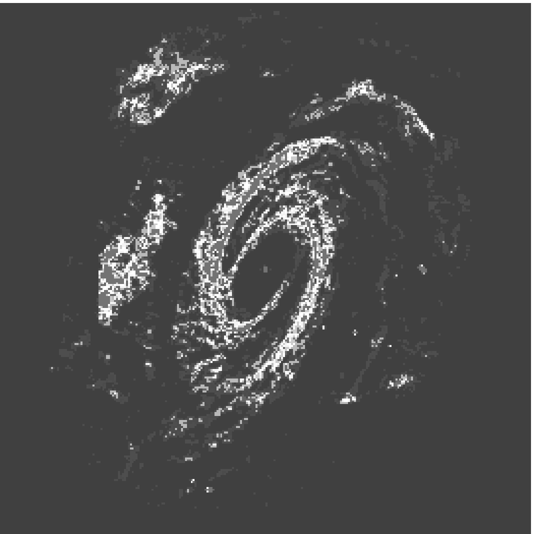
M81 spiral structure at different wavelengths

UV: hot stars



Visible: stars + obscuration

Near IR: late-type stars



21 cm: HI

Old red population shows small but real spiral density enhancement.

Passage of gas through spiral arms

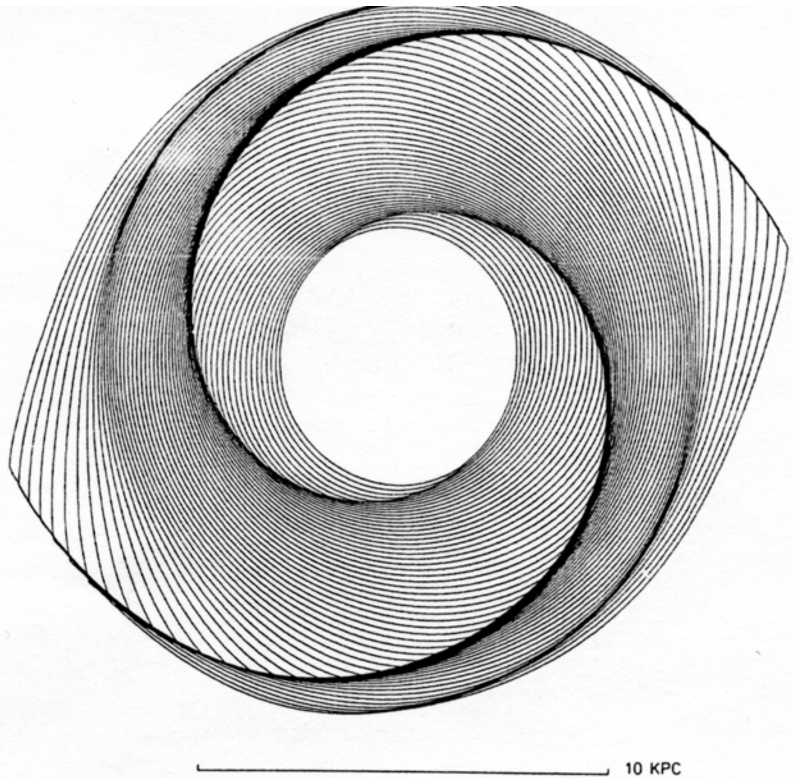
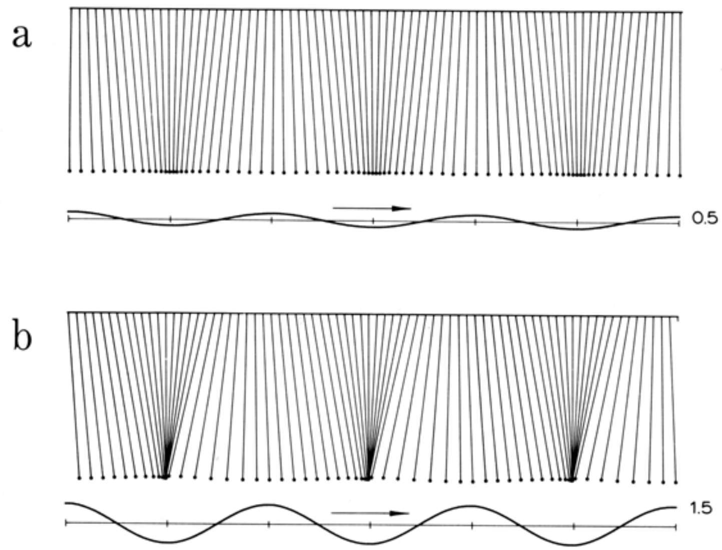


Figure 12.33. The streamlines of gas from a theoretical model of the spiral galaxy M81 (NGC3031, type Sb). (From H. Visser, *Astr. Ap.*, **88**, 1980, 159.)

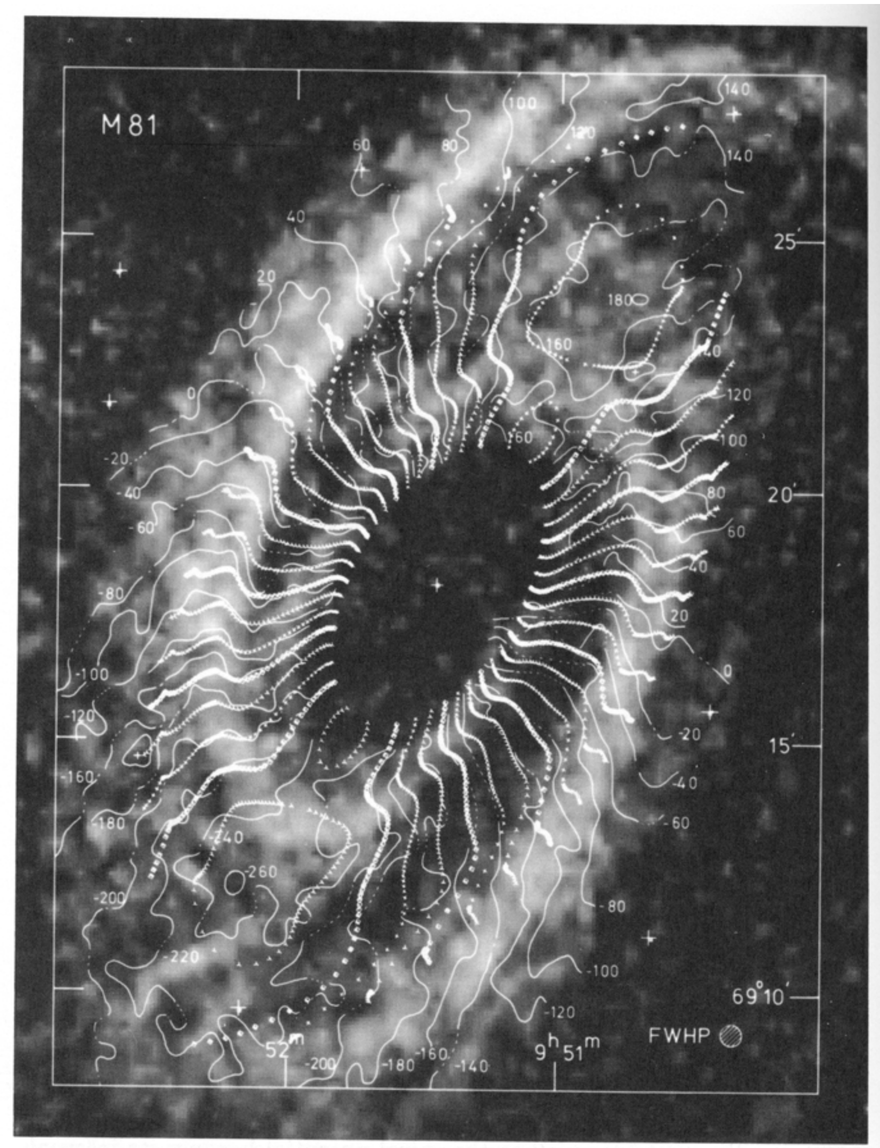
Calculated streamlines for gas

Response of gas to density waves

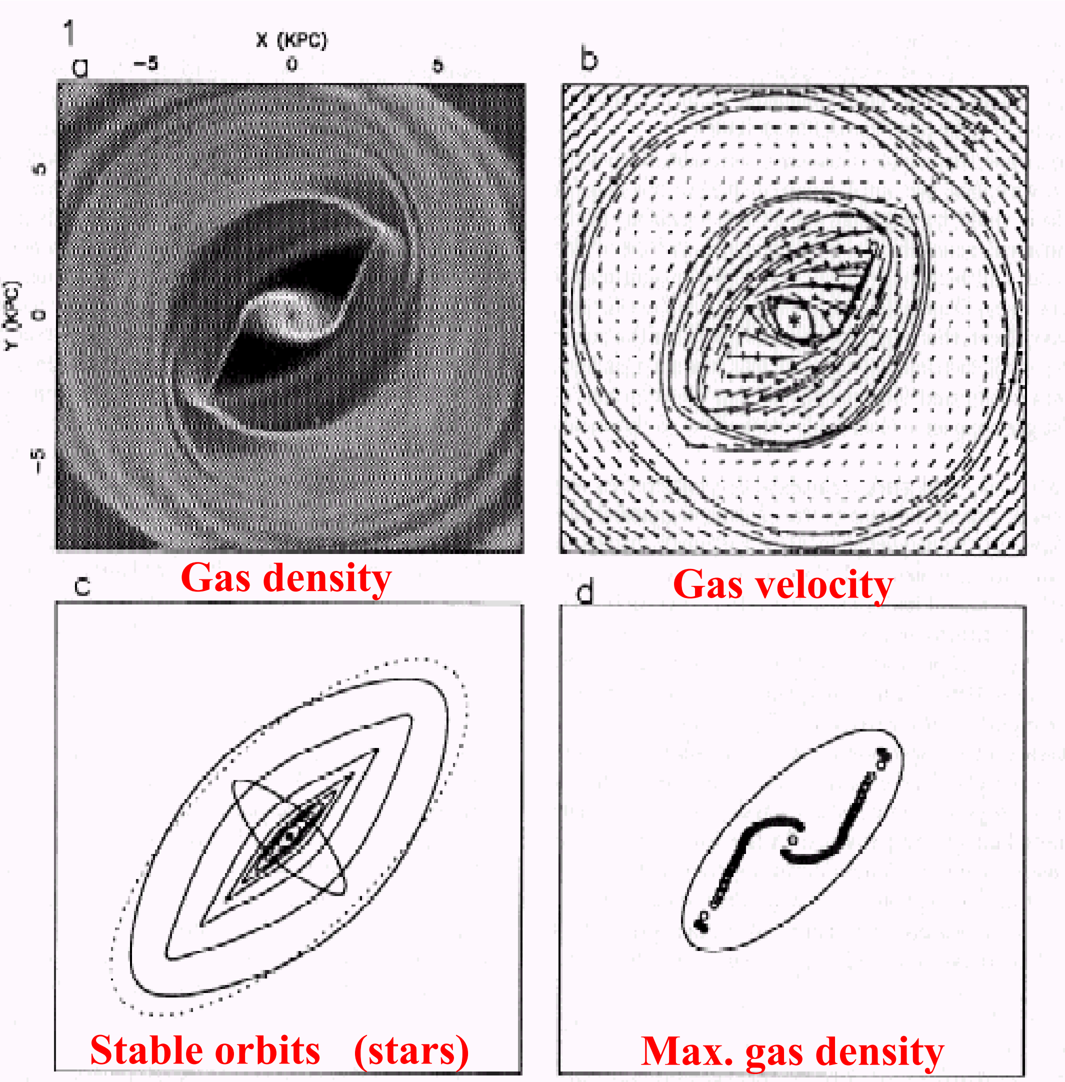


- Simple pendulum model
  - Each pendulum = 1 gas cloud
  - For large amplitude forcing, pendulums collide.
  - ➔ shock fronts in spiral arms

- HI map (right) shows velocity jumps at spiral arms.

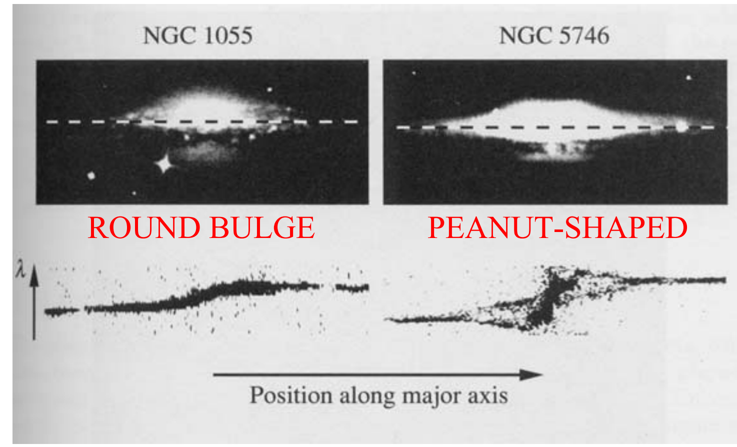


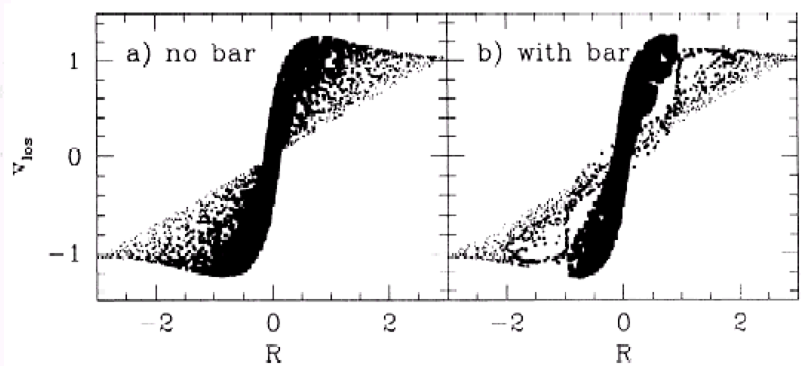
Orbits in Barred Spirals



Kinematics of gas, in [NII]

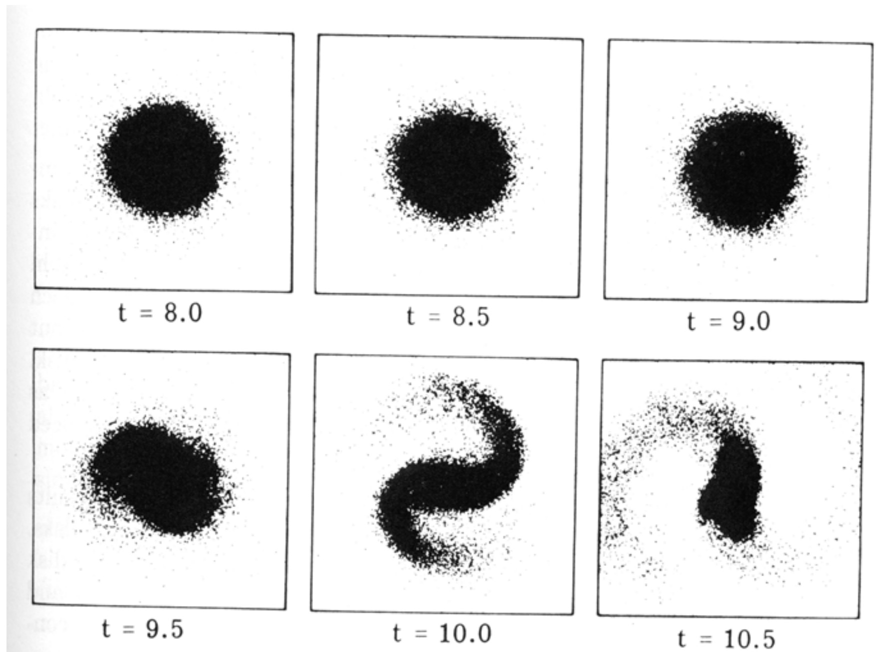
[BM] Fig 4.60



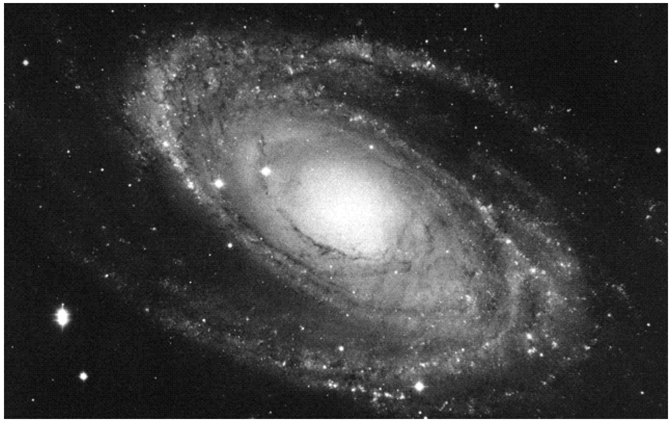


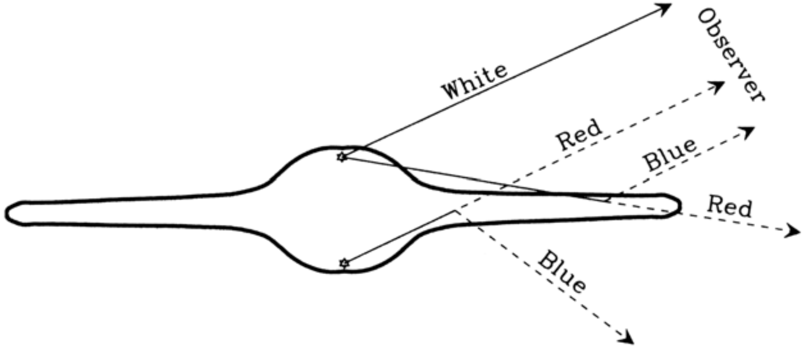
- Gas avoids "co-rotation" radius in barred potential.
- Causes "Fig-8" shape in rotation curve.

Bars appear to be easily excited instability in disks

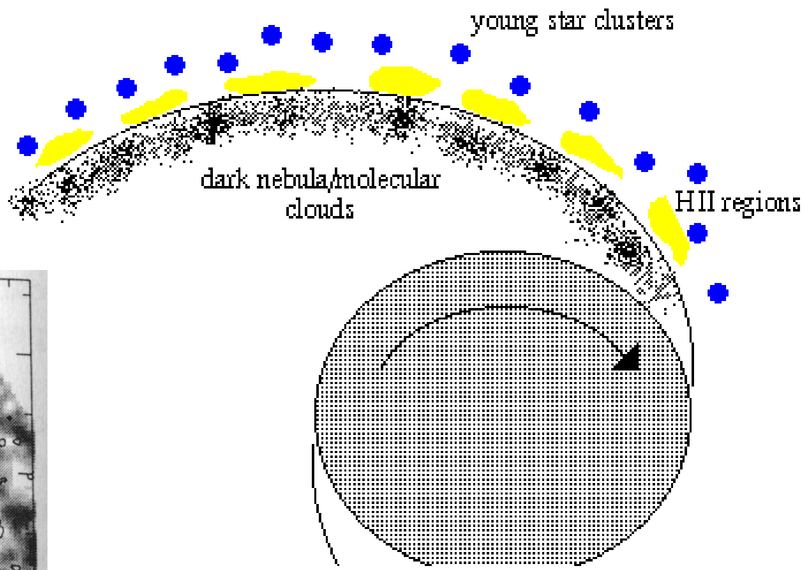


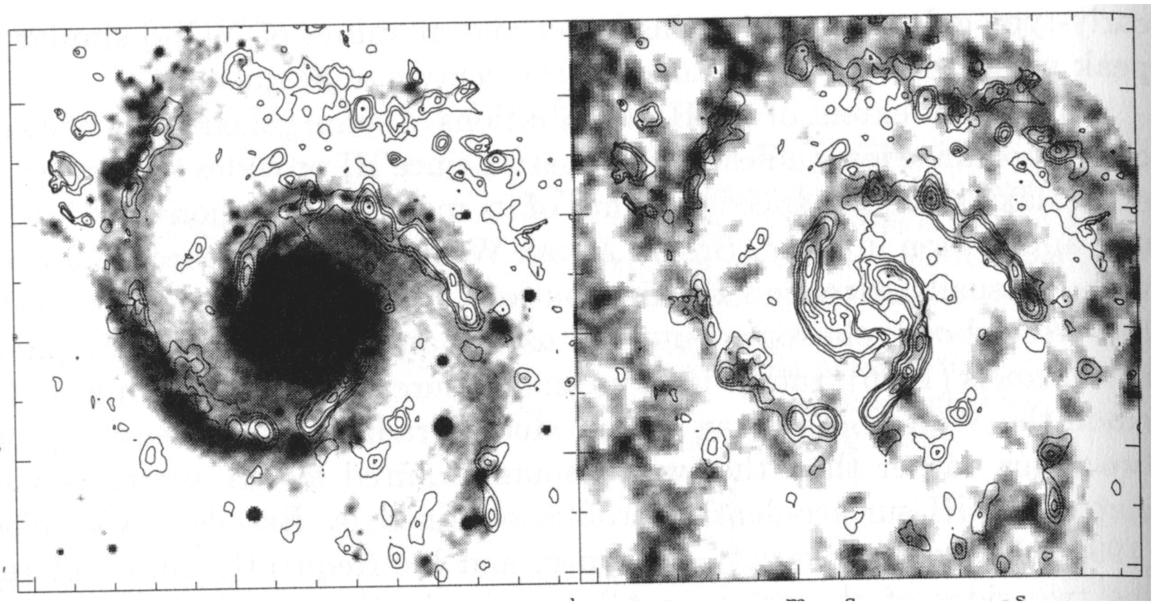
Trailing vs. leading spirals
Which is the near side of the galaxy?





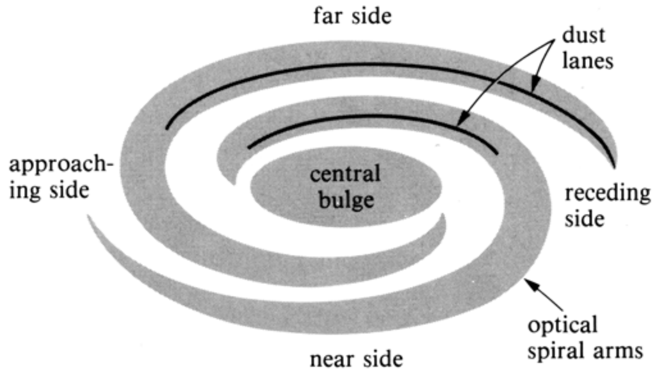
Molecular clouds on inner edges of arms





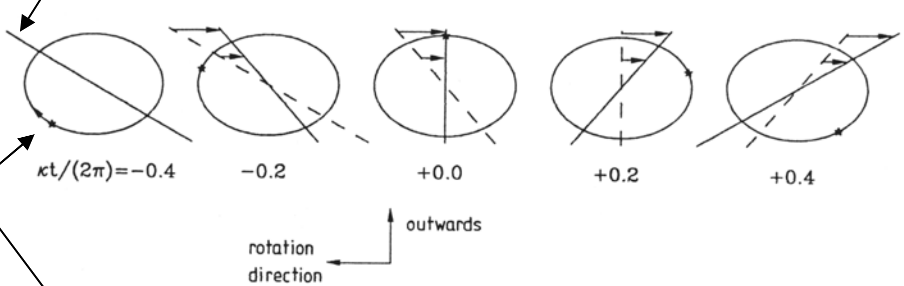
CO contours over red image

CO contours over 21 cm map

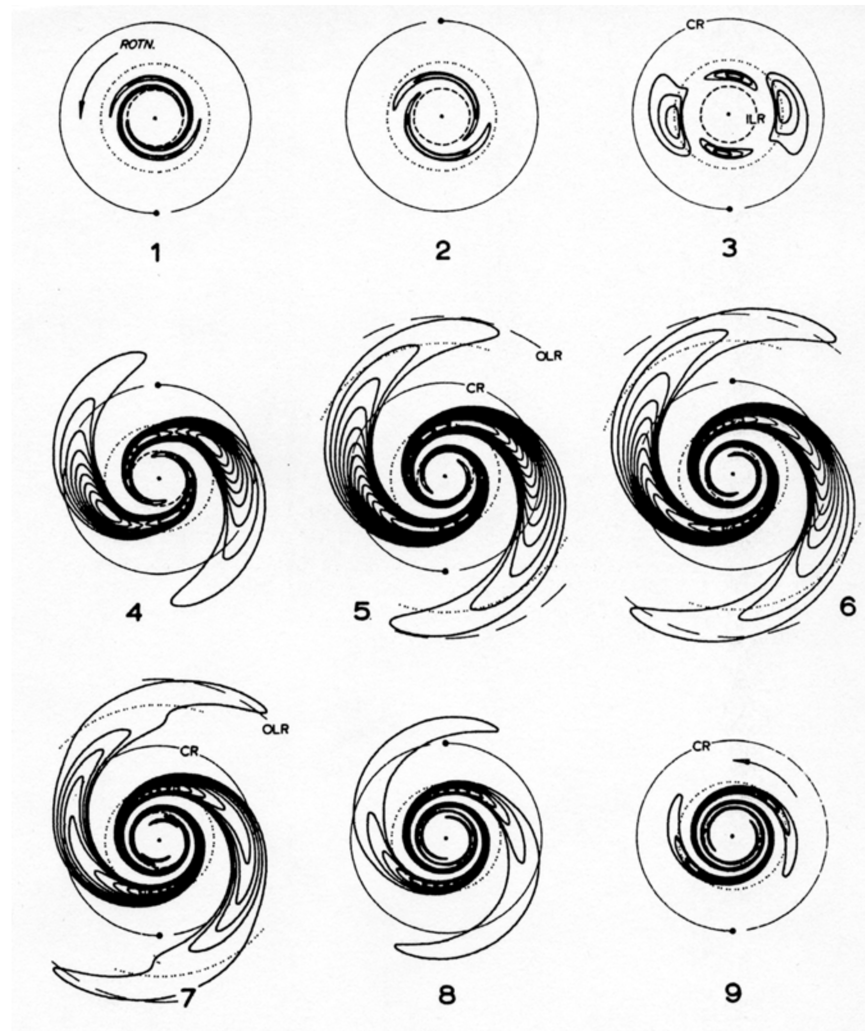


Swing Amplification

Position of (leading) spiral density enhancement



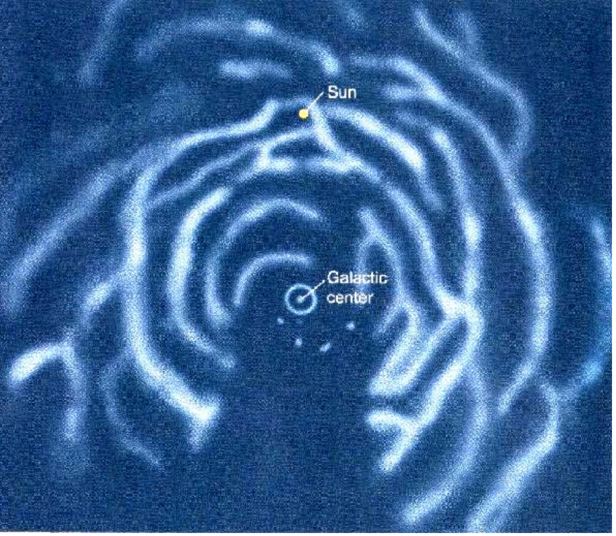
Epicyclic orbit of star



Automatically converts any leading spirals into much stronger trailing spirals.

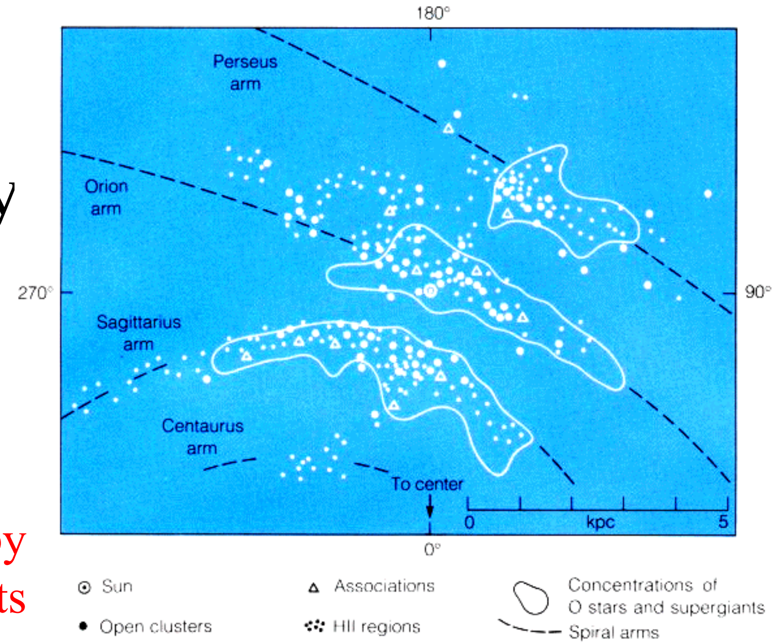
Spiral Structure of the Milky Way

Hard to measure, because we are inside it.

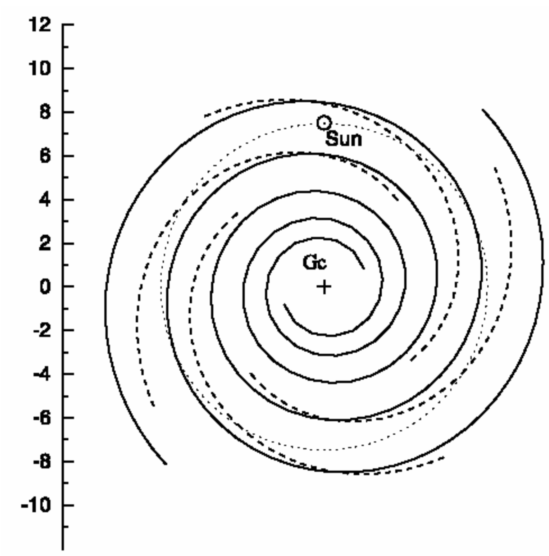


From HI (21 cm observations) assuming circular rotation.

Map of nearby young objects



- New model
  - Lepine et al (2001) ApJ 546, 234.
- ➔ mix of
  - 2-armed mode
  - 4-armed mode
- Sun at ~ co-rotation radius.



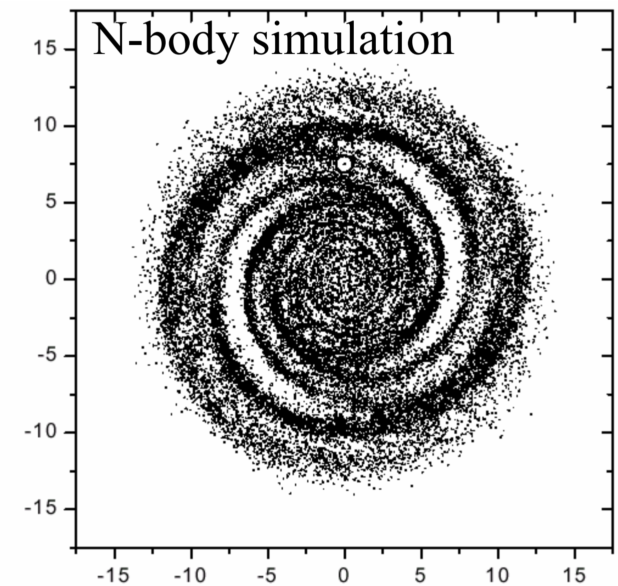


FIG. 3.—Visible structure of the Galaxy derived for the best model (superposition of 2 + 4 self-sustained wave harmonics) by means of cloud-particle simulation. The scale is indicated in kpc. Note that the model is not valid for radii smaller than about 2.5 kpc.

Summary: Density Waves?

- Evidence showing density waves *do* occur.
  - Old, red stars show spiral density perturbation.
  - Molecular clouds form on inner edges of spiral arms.
  - HI gas flow shows discontinuity due to shocks at inner edges of spiral arms.
  - Bright young stars also in narrow arms.
    - Observed width $\Delta\theta \sim t_*(\Omega - \Omega_p)$, as predicted.
- Are these waves self-sustaining over 10^{10} years? Problems:
  - Lin-Shu theory is linear; does not predict whether waves will grow or decay.
  - How are density waves initially formed?
- The usual interpretation
  - Density waves need a driving force
    - Satellite galaxy at co-rotation radius (M51)
    - Bars
  - Otherwise, act to prolong life of transitory phenomena.
  - Other mechanisms probably also important.
    - Swing-amplification efficiently builds up temporary trailing spirals.