Astronomy 802, STARS

Homework # 5, Due Thursday, Jan. 29, 2004

Show All Your Work

I will be out of town Monday - Wednesday, but will be in email contact to answer any questions you may have.

For the first order wave equation

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0 \tag{1}$$

(which is the linearized form of the Burger's equation) calculate, to lowest order in the error terms (that is the largest error terms in the limit that $\Delta t \to 0$ and $\Delta x \to 0$):

1. The actual differential equation that is solved (by using Taylor expansions)
2. The amplifcation factor, G,

for the two finite difference schemes:

1. Explicit time - space centered

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} + c\frac{u_{j+1}^n - u_{j-1}^n}{2\Delta x} = 0 \tag{2}$$

2. Implicit time - space centered

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} + c\frac{u_{j+1}^{n+1} - u_{j-1}^{n+1}}{2\Delta x} = 0 \tag{3}$$

Plot the magnitude, $|G|$, and phase speed $c_{\text{ph}} = \theta/ckt$, as a function of $k\Delta x = [0, \pi]$, for several values of $\mu \equiv c\Delta t/\Delta x = [0.1 - 2]$.