

Worksheet #3 - PHY102 (Spr. 2004)

Matrices and linear algebra

Last week we did vector operations with *lists*. This week we introduce you to matrices, their representation using lists and to some of the matrix operations which Mathematica is able to do.

Let us first see how to represent matrices in Mathematica as a *list*. Type `m={a,b},{c,d}`. Now `type MatrixForm[m]`. Note that all of the *Mathematica operations* must be applied to the *list* form of the matrix (not the matrix form). You can see that you get the matrix m with its elements a, b, c, d in the usual form. You can think of this matrix as consisting of two row vectors (a,b) and (c,d) . Type `m[[1]]` and check you get the first row vector (a,b) . Now type `m[[1,1]]`; this will give you the 1st element of the first vector, namely a (notice that `m[[1]][[1]]` also does the same thing). Likewise, to access the element d , type `m[[2,2]]`. As you have done with vectors, you can perform algebraic operations on matrices. You can multiply a matrix with a vector. To see this, type `r={x,y}`. In order to take a dot product of the matrix m with this vector r , Type `m.r` (or `Dot[m,r]`). Now type `Dimensions[m]`. The output $(2,2)$ verifies that the matrix m is a 2×2 matrix.

At times, you need to get the *transpose* of a matrix, which is obtained by exchanging its off-diagonal elements (in this case the elements c and d). Type `t=Transpose[m]`. Now type `MatrixForm[t]`. You see that the matrix has diagonal elements the same but the elements c and d got interchanged with respect to the original matrix m . Often we require the *determinant* of a matrix, which is a scalar quantity constructed from the elements. Type `Det[m]` which will give you the *determinant* of the matrix m . Now type `Det[t]` and verify that the *determinant* is the same for the transposed matrix. A *diagonal matrix* has all off-diagonal elements set to zero. Type `DiagonalMatrix[{e,f}]`. Now type `MatrixForm[DiagonalMatrix[{e,f}]]` (you can type this by taking the cursor to the end of the output and pushing the return key. Mathematica will immediately give you a replica of the output

which you can use as partial input for further operations. This is a convenient way of avoiding the usual cut and paste).

The “Inverse” of a matrix is the one which when multiplied by the original matrix produces a unit diagonal matrix (unit matrix, as it is often called). Type “mi=Inverse[m]”. Now take the product “h=m.mi” and check that h is indeed a unit matrix (you may have to perform “Simplify” on h). In general any $n \times n$ matrix has n eigenvalues and n eigenvectors. Type “Eigenvalues[m]” followed by “Eigenvectors[m]” to see what they are for the matrix m.

Assignment 3. - Hand in by Monday Feb. 9th

Problem 1. A 2×2 matrix A is constructed from the following rows: (5,3) and (2,1).

- (i) Write it in matrix form.
- (ii) Find its determinant.
- (iii) Find its transpose.
- (iv) Find its inverse A^{-1} .
- (v) Check $A \cdot A^{-1}$ is a unit matrix.
- (vi) Find its eigenvalues and eigenvectors.

Problem 2.

- (i) Find the solution to the following set of equations:

$$2x - y + 2z = 2$$

$$-x + 5y + z = 1$$

$$2x + y + 6z = 1$$

(Hint: Write it as a matrix equation $Ax=b$, which has solution $x = A^{-1}b$). Check that you have the solution by evaluating $Ax - b$.

- (ii) Try to find the solution to the following set of equations:

$$1x + 2y - z = -0.4$$

$$1.3x - 3.2y + 1.3z = 1$$

$$-2.5x - 5y + 2.5z = 1$$

What does mathematica tell you? What does this mean? From a mathematical point of view, explain why this set of equations does not have a unique solution. Write your answers in a text cell.