Worksheet #5 - PHY102 (Spr. 2004)

Solving Equations and Differential Equations II

Due Monday Feb. 23

Last week you learned about the following functions which are among the most important in Mathematica:

Solve - solves polynomial equations analytically
NSolve - solves polynomial equations numerically
FindRoot - solves all equations numerically
DSolve - solves differential equations analytically
NDSolve - solves differential equations numerically

Once you know what these routines do and how to use them, you have a very powerful set of tools for solving problems in physics. However the hardest part of physics is to set up the mathematical description of the problem, and that you still need to do that by hand. This worksheet is intended to help you learn some more about setting up and solving physics problems.

Problem 1.

(i) A ball is falling vertically through a fluid. Apart from gravity, a drag force F_d acts on the ball. The drag force opposes the motion and increases in proportion to the speed ($\vec{F_d} = -k\vec{v}$, where k is a *drag coefficient* that depends on the fluid). Find and plot the time dependence of the position and velocity of a $100g$ ball which is released from rest at $t = 0$, in a fluid with drag coefficient $k = 0.02$. Choose a time range which shows the terminal velocity of the ball. Don't forget that velocity and position are vectors in general - you can define them as lists in Mathematica and make the equations vector equations. Refer to worksheet 2 for reference.

(ii) Now consider a ball which has an initial velocity $v_x = 10m/s$ in the horizontal direction. Again find and plot the position and velocity of the particle.

(iii) Consider a cannon on a $500m$ high hill. Assuming that the cannon fires $10kg$ cannonballs horizontally with initial velocity $500m/s$, find the range of the cannon as a function of the drag coefficient of the air (let the drag coefficient vary from 0 to 1). This problem is difficult if we assume the drag force affects both the vertical *and* horizontal motion as it must in reality. We will learn later in the semester how to solve it properly using Mathematica. To make it more straightforward (though less physical) for now assume that the drag force acts only on the horizontal motion and there is no drag force acting on the vertical motion.

Problem 2.

You lift a box of mass $30kg$ vertically a height of h meters. However you decide that you want your little brother to lift the other 30 boxes that must be lifted to the same height. Since he is weaker than you, you kindly put an inclined plane (at angle θ to the horizontal) in place to assist him.

(i) If there is no friction, use mathematica to evaluate the integral $\int \vec{F}.\vec{dr}$ and hence prove that you and he do the same amount of work per box.

(ii) Now consider adding friction $f = \mu x N$ which increases as the box moves up the plane. Here x is distance along the inclined plane, μ is the friction coefficient and N is the force normal to the inclined plane. How much additional work does your brother do (per box) due to the friction on the inclined plane? (Solve this problem analytically using Mathematica).