Physics 321 – Spring 2004

Homework #6, due at beginning of class Wednesday Mar 3.

1. [4 pts] Marion & Thornton, problem 3-20. Do this problem by hand (i.e., using algebra, not using a computer). You need to find the two angular frequencies on either side of the resonance (call them ω_1 and ω_2) where the velocity amplitude is equal to the maximum velocity (on resonance) divided by $\sqrt{2}$, so the kinetic energy has half its maximum value. (This procedure finds the "Full Width at Half Maximum" (FWHM) of the resonance, $\omega_1 - \omega_2$, which is a common way to characterize its width.)

2. [4 pts] Consider the harmonic oscillator with damping: $\ddot{x} + 2\beta\dot{x} + x = 0$ in simple units. Make plots of some possible motions in phase space, i.e., in the (x, p) plane where p is the momentum. (Since the mass has been set equal to 1, $p = \dot{x}$.)

   Make your plots for (a) $\beta = 0.01$, (b) $\beta = 0.20$.

   You can use the Mathematica command `ParametricPlot[{x,y},{t,0,2}]`, or perhaps better, `ParametricPlot[{x,y},{t,0,2},Plotrange->All]`. You will have to choose the range of t and the initial conditions to make nice pictures.

3. [4 pts] A hook is at height y above the floor, where y is constant for all negative times: $y = y_0$ for $t < 0$. For positive times, y oscillates: $y = y_0 + A \sin \omega t$ for $t > 0$. A mass M hangs from an ideal spring attached to this hook. The mass is at height x above the floor. The mass hangs motionless at $x = x_0 = y_0 - Mg/k$ for $t < 0$, where k is the spring constant. Let $\omega_0 = \sqrt{k/M}$ as usual.

   (a) Find the motion $x(t)$ of the mass for $t > 0$ if $\omega = 2\omega_0$.

   (b) Find the motion $x(t)$ of the mass for $t > 0$ if $\omega = \omega_0$. (You can do this by first finding $x(t)$ for arbitrary ω and then carefully taking the limit $\omega \to \omega_0$; or if you're chicken, you can set $\omega \to \omega_0$ in the equation of motion and solve it.)

4. [4 pts] A driven harmonic oscillator obeys the equation $\ddot{x} + x = t\,(A - t)$ for $0 < t < A$. Given the initial conditions $x = \dot{x} = 0$ at $t = 0$, find the subsequent motion $x(t)$ during the time interval $0 < t < A$.

5. [4 pts] A damped driven harmonic oscillator obeys the equation $\ddot{x} + 2\beta\dot{x} + x = t\, e^{-\alpha t}$ for $t > 0$, where $0 < \beta < 1$ and α is a positive constant.

   (a) Given the initial conditions $x = \dot{x} = 0$ at $t = 0$, find the subsequent motion $x(t)$. Hint: as is so often the case, the easiest way to solve the differential equation is to guess the answer.

   (b) Plot your solution $x(t)$ for $\alpha = 0.2$, $\beta = 0.1$.

6. [4 pts] Marion & Thornton, problem 3-32.

7. [4 pts] Marion & Thornton, problem 3-43.

(Last updated 2/25/2004.)