Physics 410 - 2004

Thermal Physics

Problem Set 6

1. Evaluate the partition function and the free energy of an ideal gas from the classical expression for the partition function (5 pt)

2. Calculate the partition function of a *classical* harmonic oscillator of mass m and angular frequency ω_0 at temperature τ. Compare the classical result with the limit of the partition function of the quantum oscillator for $\hbar\omega_0 \ll \tau$. (6 pt)

3. Consider an ideal gas of molecules with an electric dipole moment $\mathbf{p}_0$, which can point in an *arbitrary* direction. The gas is placed into a uniform electric field $\mathbf{E}$. The energy of a molecule depends on an angle between the vectors $\mathbf{p}_0$ and $\mathbf{E}$. The gas density is n. Neglect the interaction between the molecules. Find the partition function using the classical approach [*Hint*: calculate the partition function for one molecule Z_1 first; use spherical coordinates, i.e., assume that the gas is inside a sphere of a large radius R, which you will then have to express in terms of the volume] (4 pt) Show that the polarization of the gas (the dipole moment per unit volume) is

$$P = np_0 \left[\coth \frac{p_0 E}{\tau} - \frac{\tau}{p_0 E}\right]$$

   (4 pt)

4. Heat capacity for constant pressure is given by the expression

$$C_p = \tau \left(\frac{\partial \sigma}{\partial \tau}\right)_p .$$

   Using Maxwell relations, show that

$$[\partial C_P/\partial p]_\tau = -\tau[\partial^2 V/\partial \tau^2]_p \text{ (6 pt)}$$

5. Consider a quantum oscillator of angular frequency ω_0 in thermal equilibrium at temperature τ. Assume that the oscillator energy in the sth state is $\hbar\omega_0\left(s+\frac{1}{2}\right)$. Calculate $\langle s\rangle$. (5 pt)

You need to have 25 points