Physics 492 Homework XII, due Fri Apr 23

Reading: Chapter 12

Problems:

1. Accelerator problem

In a high-energy physics accelerator, particles of charge $\pm e$, energy E and mass m travel around a ring of radius R in a magnetic field $\vec{B} = B\,\hat{k}$.

(a) Derive a formula for R in terms of e, m, B and E.

HINT: For a circular orbit

$$\begin{aligned} \vec{r}(t) &= R\,(\hat{\imath}\cos\omega t + \hat{\jmath}\sin\omega t) \\ \vec{v}(t) &= \omega\,R\,(-\hat{\imath}\sin\omega t + \hat{\jmath}\cos\omega t) \end{aligned}$$

where $\omega = v/R$. The equation of motion is

$$\frac{d\vec{p}}{dt} = e\,\vec{v}\times\vec{B}, \qquad \text{where} \quad \vec{p} = \frac{m\,\vec{v}}{\sqrt{1 - v^2/c^2}}$$

(b) The Fermilab Tevatron is an accelerator for protons and antiprotons at particle energy of 0.9 TeV. The radius of the accelerator is 1 km. Determine the magnetic field B.

2. Williams, Problem 12.2.

3. Williams, Problem 12.18.
Hint: If the neutrino mass is zero, then all neutrinos travel at the speed of light, irrespective of neutrino energy. If the neutrino mass is nonzero, then the speed v and energy E are related by

$$E = \frac{m\,c^2}{\sqrt{1 - v^2/c^2}}$$

Assume that the energy spread of the detected neutrinos is from 4 to 12 MeV.

4. Williams, Problem 12.6.

5. Williams, Problem 12.8. The muon lifetime can be calculated from the Fermi theory of β-decay. Neglecting the electron mass (a valid approximation since $m_e \ll m_\mu$) the result is

$$\frac{1}{\tau_\mu} = \frac{G_F^2\,c^4}{192\,\pi^3\,\hbar^7}\,m_\mu^5\,;$$

see for comparison Eq. (12.23). Draw Feynman diagrams for the β-decay of μ^+ and for the weak decays (leptonic and hadronic) of D^+. Is the number of possible final states the same in the D^+-decay as in the μ-decay? How does the number of possible states affect the decay rate? Use a formula analogous to the one above to estimate the D^+-lifetime. Your estimate should be somewhat smaller than the experimental value

$$\tau(D^+) = (10.57 \pm 0.15) \times 10^{-13}\,\mathrm{s}$$

The discrepancy is because the final state will include a kaon, whose mass cannot be really neglected. Get a better estimate for the lifetime by replacing m_D in the formula by $m_D - m_K$.

Note: The deadline for the term paper has been advanced to Monday, April 26.