Physics 492 homework VI, due Fri Feb 27

Reading: Chapters 6, 7.1-3
Problems:

1. Use the masses in the Table of Isotopes in the CRC Handbook of Chemistry and Physics (on reserve in Physics Library) or on the Web (http://www.nndc.bnl.gov/nndcscr/ masses/MASS_RMD.MAS95) to show that ${ }^{197} \mathrm{Au}$ is nominally unstable with respect to $\alpha$ decay. Calculate the kinetic energy of an $\alpha$ particle that would be emitted in the decay. (Note: Because of the recoil given to the daughter nucleus, the kinetic energy is slightly less than the $Q$-value for the decay.) Using the empirical Geiger-Nutall law, $\log _{10} t_{1 / 2} \simeq$ $a+b Q^{-1 / 2}$, with $a \simeq-1.61 Z_{D}^{2 / 3}-21.4$ and $b \simeq 1.61 Z_{D}$, estimate the half-life for the $\alpha$ decay of gold. The time in the empirical law is in seconds and the $Q$-value is in MeV . How does that half-life compare to the age of Universe? How does the $Q$-value compare to the one obtained in Problem 5.1 from Williams?
2. Use the tunneling formula derived in class, and given in the handout, to justify the empir-th ical Geiger-Nutall law in Problem 1, including the values of the numerical coefficients there. The $\alpha$-particle velocity inside the parent nucleus may be assumed to take some representative value, such as outside the nucleus or larger. Note that the nuclear $Z$ and $A$ are correlated with each other around the line of stability and approximately proportional to each other for heavy nuclei.
3. Consider the strongly deformed nucleus ${ }^{252} \mathrm{Fm}$ with the deformation parameter $\epsilon=0.3$. That is, the nucleus is shaped like an ellipsoid of revolution with semimajor axis $a^{\prime}=R(1+\epsilon)$ and semiminor axis $a=R /(1+\epsilon)^{1 / 2}$, where $R \simeq r_{0} A^{1 / 3}$ is the mean radius. Using a potential of the form suggested in the figure below, and following one-dimensional barrierpenetration considerations, estimate the relative probabilities of polar and equatorial emission of $\alpha$ particles.



In a deformed nucleus, $\alpha$ particles escaping from the poles enter the Coulomb barrier at the larger separation $a^{\prime}$, and must therefore penetrate a lower, thinner barrier. It is therefore more probable to observe emission from the poles an from the equator.
4. A typical induced fission reaction is

$$
n+{ }_{92}^{235} U \rightarrow{ }_{36}^{92} K r+{ }_{56}^{142} B a+2 n .
$$

(a) Estimate the mass energy released, using the Weizsäcker semi-empirical mass formula.
(b) Calculate the mass energy released, using the exact atomic masses in the Table of Isotopes.
(c) Calculate the total mass energy, in joules, released when 1 kg of ${ }^{235} \mathrm{U}$ undergoes fission.

Note: nuclear mass $=$ atomic mass $-Z m_{e}$, $1 \mathrm{u}=931.494 \mathrm{MeV} / \mathrm{c}^{2}$, mass excess $=$ atomic mass $-\mathrm{A} \times 1 \mathrm{u}$.

