Problem Set #3

PHY 854, Spring Semester, 2004

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These problems are due by Wednesday of Finals Week (May 5) to my mailbox on the first floor of BPS. Note, $\hbar = c = 1$.

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Problem 16 In class we derived the differential cross section, $d\sigma/d\Omega$, for electrons which scatter from a Coulomb field. In that presentation, I treated the spinors directly. Derive the same thing under the same conditions, except this time use the trace-techniques for averaged and summed electron helicities.

Problem 17 For the Compton problem, we found that the summed/averaged invariant amplitude-squared becomes

$$\bar{\sum_i}\sum_f |T|^2 = \frac{e^4}{2} Tr\left\{ (\not p' + m)\left[\frac{1}{2p\cdot k}\not\varepsilon'\not k\not\varepsilon + \frac{1}{2p\cdot k'}\not\varepsilon\not k'\not\varepsilon'\right] \times (\not p + m)\left[\frac{1}{2p\cdot k}\not\varepsilon\not k\not\varepsilon' + \frac{1}{2p\cdot k'}\not\varepsilon'\not k'\not\varepsilon\right]\right\}.$$

Show that the term proportional to $\frac{1}{(2p\cdot k')^2}$ is:

$$8(p\cdot k)(p\cdot k')\left[p'\cdot k' - 2\left(k'\cdot\varepsilon\right)^2\right].$$

Problem 18 For the decay of the W in its rest frame, show that the differential decay rate for a longitudinally polarized W is

$$\frac{d\Gamma_0}{d\Omega} = \frac{G_F M_W^3}{16\pi^2\sqrt{2}}\sin^2\theta_W$$

Recall that the relationship between the Fermi constant and the W-fermion coupling strength is $g^2 = \frac{8M_W^2 G_F}{\sqrt{2}}$.

Problem 19 Consider the scattering of electron-neutrinos from electrons through the exchange of an intermediate W boson, the so-called "charged current" process.

(a) What is the diagram?

(b) Show that the differential cross section, in the center of momentum frame is given by $\frac{d\sigma}{d\Omega} = \frac{G_F s}{4\pi^2}$. Here the coupling is through an intermediate W boson with the vertex element which is the same as that for the W decay, namely $\frac{-ig}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)$. Neglect the mass of the electron and presume the neutrino to be (still) massless.

(c) In actuality, there is a neutral current process for this same final state with the exchange of a massive, neutral intermediate vector boson, the Z^0. What is the diagram for this process? How does it combine with the charged current process? This is a part of the mechanism of matter oscillations for neutrinos, the so-called MSW effect.

Problem 20 In class we calculated the total cross section for the process $e^+e^- \to \mu^+\mu^-$ in which the muon was presumed to be massless. In terms of the β of the outgoing fermion and s, what is the total cross section for the process $e^+e^- \to \tau^+\tau^-$ in which the mass of the only the tau leptons cannot be neglected? Do the same thing for c quarks and b quarks and plot them on one graph. Perhaps you can see why there was such confusion in 1974-5 when both charm and the tau were nearly simultaneously discovered.

Problem 21 Calculate the ratio of leptonic decay widths for the charged pion for two assumptions about the spacetime nature of its matrix element. That is, calculate:

$$R \equiv \frac{\Gamma(\pi \to \mu\nu_\mu)}{\Gamma(\pi \to e\nu_e)}$$

for two choices of possible physics,

a) vector-axial vector: $T = i\frac{G}{\sqrt{2}}P^\mu \bar{u}_{e,\mu}(k_1)\gamma_\mu(1-\gamma_5)v_{\nu_e,\nu_\mu}(k_2)$ and
b) scalar-pseudoscalar: $T = ig\bar{u}_{e,\mu}(k_1)(1-\gamma_5)v_{\nu_e,\nu_\mu}(k_2)$.

Call the 4-momenta of the electron (muon) and its neutrino k_1 and k_2, respectively. Look up the experimental results for the decay rates and decide which model works best.