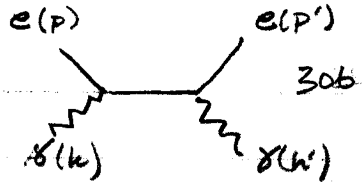
23 Compton - contd

label momenta



so, back to Compton scattering...

$$J^{(2)}(e\gamma \to e\gamma) = \frac{(-ie)^2}{2}\int d^4x_1 \int d^4x_2 \, \langle e\gamma | \gamma^{\mu}_{ij}\gamma^{\nu}_{\ell m}$$

$$\Big\{ \bar{\Psi}^-_i(x_1)\Psi^+_m(x_2)A_\mu(x_1)A_\nu(x_2)\, \underbrace{\Psi_j(x_1)\bar{\Psi}_\ell(x_2)}$$

$$- \bar{\Psi}^-_\ell(x_2)\Psi^+_j(x_1)A_\mu(x_1)A_\nu(x_2)\, \underbrace{\bar{\Psi}_i(x_1)\Psi_m(x_2)} \Big\} |e\gamma\rangle$$

$$\Psi^+_m(x) = \sum_r \int dP_1 \, a^{(r)}(P_1)\, u^{(r)}_m(P_1)\, e^{-iP_1\cdot x}$$

$$\bar{\Psi}^-_n(x) = \sum_s \int dP_2 \, \bar{u}^{(s)}_n(P_2)\, a^{(s)+}(P_2)\, e^{iP_2\cdot x}$$

$$A^+_\nu(x) = \int dK_1 \sum_\lambda \epsilon_{\nu(\lambda)}(k_1)\, a_{(\lambda)}(k_1)\, e^{-ik_1\cdot x}$$

$$A^-_\mu(x) = \int dK_2 \sum_{\lambda'} \epsilon_{\mu(\lambda')}(k_2)\, a^+_{(\lambda')}(k_1)\, e^{ik_2\cdot x}$$

Put all this stuff, plus the propagators, back... first separate out the Fock space terms for electrons, then do photons...

$$J^{(2)}(e\gamma \to e\gamma) = -\frac{e^2}{2}\int d^4x_1 \int d^4x_2 \Big\{ \langle\gamma| A_\mu(x_1)A_\nu(x_2)|\gamma\rangle \Big[$$

$$\cdot \sum_{r,s}\int dP_1 \int dP_2 \langle 0| a^{(f)}(p')a^{(s)+}(P_2)a^{(r)}(P_1)a^{(g)+}(P)|0\rangle \, \bar{u}^{(s)}_i(P_2)u^{(r)}_m(P_1)\, e^{iP_2\cdot x_1}e^{-iP_1\cdot x_2}$$

$$\cdot \int \frac{d^4q}{(2\pi)^4}\left(\frac{i}{\not{q}-m}\right)_{j\ell} e^{-iq\cdot(x_1-x_2)}$$

$$\bar{u}^{(s)}_\ell(P_2)\, u^{(r)}_j(P_1)$$

$$+ \sum_{r,s}\int dP_1 \int dP_2 \langle 0| a^{(f)}(p')a^{(s)+}(P_2)a^{(r)}(P_1)a^{+(g)}(p)|0\rangle \, e^{iP_2\cdot x_2}e^{-iP_1\cdot x_1}$$

$$\cdot \int \frac{d^4q'}{(2\pi)^4}\left(\frac{i}{\not{q}'-m}\right)_{mi} e^{-iq\cdot(x_2-x_1)} \Big]\Big\} \gamma^{\mu}_{ij}\gamma^{\nu}_{\ell m}$$

where $\psi_a(y)\bar{\psi}_b(x) = -\bar{\psi}_b(x)\psi_a(y)$

Look at $\overset{1st}{\wedge}$ electron Fock space term. Do the standard trick to turn $a(n)a^{+}(m)$ into $\{a(n), a^{+}(m)\}$ – twice

$$\langle 0| -- |0\rangle_{\text{electron}_1} = (2\pi)^3 2E'\delta(\vec{p}\,'-\vec{P}_2)(2\pi)^3 2E\,\delta(\vec{p}-\vec{P}_1)\,\delta_{fs}\,\delta_{gr}$$
$$\times \langle\gamma|:AA:|\gamma\rangle\, \underbrace{\psi_j\,\bar{\psi}_k}$$

2nd – same, except for matrix indices -

$$\langle 0| -- |0\rangle_{\text{electron}_2} = (2\pi)^3 2E'\delta(\vec{p}\,'-\vec{P}_2)(2\pi)^3 2E\,\delta(\vec{p}-\vec{P}_1)\,\delta_{fs}\,\delta_{gr}$$
$$\times \langle\gamma|:AA:|\gamma\rangle\, \underbrace{\bar{\psi}_i\,\psi_m}$$

Now, the $\int dP_1$ and $\int dP_2$ integrations can be done and the $\sum_r\sum_s$ sum collapsed,

$$J^{(2)}(e\gamma\to e\gamma) = -\frac{e^2}{2}\int d^4x_1\int d^4x_2\ \Big\{\Big[\bar{u}^{(f)}_i(p')u^{(g)}_m(p)\,e^{ip'\cdot x_1}e^{-ip\cdot x_2}$$
$$\cdot \int\frac{d^4q}{(2\pi)^4}\left(\frac{i}{q-m}\right)_{jk} e^{-iq\cdot(x_1-x_2)}$$
$$+\ \bar{u}^{(f)}_k(p')u^{(g)}_j(p)\,e^{ip'\cdot x_2}e^{-ip\cdot x_1}\int\frac{d^4q'}{(2\pi)^4}\left(\frac{i}{q'-m}\right)_{mi} e^{-iq'\cdot(x_2-x_1)}\Big]$$
$$\cdot \langle\gamma|A^{+}_{\mu}(x_1)A_\nu(x_2)|\gamma\rangle\Big\}\ \gamma^{\mu}_{ij}\,\gamma^{\nu}_{km}$$

The photon terms go the same way --
for example, $\langle\gamma|:A_\mu(x_1)A_\nu(x_2):|\gamma\rangle = \langle\gamma|A^{-}_{\nu}(x_2)A^{+}_{\mu}(x_1) + A^{-}_{\mu}(x_1)A^{+}_{\nu}(x_2)|$
(2 terms)
and

$$A^{+}_{\nu}(x_2)|\gamma\rangle = \int dK_1\sum_{\lambda_1}\epsilon_{\nu(\lambda_1)}(k_1)\,e^{-ik_1x_2}\,a_{(\lambda_1)}(k_1)\,a^{+}_{(\lambda)}(k)|0\rangle$$

do the same trick on $a(\ell)a^{+}(m) \longrightarrow [a(\ell), a^{+}(m)]$

.. twice (which cancels the funny minus signs) --

So, we get

$$A^{+}_{\nu}(x_2)|\gamma\rangle = \int dk_1 \sum_{\lambda_1} \epsilon_{\nu(\lambda_1)}(k_1)\, e^{-ik_1\cdot x_1} (2\pi)^3 2\omega_1\, \delta_{\lambda\lambda_1}\, \delta(\vec{k}-\vec{k}_1)\, |0\rangle$$

and we can do the momentum integrals giving

$$\langle \gamma(k'\lambda')|:A_\mu(x_1)A_\nu(x_2):|\gamma(k\lambda)\rangle =$$

$$\epsilon_{\mu(\lambda')}(k')\,\epsilon_{\nu(\lambda)}(k)\, e^{ik'\cdot x_1} e^{-ik\cdot x_2} + \epsilon_{\nu(\lambda')}(k')\,\epsilon_{\mu(\lambda)}(k)\, e^{ik'\cdot x_2} e^{-ik\cdot x_1}$$

and therefore, 4 terms overall.

$$J^{(2)}(e\gamma\to e\gamma) = -\frac{e^2}{2}\int d^4x_1 \int d^4x_2 \int \frac{d^4q}{(2\pi)^4}\, \gamma^\mu_{ij}\,\gamma^\nu_{\ell m}$$

$$\cdot\left\{ \bar{u}^{(f)}_i(p')\, u^{(g)}_m(p)\, e^{ip'\cdot x_1} e^{-ip\cdot x_2} \left(\frac{i}{\not q - m}\right)_{j\ell} e^{-iq\cdot(x_1-x_2)}\right.$$

$$\left. + \;\bar{u}^{(f)}_\ell(p')\, u^{(g)}_j(p)\, e^{ip'\cdot x_2} e^{-ip\cdot x_1} \left(\frac{i}{\not q - m}\right)_{mi} e^{-iq\cdot(x_2-x_1)}\right\}$$

$$\cdot\left\{ \epsilon_{\mu(\lambda')}(k')\,\epsilon_{\nu(\lambda)}(k)\, e^{ik'\cdot x_1} e^{-ik\cdot x_2} + \epsilon_{\nu(\lambda')}(k')\,\epsilon_{\mu(\lambda)}(k)\, e^{ik'\cdot x_2} e^{-ik\cdot x_1}\right\}$$

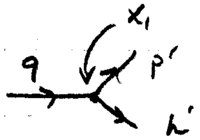
All of this is tied together through the exponentials —

$$J^{(2)} = (e_1+e_2)(\gamma_1+\gamma_2) = e_1\gamma_1 + e_2\gamma_1 + e_2\gamma_1 + e_2\gamma_2$$

$$= ① + ② + ③ + ④$$

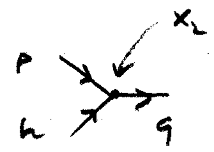
notice $\int d^4x_1\, e^{ip'\cdot x_1}\, e^{-iq\cdot x_1}\, e^{ik'\cdot x_1}$ in ①

$$= (2\pi)^4\delta(p'-q+k')$$
$$= (2\pi)^4\delta(q-p'-k')$$



4 momentum
kirchoff's law. —

and $\int d^4x_2\, e^{-ip\cdot x_2}\, e^{iqx_2}\, e^{-ikx_2} = (2\pi)^4\delta(q-p-k)$



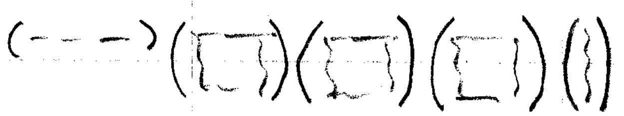
then, $\int d^4q$ integral can be trivially done.

$$\int d^4q\,(2\pi)^4(2\pi)^4\delta(q-p'-k')\,\delta(\underbrace{q-p-k}_{\longrightarrow}) \Rightarrow q=p+k$$
$$= (2\pi)^8\delta(p+k-p'-k')$$

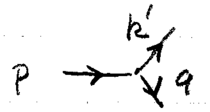
and

$$① = -\frac{e^2}{2}\Big\{(2\pi)^4\delta(p+k-p'-k')\,\underline{\bar{u}_i^{(f)}}(p')\,\underline{\gamma^\mu_{ij}}\,\epsilon_{\mu(\lambda')}(k')\left(\underline{\frac{i}{\not k+\not p-m}}\right)_{jl}$$
$$\cdot\,\underline{\gamma^\nu_{lm}}\,\epsilon_{\nu(\lambda)}(k)\,\underline{u_m^{(i)}}(p)\Big\}$$

this has the matrix structure in Dirac space,

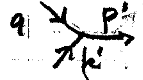

= a number

Look at ② = $e_2 \gamma_1$



The spacetime integrals give $(2\pi)^4 \delta(q-p+k') \Rightarrow q = p-k'$

$(2\pi)^4 \delta(-q-k+p') \Rightarrow \delta(q+k-p')$



$q = p'-k$

$$② = -\frac{e^2}{2}\Big\{ (2\pi)^4 \delta(p-k'+k-p')\, \overline{u}^{(f)}_{\ell}(p')\, \gamma^{\nu}_{\ell m} \left(\frac{i}{\not p-\not k'-m}\right)_{mi} \gamma^{\mu}_{ij}\, u^{(q)}_j(p) \Big\}$$

$$\cdot \left[\epsilon_{\mu(\lambda')}(k')\, \epsilon_{\nu(\lambda)}(k)\right]$$

Keep this up and we would find ① = ④ & ② = ③

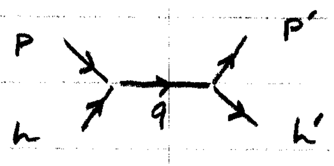
So, $\quad J^{(2)} = 2\left[① + ② \right]$

$$= -e^2 (2\pi)^4 \delta(p+k-p'-k')\, \overline{u}^{(f)}(p') \Big[\not\epsilon_{(\lambda')}(k') \frac{i}{\not p+\not k-m} \not\epsilon_{(\lambda)}(k)$$

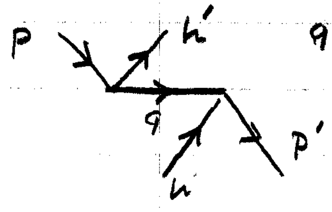
$$+ \not\epsilon_{(\lambda)}(k) \frac{i}{\not p-\not k'-m} \not\epsilon_{(\lambda')}(k') \Big] u^{(s)}(p)$$

Look at what the "momentum flow" results imply at the 2 vertices. –

From ①



From ②

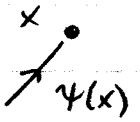


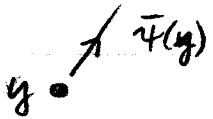
$q = p-k' \Rightarrow p = q+k'$

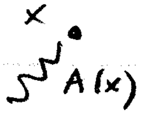
$q = p'-k \Rightarrow p' = q+k$

Earlier, I attached a graphical meaning to the Wick expansion terms. Let's recap that according to what we've calculated.

$$\underbrace{\bar{\Psi}(x)\Psi(y)} = \langle 0 | T[\bar{\Psi}(x)\Psi(y)] | 0 \rangle$$

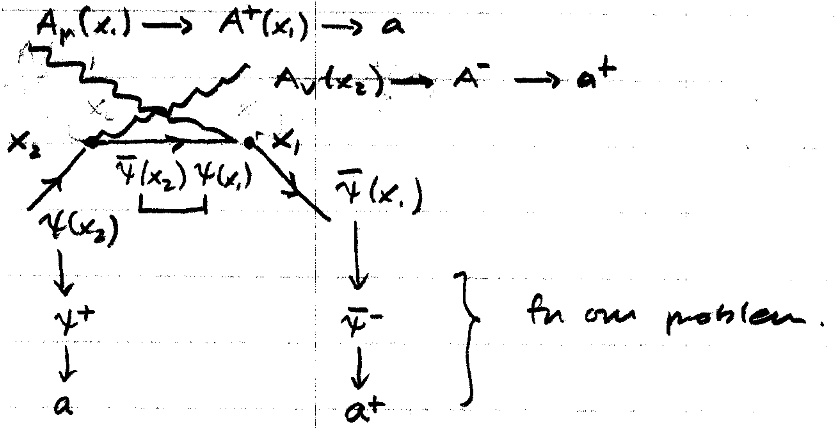




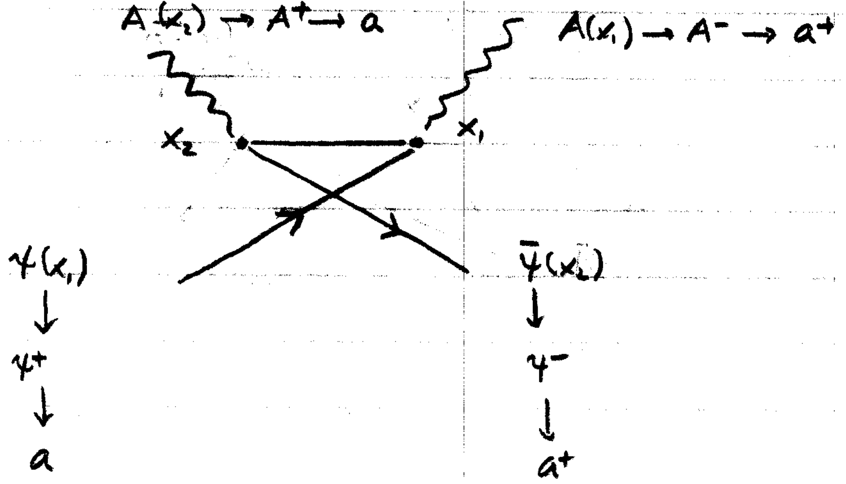


So, the first (① or ③) graph would be

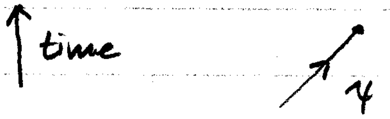
generally:



and ② or ④

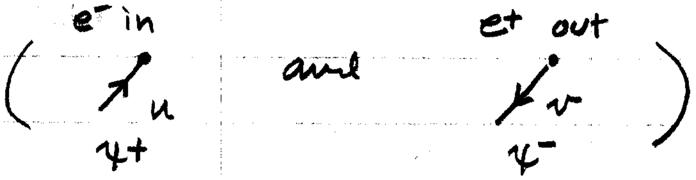


Remember the Feynman interpretation

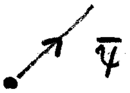


ψ^+ both annihilates electrons (a)
ψ^- and creates positrons $(b^\dagger)$

|||

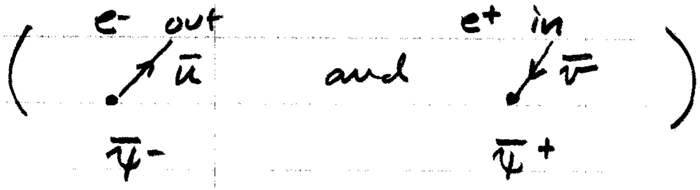


likewise



annihilates positrons (b)
& creates electrons $(a^\dagger)$

|||



So, our process can be relabeled with the frequency components indicated.

The really useful Feynman rules eliminate <u>all</u> of the previous calculation and go directly to momentum space.

Standard Model on ~~one~~ ~~two~~ three pages

primordial group conserved: $SU(2) \otimes SU(3) \otimes U(1)$

weak isospin

color

weak hypercharge.

$T_{universe} > 10^{15}$ K

$\tau_{universe} < 10^{-12}$ s

constituents are:

g^μ SU(3) octet, massless

$\vec{B}^\mu$ SU(2) triplet, massless 2dof

a^μ U(1) singlet, massless 2dof

appearance in problem $\longrightarrow$ $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$, $\Phi^+ = \begin{pmatrix} \phi^- \\ \phi^0 \end{pmatrix}$, massless

$-\frac{\lambda}{4}(\varphi^4 - 2a\varphi^2 + a^2)$

$-\frac{\lambda}{4}\varphi^4 + \frac{\lambda a}{2}\varphi^2 - \frac{\lambda a^2}{4}$

wrong sign for mass if λ & $a > 0$

as many massless quarks & leptons as needed + built-in V-A weak interactions

@ $T \sim 10^{15}$ K, a 2nd order phase transition...

$B^{\mu 1} + \phi^+ \longrightarrow W^+$, massive

$B^{\mu 2} + \phi^- \longrightarrow W^-$, massive

propagators of weak interactions 3dof

$B^{\mu 3} + \phi^0 \longrightarrow W^0$ 3dof

$g \longrightarrow g$ no change, SU(3) unbroken.

$\phi^0 \longrightarrow h$, massive Higgs Boson

& also: $\begin{pmatrix} A^\mu \\ Z^\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} a^{\mu 1} \\ W^{\mu 0} \end{pmatrix}$

$\uparrow$

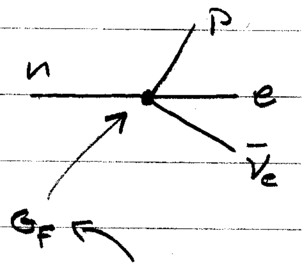
mass eigenstates $\longrightarrow$ QED interactions + new weak-electromagnetic interactions

mixing angle, Θ_W, is a measurable ("Weinberg angle")

$\Theta_W \sim 28°$

Historically, the weak interactions were characterized by Fermi's original idea:

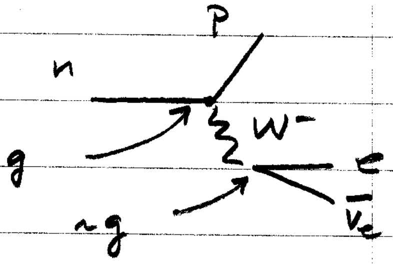
old way:



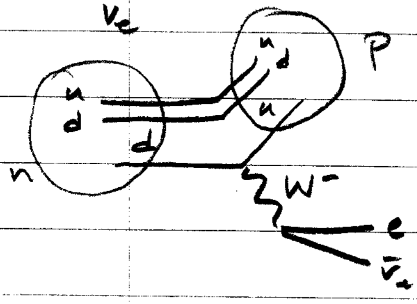
strength characterized by Fermi constant

$$G_F = 1.03 \times 10^{-5} m_p^{-2}$$

modern way: exploit the virtual "exchange" of W--



even more modern



unification links various constants..

$$g^2 = \frac{8M_W^2 G_F}{\sqrt{2}}$$ easy to see for heavy W propagator

$$g \sin\theta_W = e$$ ← explicit weak & electromagneti

The leptons and quarks align themselves into $SU(2)$ doublets and singlets ... $T_f = 1/2$

$$T_f^3 = \begin{array}{c} +1/2 \\ -1/2 \end{array} \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \begin{pmatrix} c \\ s \end{pmatrix}_L \begin{pmatrix} t \\ b \end{pmatrix}_L \qquad u_R, d_R, c_R, s_R, t_R, b_R$$

$$\begin{array}{c} +1/2 \\ -1/2 \end{array} \quad \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

$$\underbrace{\hspace{5cm}}_{\text{weak interactions}}$$

$$\underbrace{\hspace{10cm}}_{\text{electromagnetic interactions (except } \nu\text{'s)}}$$