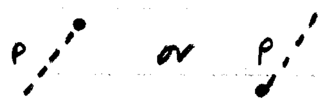
We're always calculating:

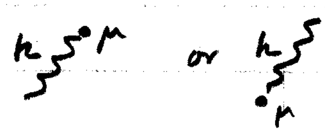
$$S_{fi} = \langle f | S^{(n)} | i \rangle = \delta_{fi} + \underbrace{(2\pi)^4 \delta(p - \sum_{i=1}^{n} k_i)\, T_{fi}^{(n)}}_{\text{what I've called } \mathcal{T}^{(n)}}$$

1. The invariant amplitude for the transition $T_{fi}^{(n)}$ is obtained by drawing all topologically distinct diagrams <u>without</u> disconnected bubbles <u>or</u> loops on <u>external</u> lines.
   - There will be as many vertices as order, n.
   - All fermion lines must be continuous.

2. For each external line, insert:
   - SPIN ϕ

      insert 1.

   - SPIN 1

      insert $\epsilon_\mu(k)$

   - SPIN 1/2

      insert $u^{(s)}(p)$, initial state, entering f

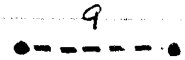
      " $\bar{u}^{(s)}(p)$, final state, leaving f

      " $\bar{v}^{(s)}(p)$, initial state, leaving $\bar{f}$

      " $v^{(s)}(p)$, final state, entering $\bar{f}$

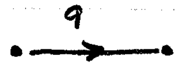
3. For each internal field (propagators)

- SPIN ϕ



insert $i\Delta_F(q) = \dfrac{i}{q^2 - m^2 + i\eta}$

- SPIN 1/2



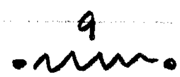
$iS_F(q) = \dfrac{i}{\not{q} - m + i\eta}$

- SPIN 1 massless

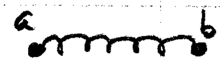
$\left(\begin{array}{ll} \zeta = 1 & \text{Feynman gauge} \\ = 0 & \text{Landau gauge} \\ = \infty & \text{Unitary gauge (Massive spin 1)} \end{array} \right.$

$iD_F(q)_{\mu\nu} = \dfrac{-i\left[g_{\mu\nu} + (\zeta - 1) q_\mu q_\nu / q^2\right]}{q^2 + i\eta}$

- SPIN 1 massive



$iD_F(q)_{\mu\nu} = \dfrac{i\left[-g_{\mu\nu} + (\zeta - 1) q_\mu q_\nu / (q^2 - M^2)\right]}{q^2 - M^2 + i\eta}$

- SPIN 1 gluon  $iD_F(q)_{\mu\nu} = -i\,\delta_{ab}\dfrac{\left[g_{\mu\nu} + (\zeta - 1) q_\mu q_\nu / q^2\right]}{q^2 + i\eta}$

4. Conserve overall 4-momentum

5. For any internal 4-momentum not constrained by 4-momentum conservation at each vertex, an integration must be performed,

$$\int_{-\infty}^{\infty} \frac{d^4p}{(2\pi)^4} \quad \to \text{loops.}$$

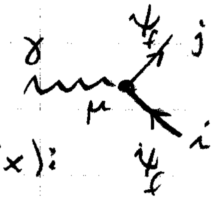
6. For every closed fermion loop, multiply by -1

7. For graphs which differ by an exchange of 2 external, identical fermion lines, a factor of -1 between them. Also, for exchange of initial particle (antiparticle) and final antiparticle (particle) lines.

8. For technical reasons, multiply entire amplitude by $(-i)$.

The rest of the rules deal with the couplings of one field with another → the models impact this part.

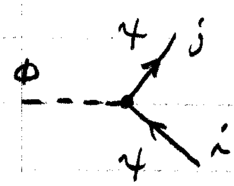
i) photon - fermion



$$\boxed{-iQ_f\gamma_\mu\delta_{ij}}$$

$$\mathcal{L} = Q_f :\bar{\psi}_j(x)\gamma^\mu\psi_i(x)A_\mu(x):$$

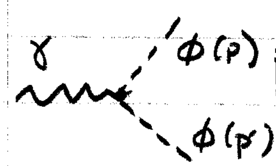
ii) pseudoscalar - fermion ("Yukawa coupling", originally πN)



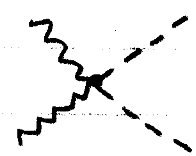
$$\boxed{g_0\gamma_5}$$

$$\mathcal{L} = -ig_0 :\bar{\psi}_j(x)\gamma_5\psi_i(x)\phi:$$

iii) scalar electrodynamics



$$\boxed{-ie(p+p')_\mu}$$

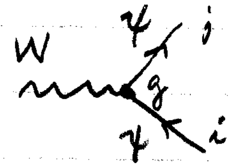


$$\boxed{2ie^2 g_{\mu\nu}}$$

$$\mathcal{L} = -ie :\phi^\dagger \overleftrightarrow{\partial_\mu}\phi A^\mu: + e^2 :A_\mu A^\mu \phi^\dagger\phi:$$

iv) Gauge couplings

classical weak interaction

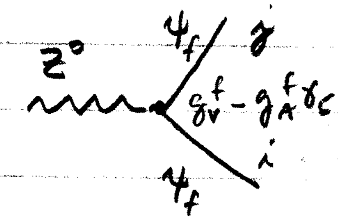


$$\boxed{\frac{-ig}{2\sqrt{2}} \gamma^\mu (1-\gamma_5)}$$

$$\mathcal{L} = -i \frac{g}{2\sqrt{2}} :\bar{\psi}_j \gamma^\mu (1-\gamma_5) \psi_i W_\mu : \quad i \neq j$$

note $\quad \dfrac{G_F}{\sqrt{2}} = \dfrac{g^2}{8M_W^2}$

electroweak neutral current



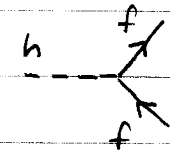
$$\boxed{\frac{-ig}{2\cos\theta_W} \gamma^\mu (g_V^f - g_A^f \gamma_5) \delta_{ij}}$$

$$\mathcal{L} = -i \frac{g}{\cos\theta_W} :\bar{\psi}_j \gamma^\mu \left[\tfrac{1}{2}(1-\gamma_5) T^3 - \sin^2\theta_W Q_f \right] \psi_i Z_\mu :$$

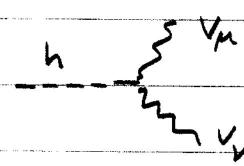
where $\quad g_V^f = T_f^3 - 2\sin^2\theta_W Q_f$

$\qquad\qquad g_A^f = T_f^3$

vii) Higgs Boson couplings



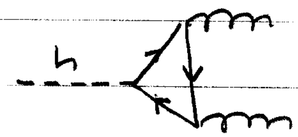
$-\frac{ig\,m_f}{2M_W}$ important

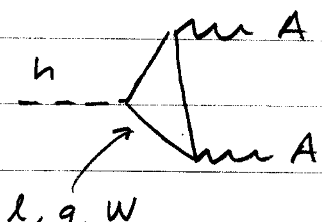


$iM_V\, g_{VH}\, g^{\mu\nu}$

$g_{Vh} = g \qquad V = W$

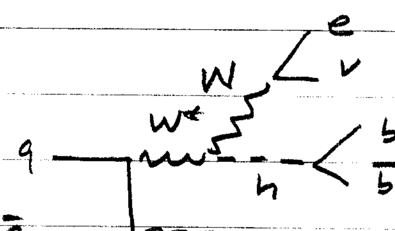
$\quad = \frac{g}{\cos^2\theta_W} \qquad V = Z$



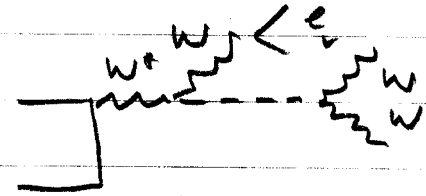


complicated

Fermilab discovery channels:



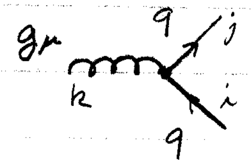
low mass Higgs ~ 120 GeV or less



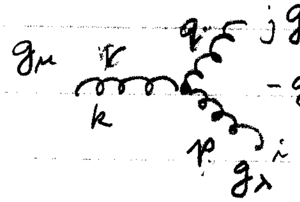
higher mass Higgs ~ > 160 GeV

v) gluon couplings

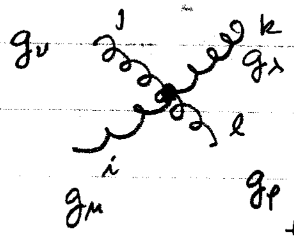
SU(3) generators $[T_a, T_b] = i f_{abc} T_c$ 321



$$-i g_s T^k_{ji} \gamma^\mu$$

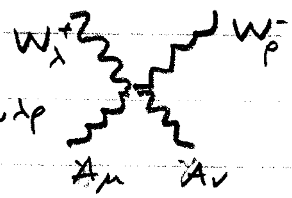


$$-g_s f^{ijk} \left[(p-q)_\lambda g_{\mu\nu} + (q-r)_\mu g_{\nu\lambda} + (r-p)_\nu g_{\lambda\mu} \right]$$

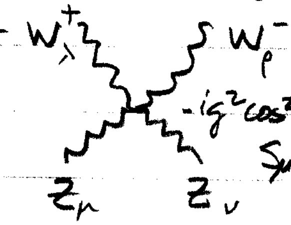


$$-i g_s^2 \left[f^{ijm} f^{k\ell m} (g_{\mu\lambda} g_{\nu\rho} - g_{\mu\rho} g_{\nu\lambda}) + f^{ikm} f^{\ell jm} (g_{\mu\rho} g_{\nu\lambda} - g_{\mu\nu} g_{\lambda\rho}) + f^{ikm} f^{j\ell m} (g_{\mu\nu} g_{\lambda\rho} - g_{\mu\lambda} g_{\nu\rho}) \right]$$

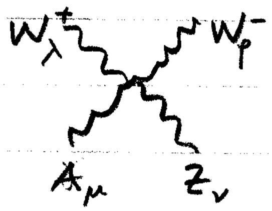
vi) quadralinear couplings



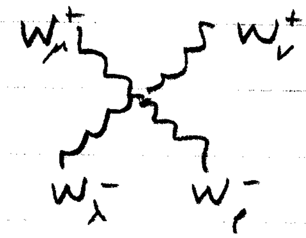
$$-ie^2 S_{\mu\nu,\lambda\rho}$$



$$-ig^2 \cos^2\theta_W \, S_{\mu\nu,\lambda\rho}$$



$$-ieg \cos\theta_W \, S_{\mu\nu,\lambda\rho}$$



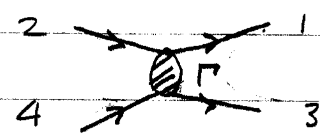
$$ig^2 S_{\mu\nu,\lambda\rho}$$

where

$$S_{\mu\nu,\lambda\rho} \equiv 2 g_{\mu\nu} g_{\lambda\rho} - g_{\mu\lambda} g_{\nu\rho} - g_{\mu\rho} g_{\nu\lambda}$$

Sometimes one calculation can serve the needs of another when fermions are involved due to the completeness of the Dirac equation solutions.

Suppose we have $\quad \mathcal{L}_I = \sum_i c_i \, (\bar{\psi}_1 \Gamma_i \psi_2)(\bar{\psi}_3 \Gamma_i \psi_4) + h.c.$

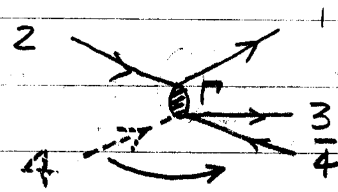


$\xrightarrow{\text{represent}} \quad \begin{pmatrix} 2 \\ 4 \end{pmatrix} \to \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$$2+4 \to 1+3$$

The same Lagrangian density would service the process

$$2 \to 1+3+\bar{4}$$



initial entering fermion, u/ψ^+ — final entering antifermion, v/ψ^-

Lots of shufflings are possible

$$\begin{pmatrix} 2 \\ 4 \end{pmatrix} \to \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 2 \\ \bar{1} \end{pmatrix} \to \begin{pmatrix} \bar{4} \\ 3 \end{pmatrix} \quad \text{etc.}$$

This is called Crossing Symmetry $\to$ a rearrangement of the ψ's and $\bar{\psi}$'s.

call $\quad K(1234) \equiv \bar{\psi}_1 \Gamma_i \psi_2 \bar{\psi}_3 \Gamma_i \psi_4 \quad$ for $\begin{pmatrix}2\\4\end{pmatrix} \to \begin{pmatrix}1\\3\end{pmatrix}$

then $\quad \begin{pmatrix}2\\4\end{pmatrix} \to \begin{pmatrix}3\\1\end{pmatrix}$ would be

$$K(3214) = \bar{\psi}_3 \Gamma_i \psi_2 \bar{\psi}_1 \Gamma_i \psi_4$$

Remember that there are 5 independent quantities for the Γ_i and 5 Lorentz scalars for $K(1234)$

$$\Gamma_i = \left\{ 1, \gamma^\mu, \frac{\sigma^{\mu\nu}}{\sqrt{2}}, \gamma^\mu\gamma^5, \gamma^5 \right\} \qquad \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$$

The Fierz Reordering Theorem says that

$$K_i'(3214) = \sum_{j=1}^{5} \lambda_{ij} K_j(1234)$$

where

$$\lambda_{ij} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 4 & -2 & 0 & 2 & -4 \\ 6 & 0 & -2 & 0 & 6 \\ 4 & 2 & 0 & -2 & -4 \\ 1 & -1 & 1 & -1 & 1 \end{pmatrix}$$

$j = 1 \;\; 2 \;\; 3 \;\; 4 \;\; 5$

If the Γ_j' in $K_j'(1234)$ are S, V, T, A, P, then the Γ_i in $K_i(3214)$ are related

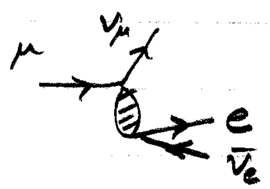
$$S' = -1/4\,(S + V + T + A + P)$$
$$V' = -1/4\,(4S - 2V + 2A - 4P)$$
$$T' = -1/4\,(6S - 2T + 6P)$$
$$A' = -1/4\,(4S + 2V - 2A - 4P)$$
$$P' = -1/4\,(S - V + T - A + P)$$

Only the following combinations are invariant wrt the Fierz reordering:

$$V' - A' = V - A$$
$$S' - T' + P' = S - T + P$$

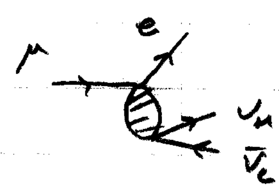
This was of interest in 1955 when it was enunciated when the nature of the weak interaction coupling was in dispute.

Calculations of, say, muon decay



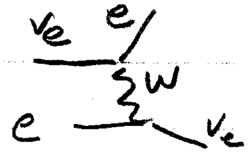
("charge exchange" form)

were made much simpler for a general S, T, V, S, A case in the following



("charge retention" form)

Later it allowed one to relate the charged current process



to the neutral current process