Lecture 15 Spin 1 Quantization

Classically:

$$\mathcal{L}(x) = -1/4\, F^{\mu\nu}F_{\mu\nu} \qquad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

and

conjugate to $A^i(x)$ (index up)

$$\pi^\mu(x) \Rightarrow \qquad \pi^i(x) = \frac{\partial \mathcal{L}}{\partial \dot{A}_i} = E^i(x) \quad \text{and}$$

$$\pi^0(x) = \frac{\partial \mathcal{L}}{\partial \dot{A}_0} = 0 \Rightarrow \text{there are no } \frac{\partial A^0}{\partial t}$$

terms in $\mathcal{L}$.

To quantize . .

$$[A^\mu, A^{\nu\prime}] = [\pi^i, \pi^{j\prime}] = [\pi^i, A^{0\prime}] = 0 \quad \text{no problem.}$$

$$[\pi^i(x,t), A_j(x',t)] = -\delta^i_j\, \delta(\vec{x}-\vec{x}') \quad \text{is a problem.}$$

why?

$$[\pi^i(x,t), A_j(x',t)] = -[\pi^i(x,t), A^j(x',t)]$$
$$= -[E^i, A^{j\prime}] = -i\,\delta^{ij}\,\delta(\vec{x}-\vec{x}')$$

For no sources $\vec{\nabla}\cdot\vec{E} = 0$ so

$$\left[\frac{\partial E^i(x,t)}{\partial x^i}, A^j(x',t)\right] = i\,\delta^{ij}\frac{\partial}{\partial x^i}\delta(\vec{x}-\vec{x}') \neq 0$$

you think, what about the Lorentz Gauge?

$$\partial_\mu A^\mu = 0$$

This indeed takes # independent dof from $4 \to 3$

The field vector must be arbitrary up to $\frac{\partial \theta}{\partial x_\mu}$

$$A^\mu \longrightarrow A^{\mu\prime} = A^\mu + \frac{\partial \theta}{\partial x_\mu}$$

and want $A^{\mu\prime}$ to satisfy Lorentz Gauge $\Rightarrow$

$$\frac{\partial^2 \theta}{\partial x^\mu \partial x_\mu} = 0$$

Can impose: $\quad \frac{\partial \theta}{\partial x^0} \equiv -\phi = -A^0 \quad \Rightarrow \quad \phi' = 0$

Then $\quad \frac{\partial A'^\mu}{\partial x^\mu} = \underbrace{\frac{\partial A'^0}{\partial x^0}}_{0} + \frac{\partial A'^i}{\partial x^i} = 0$

$\Rightarrow \quad \vec{\nabla} \cdot \vec{A}' = 0$ and we recover the Coulomb gauge

So, # dof, $3 \to 2$ .. which is the physical situation for photons.

This is the way photons are. So, we can start with Coulomb Gauge, or recover the Coulomb gauge.

So, $\quad \vec{\nabla} \cdot \vec{A} = 0 \quad \Rightarrow \quad 2$ dof.

So, we're back to the problem from a different perspective

$$-\left[E^i(x,t), \frac{\partial A^j}{\partial x^{j\prime}}(\vec{x}^{\prime},t)\right] = -i\,\delta^{ij}\frac{\partial}{\partial x^{j\prime}}\,\delta(\vec{x}-\vec{x}^{\prime}) \neq 0 \text{ still.}$$

$$\underset{\displaystyle 0}{\overset{\parallel}{}}$$

So, the quantization condition must be modified. to enforce RHS $\to 0$

$$[A_j(x\,t), \pi^i(x'\,t)] = \Big[-i\,\eta^i{}_j \underbrace{\int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}}_{\delta(\vec{x}-\vec{x}')}\Big]$$

some 2nd rank tensor.

now force RHS's gradient $\to 0$

$$\frac{\partial}{\partial x^i}[\quad] = -i\,\eta^i{}_j \int \frac{d^3k}{(2\pi)^3}\, i k_i\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} \to 0$$

$\Rightarrow$ choose

$$\eta^i{}_j\, k_i = 0$$

which is satisfied by

$$\eta^i{}_j = \delta^i{}_j - \frac{k^i k_j}{k^2} \qquad \text{i.e.}$$

$$k^j \eta^i{}_j = k^i - \frac{k^i k^2}{k^2} = 0$$

Define $\quad \delta^{tr\,i}{}_j(\vec{x}-\vec{x}') \equiv \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} \left(\delta^i{}_j - \frac{k^i k_j}{k^2}\right)$

and

$$[A_j(x,t), \pi^i(x',t)] = -i\,\delta^{tr\,i}{}_j(\vec{x}-\vec{x}')$$

This, in some sense, says quantize $\vec{A}$ as it is.

Then, in the standard way,

$$\vec{A}(x) = \int dk \sum_{\lambda=1}^{2} \vec{\epsilon}_\lambda(k)\left[a_\lambda(\vec{k})e^{-ik\cdot x} + a^\dagger_\lambda(\vec{k})e^{ik\cdot x}\right]$$

Then, $\quad \vec{\nabla}\cdot\vec{A} = 0 \quad \Rightarrow \quad \vec{k}\cdot\vec{\epsilon}_\lambda = 0 \quad \Rightarrow$ transverse photons ✓

BUT, this is unsatisfying as it's not manifestly Lorentz covariant. Moreover, there is essentially no canonical momentum to A^0 which one expects.

Indeed, one must have a quantization condition

$$[\pi_\mu(x,t), A_\nu(x',t)] = -i\,g_{\mu\nu}\,\delta(\vec{x}-\vec{x}')$$

while $A_\nu \rightarrow \hat{A}_\nu$, an operator, and one expects

$$\pi_\mu \rightarrow \hat{\pi}_\mu \quad - \quad \hat{A}_\nu \text{ commutes with } \pi_0 \text{ since } \pi_0 = 0$$

so, $\pi_i \rightarrow \hat{\pi}_i$ okay
π_0 is like a c-number $\Big\}$ modify $\mathcal{L}$

According to Fermi –

need to maintain $\Box A_\mu = 0$, the equation of motion.

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\lambda}{2} (\partial_\mu A^\mu)^2$$ "gauge-Fixing" term.

$$\frac{\partial A^\mu}{\partial x^\mu} = g_{\mu\nu} \frac{\partial A^\mu}{\partial x_\nu} \Rightarrow \left\{ \begin{array}{l} (\partial_\mu A^\mu)^2 = \\ -\frac{1}{2} g_{\mu\nu} \frac{\partial A^\mu}{\partial x_\nu} \frac{\partial A^\alpha}{\partial x^\alpha} \end{array} \right.$$

so,

$$\frac{\partial}{\partial x_\alpha} \left(\frac{\partial \mathcal{L}}{\partial A^\beta / \partial x_\alpha} \right) - \underset{\text{still} = 0}{\underset{\uparrow}{\frac{\partial \mathcal{L}}{\partial A^\beta}}} = 0$$

now leads to

$$\frac{\partial \mathcal{L}}{\partial (\partial A^\mu / \partial x_\nu)} = -F_{\nu\mu} - g_{\mu\nu} \partial_\alpha A^\alpha$$

so
$$\frac{\partial}{\partial x_\mu} (F_{\nu\mu} - g_{\mu\nu} \partial_\alpha A^\alpha) = \frac{\partial}{\partial x_\mu} \left(\frac{\partial A_\mu}{\partial x^\nu} - \frac{\partial A_\nu}{\partial x^\mu} - g_{\mu\nu} \frac{\partial A^\alpha}{\partial x^\alpha} \right)$$

$$= \partial^\mu (\partial_\nu A_\mu - \partial_\mu A_\nu - g_{\mu\nu} \partial_\alpha A^\alpha)$$

$$= \partial^\mu \partial_\nu A_\mu - \partial^\mu \partial_\mu A_\nu - \partial_\nu \partial_\alpha A^\alpha$$

$$= -\partial^\mu \partial_\mu A_\nu + \partial_\nu (\partial^\mu A_\mu - \partial_\alpha A^\alpha)$$

$\Box A_\nu = 0$ ✓ still get equations of motion. that's good.

The extra term is called the Gauge Fixing term..

sometimes this is made general

like a Lagrange multiplier – a constraint

$$\mathcal{L}(x) = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\lambda}{2} (\partial_\mu A^\mu)^2$$

w/ equation of motion:

$$\Box A^\mu - (1-\lambda)\, \partial^\mu (\partial_\alpha A^\alpha) = 0 \qquad \text{(not Maxwellian)}$$

$\lambda = 1$ "Feynman Gauge" (or "Fermi gauge")

$= 0$ "Coulomb Gauge"

<u>now</u> $$\pi^\mu = \frac{\partial \mathcal{L}}{\partial \dot{A}_\mu} = F^{\mu 0} - 2\left(\tfrac{\lambda}{2}\right)\left(\frac{\partial A^\alpha}{\partial x^\alpha}\right) \frac{\partial\, \partial_\rho A^\rho}{\partial(\partial_\nu A^\nu)}\, g^{\mu 0}$$

$$= F^{\mu 0} - \partial_\alpha A^\alpha g^{\mu 0}$$

$$\pi^0 = F^{00} - \partial_\alpha A^\alpha = -\partial_\alpha A^\alpha$$

<u>but</u>, this is <u>zero</u> in the L.G. ~~Stay tuned~~ — ~~of that is to relax the L.G. as a strict operator identity* to a statement about measurables.~~ –

$$\langle \psi | \partial_\mu A^\mu | \psi \rangle = 0 \longrightarrow \text{L.G. and Maxwell's Eq still hold as a classical limit.}$$

real states

classical vector field

* worked for $\partial_\mu A^\mu = 0$ okay – operator, not okay

So, now we deal with a covariant expansion -

$$A_\mu(x) = \int dK \sum_{\lambda=0}^{3} \epsilon_{\mu(\lambda)}(k)\left[a_\lambda(k)e^{-ik\cdot x} + a_\lambda^\dagger(k)e^{ik\cdot x}\right] \quad *$$

all 4

convention: $\epsilon^\mu_{(0)}$ = timelike vector

$\epsilon^\mu_{(i)}$ = spacelike vector.

orthonormality:

$$\epsilon_{(\lambda)}\cdot\epsilon_{(\lambda')} = \epsilon_{(\lambda)\mu}\cdot\epsilon^\mu_{(\lambda')}$$

$$= \epsilon_{(\lambda)\mu}\, g^{\mu\nu}\, \epsilon_{(\lambda')\nu}$$

$$= \epsilon_{(\lambda)0}\,\epsilon_{(\lambda')0} - \vec{\epsilon}_{(\lambda)}\cdot\vec{\epsilon}_{(\lambda')} \text{ and demand}$$

$$= g_{\lambda\lambda'} = -\zeta_\lambda\,\delta_{\lambda\lambda'} \quad \text{where } \zeta_0 = -1,\ \zeta_i = +1$$

completeness:

$$\sum_\lambda \zeta_\lambda\, \epsilon^\mu_{(\lambda)}\,\epsilon^\nu_{(\lambda)} = -g^{\mu\nu}$$

How do these look?

presume momentum along 3 axis -

$$k^\mu = (k, 0, 0, k)$$

and we can define

$$\epsilon_{(0)} = (1,0,0,0) \qquad \epsilon_{(1)} = (0,1,0,0) \qquad \epsilon_{(2)} = (0,0,1,0)$$

scalar or timelike photon — physical photon

$$\epsilon_{(3)} = (0,0,0,1)$$

longitudinal photon

The commutation relations for the a's we can hint at. -

go back

$$\pi^\mu = \frac{\partial \mathcal{L}}{\partial(\dot{A}_\mu)} = F^{\mu 0} - g^{\mu 0}\left(\frac{\partial A^\nu}{\partial x^\nu}\right)$$

or $$\pi^0 = F^{00} - g^{00}\left(\frac{\partial A^\nu}{\partial x^\nu}\right) = 0 - \frac{\partial A^0}{\partial x^0} + \vec{\nabla}\cdot\vec{A} = -\partial_\mu A^\mu$$

L.G $= 0$!

$$\pi^i = F^{i0} - 0 = \frac{\partial A^0}{\partial x_i} - \frac{\partial A^i}{\partial x_0}$$

hmm
that's not good.

so want $[A_\mu(x,t), \pi_\nu(x',t)] = i\, g_{\mu\nu}\, \delta(\vec{x}-\vec{x}')$

from $$[A_\mu(x,t), A_\nu(x',t)] = 0$$

$$\frac{\partial}{\partial x_i} \longrightarrow \left[\frac{\partial A_\mu}{\partial x_i}(x,t), A_\nu(x',t)\right] = 0$$

part of $[A,\pi]$ story

$\rightarrow$ spatial derivatives commute at equal time.

So, from

$$[A_\mu(x,t), \pi_\nu(x',t)] = i\, g_{\mu\nu}\, \delta^3(\vec{x}-\vec{x}')$$

only the time derivatives of A inside π_ν won't commute, so

$$[A_\mu(x,t), \dot{A}_\nu(x',t)] = -i\, g_{\mu\nu}\, \delta^3(\vec{x}-\vec{x}')$$

So, what's wrong?

We have $\quad [\hat{A}^{\mu}_{\mu}(x,t), \hat{\pi}_{\nu}(x't)] = -i g_{\mu\nu}\, \delta(\vec{x}-\vec{x}')$

and $\quad [\hat{A}_{\mu}(x,t), \hat{A}_{\nu}(x't)] = 0$

If we're truly quantized, we would like for L.G. condition:

$$\partial_\mu A^\mu = 0 \longrightarrow \partial_\mu \hat{A}^\mu = 0$$

But look..

$$[\hat{A}_\mu, \hat{\pi}_\nu] = [\hat{A}_\mu, \hat{F}'_{\nu 0} - g_{\nu 0}(\partial'_\alpha A'^\alpha)]$$

$$= [\hat{A}_\mu, \partial'_\nu A'_0 - \partial_0 A'_\nu - g_{\nu 0}(\partial'_\alpha A'^\alpha)] = -i g_{\mu\nu}\, \delta(\ )$$

look at $\nu = i$

$$[\hat{A}_\mu, \partial'_i A'_0 - \partial_0 A'_i - 0] = -i g_{\mu i}\, \delta(\ )$$

$$[\hat{A}_\mu, \partial'_i A'_0] - [\hat{A}_\mu, \partial_0 A'_i] = -i g_{\mu i}\, \delta(\ )$$

$$\partial'_i \underbrace{[\hat{A}_\mu, A'_0]}_{=0} - [\hat{A}_\mu, \partial_0 A'_i] =$$

$\mu = i$: $\quad - [\hat{A}_i, \partial_0 A'_i] = i\delta(\ )$

$\mu = 0$

$\quad - [A_0, \partial_0 A'_i] = 0$

hold at $\nu = 0$

$$[\hat{A}_\mu, \cancel{\partial_0 \hat{A}'_0} - \partial_0 \hat{A}'_0 - \partial'_\alpha A^{\alpha'}] = -i\, g_{\mu 0}\, \delta(\quad)$$

$\mu = 0$

$$[\hat{A}_0, -\partial'_\alpha \hat{A}^{\alpha'}] = -i\, \delta(\quad)$$

$$[\hat{A}_0, -\partial_0 \hat{A}'^0] - [\hat{A}_0, \partial'_i \hat{A}'^i] =$$

$$[\hat{A}_0, -\partial_0 \hat{A}'^0] - \partial'_i \underbrace{[\hat{A}_0, \hat{A}'^i]}_{=0} =$$

$$[\hat{A}_0, -\partial_0 \hat{A}'^0] = -i\, \delta(\quad) \qquad \star$$

$\mu = i$

$$[\hat{A}_i, \partial'_\alpha A^{\alpha'}] = 0$$

$$[\hat{A}_i, \partial_0 \hat{A}^{0'}] - \partial'_j \underbrace{[\hat{A}_i, \hat{A}^{j'}]}_{0} = 0$$

$$[\hat{A}_i, \partial_0 \hat{A}^{0'}] = 0 \qquad \star$$

all $\star \Rightarrow$

$$\star\star \quad [\hat{A}_\mu(x,t), \dot{A}_\nu(x',t)] = i\, g_{\mu\nu}\, \delta(\vec{x} - \vec{x}')$$

So, first of all, we have circumstances where $\partial_\alpha \hat{A}^\alpha = 0$ does not hold as an operator identity also, $$[\hat{A}_\mu , \hat{\pi}_\nu'] = -i g_{\mu\nu} \delta(\vec{x} - \vec{x}')$$

in particular

$$[\hat{A}_0, \hat{\pi}_0'] = -i \delta(\vec{x} - \vec{x}') \neq 0$$

$$\cancel{[\hat{A}_\mu, -\partial A}$$

$$[\hat{A}_0, \partial_\alpha' \hat{A}'^\alpha] \neq 0$$

$$\overset{\shortparallel}{0} \text{ L.C.}$$

The above is equivalent to saying

$$[\hat{A}_\mu(x,t),\ \partial'_\alpha \hat{A}^{\alpha\prime}] = -i g_{\mu 0}\, \delta(\vec{x}-\vec{x}') \neq 0$$

not consistent w/ L.G.

Also, the noncommutivity of <u>only</u> the time derivatives leads to trouble

From ★★

substitute the expansions for A_μ and $\dot{A}_\nu$ --

...

$$[\, a_{(\lambda)}(k), a^{\dagger}_{(\lambda')}(k') \,] = -g_{\lambda\lambda'}\, 2k_0\, (2\pi)^3 \delta(\vec{k}-\vec{k}')$$

negative sign is problematic - esp for timelike

consider a timelike (scalar), 1 photon state,

$$|k_{(0)}\rangle = \int dK\, f_k(x)\, a^{\dagger}_{(0)}(k)|0\rangle$$

find the norm

$$\langle k_{(0)}|k_{(0)}\rangle = \int dK \int dK'\, f_k f^{*}_{k'} \langle 0|a_{(0)}(k')\, a^{\dagger}_{(0)}(k)|0\rangle$$

Use commutation relations -

$$= \int dK \int dK'\, f_k f^{*}_{k'} \langle 0| -2k_0 (2\pi)^3 \delta(\vec{k}-\vec{k}')|0\rangle$$

$$= -\int dK \int d^3k\, f_k f^{*}_{k'}\, \delta(\vec{k}-\vec{k}') \langle 0|0\rangle$$

$$= -\int dK\, |f_k|^2 \quad \rightarrow \text{ a negative norm!}$$

for 1 time-like photon state - norm

for 2 time-like photon state + norm

3 " - norm.

→ indefinite!

To add insult to injury,

$$N_{(0)}(k) = -a^{\dagger}_{(0)}(k)\, a_{(0)}(k)$$

so that the timelike photon contribution to the energy is negative.

The way out is to demand a weakened "subsidiary condition": $\partial_\mu \hat{A}^\mu = 0$ as an operator identity is too strong — use physical states.

$$\frac{\partial \hat{A}^{(+)\mu}}{\partial x^\mu} |\psi\rangle = 0 \qquad \text{(already satisfied for the vacuum, } \hat{a}|0\rangle = 0\text{)}$$

Since then for $\langle\psi| \frac{\partial \hat{A}^\mu}{\partial x^\mu} |\psi\rangle \overset{\text{want}}{=} 0$ ✓

$$= \langle\psi| \underbrace{\frac{\partial A^{(+)\mu}}{\partial x^\mu}}_{\text{demand } = 0} + \frac{\partial A^{(-)\mu}}{\partial x^\nu} |\psi\rangle \qquad \leftarrow \hat{a}^{+} \to 0$$

$$= \langle\psi| \frac{\partial A^{(-)\mu}}{\partial x^\nu} |\psi\rangle = \langle\psi| \frac{\partial A^{(+)\mu}}{\partial x^\nu} |\psi\rangle^* = 0$$

Called Gupta-Bleuler quantization - 1950.

put expansion for $A^{(+)\mu}$ into G.B. condition above, from *

$$\sum_{\lambda=0}^{3} k^\mu \epsilon_{(\lambda)\mu}\, a_{(\lambda)}(k) |\psi\rangle = 0$$

all 4 — need to make this happen

for γ in k_3 direction, for example,

$$k^\mu = (k, 0, 0, k) \quad \text{so} \quad k \cdot \epsilon_{(1,2)} = 0 \qquad \checkmark$$

we get for G.B. condition:

$$\left[k^\mu \epsilon_{(0)\mu}\, a_{(0)}(k) + k^\mu \epsilon_{(3)\mu}\, a_{(3)}(k) \right] |\psi\rangle = 0$$

from the definition of ϵ_0 and ϵ_3,

$$k^\mu \epsilon_{(0)\mu} = - k^\mu \epsilon_{(3)\mu} \quad \text{so,}$$

$$k^\mu \epsilon_{(0)\mu} \left[a_{(0)}(k) - a_{(3)}(k) \right] \cdot |\psi\rangle = 0$$

$$\text{so} \qquad \left(a_{(0)}(k) - a_{(3)}(k) \right) |\psi\rangle = 0$$

serves as a <u>constraint</u> on the timelike and longitudinal pieces.

<u>Physical</u> <u>states</u> <u>will</u> <u>be</u> considered to be <u>admixtures</u> of <u>these 2 so</u> that the constraint holds. → there are no 1 particle physical states of only timelike photons

What does this mean practically. —

The Hamiltonian can be constructed in the standard fashion to be,

$$H = \sum_{\lambda=0}^{3} \int dK \, \zeta_\lambda \, a^+_{(\lambda)}(\vec{k})\, a_{(\lambda)}(\vec{k}) \; k_0$$

Even though $\zeta_0 = -1$ .. the energy is positive definite

ie $$\int dK\, k_0 \Big\{ \sum_{\lambda=1}^{3} a^+_{(\lambda)}(k)\, a_{(\lambda)}(k) - a^+_{(0)}(k)\, a_{(0)}(k) \Big\}$$

What about $\langle \psi | H | \psi \rangle$?

$$= \langle \psi | \int dK\, k_0 \Big\{ \sum_{i=1}^{2} a^+_{(i)}(k)\, a_{(i)}(k) + a^+_{(3)}(k)\, a_{(3)}(k) - a^+_{(0)}(k)\, a_{(0)}(k) \Big\} | \psi \rangle$$

The subsidiary condition says

$$a_{(0)}(k) | \psi \rangle = a_{(3)}(k) | \psi \rangle.$$

Its adjoint says $\quad \langle \psi | a^+_{(0)}(k) = \langle \psi | a_{(3)}(k)$

so $$\langle \psi | a^+_{(0)} a_{(0)} | \psi \rangle = \langle \psi | a^+_{(0)} a_{(3)} | \psi \rangle$$

$$= \langle \psi | a^+_{(3)} a_{(3)} | \psi \rangle$$ cancels

so
$$\langle \psi | H | \psi \rangle = \langle \psi | \int dK\, k_0 \Big\{ \sum_{i=1}^{2} a^+_{(\lambda)}(k)\, a_{(\lambda)}(k) \Big\} | \psi \rangle$$

$\longrightarrow$ only the physical states contribute: 2 dof.

A word about massive spin 1 fields --

In your problem set you'll find that for spin 1 massive fields, B_μ, that $\dfrac{\partial B^\mu}{\partial x^\mu} = 0$

THIS IS NOT A GAUGE CONDITION -- only a constraint which comes from the equation of motion - Proca Equations

An expansion of the sort

$$B_\mu = \int dK \sum_{\lambda=0}^{3} \left[\epsilon_{(\lambda)\mu}(k)\, a_\lambda(k)\, e^{-ik\cdot x} + \epsilon_{(\lambda)\mu}\, a_\lambda^\dagger(k)\, e^{ik\cdot x} \right]$$

4d

The constraint $\Rightarrow$ $\quad k \cdot \varepsilon_{(\lambda)} = 0 \quad$ 4 components $\to$ 3 ✓

From $\quad \mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{1}{2} M^2 B_\mu B^\mu$ $\Big\}$ gives K.G. equation called Proca Equation.

we get $\quad \pi^\mu = \dfrac{\partial \mathcal{L}}{\partial \dot{B}_\mu} = \partial^\mu B^0 - \dot{B}^\mu \Rightarrow \pi^0 = \partial^0 B^0 - \partial^0 B^0 = 0$ still.

now using $\partial^\mu B_\mu = 0 \Rightarrow$ we can eliminate a $B \to B^0$

so, $\quad \pi^i = -\dot{B}^i$

and

$$[B^i(x,t), \pi_j(x',t)] = i\,\delta^i{}_j\, \delta^3(\vec{x}-\vec{x}')$$

becomes,

$$[\dot{B}_i(\vec{x},t), B_j(\vec{x}',t)] = i\, g_{ij}\, \delta^3(\vec{x}-\vec{x}')$$

w/ expansion substituted,

⋮

$$[\,a_{(\lambda)}(k),\, a_{(\lambda')}(k')\,] = \delta_{\lambda\lambda'}\, 2k^0 (2\pi)^3 \delta^3(\vec{k}-\vec{k}')$$

no negative sign. $\longrightarrow$ all is good because Π_μ is different.

Polarization tensor is different.:

In the rest frame $\quad k\cdot\epsilon = 0$ is satisfied by

$$k^\mu = (M, 0, 0, 0)$$

$$\left.\begin{array}{l} \epsilon^\mu_{(1)} = (0,1,0,0) \\ \epsilon^\mu_{(2)} = (0,0,1,0) \end{array}\right\} \text{usually spherical used} \quad \begin{array}{l} \vec{\epsilon}_{(+)} = -\frac{1}{\sqrt{2}}(1, i, 0) \\ \vec{\epsilon}_{(-)} = \frac{1}{\sqrt{2}}(1, -i, 0) \end{array}$$

$$\epsilon^\mu_{(3)} = (0,0,0,1)$$

called longitudinal, but in spherical basis refered to as $\quad \epsilon^\mu_{(0)} \equiv (0,0,0,1)$

$$\left\{\begin{array}{l} k\cdot\epsilon_{(\lambda)} = 0 = k^0 \epsilon_{0(\lambda)} + \sum_i k^i \epsilon_{i(\lambda)} \\ \text{rest frame: } \underline{\text{define}}\ \epsilon_0^{(\lambda)} = 0 \Rightarrow k\cdot\epsilon = M(0) + \sum_i (0)\epsilon_i = 0 \end{array}\right.$$

Boost along 3-direction: $\quad k \to k^\mu = (E, 0, 0, \vec{k})$

for ϵ:

$$\epsilon^\mu_{(\lambda)}(k_{rest}) = (0, \vec{\epsilon}_{(\pm)})$$

$$\xrightarrow{\text{BOOST } 3} \quad \epsilon^\mu_{(\pm)}(k) = \epsilon^\mu_{(\lambda)}(k_{rest}) = (0, \vec{\epsilon}_{(\pm)})$$

what about $\lambda = 0$?

$$k \cdot \epsilon_{(0)} = E\,\epsilon^0_{(0)} - k^3\,\epsilon^3_{(0)} = 0$$

<u>construct</u> :
$$\epsilon^\mu_{(0)} \equiv (k^3/M,\ 0,\ 0,\ E/M)$$
$$= (\gamma\beta,\ 0,\ 0,\ \gamma)$$

then, $$k \cdot \epsilon_{(0)} = E\,(k^3/M) - k^3\,(E/M) = 0$$

So, we summarize

$$\epsilon^\mu_{(\pm)} = \mp\frac{(0,\ 1,\ \pm i,\ 0)}{\sqrt{2}} \qquad \text{transverse}$$

$$\epsilon^\mu_{(0)} = (\gamma\beta,\ 0,\ 0,\ \gamma) \qquad \text{longitudinal}$$

both necessary for real spin 1 massive.