Worksheet #2 - PHY102 (Spr. 2004)

Formats and List operations (Vectors)

Formating

From the toolbar do the following: “format” followed by “style”, the dropdown menu offers you lots of options. “input” is the default, and is the “format” in which you do calculations. However *Mathematica* also allows various fonts and styles of text input. In this problem set you should include a text “cell” before each problem which identifies the problem (e.g. Problem 1), and a text cell at the top of the page with your name and the number of the worksheet (e.g. “worksheet 2”).

Last week we did derivatives and integrals using the full *Mathematica* commands. However many of these commands may be entered from “palettes”. To activate a palette, from the toolbar do the following: “file” followed by “palettes”. You have several options: “basic input” is one I like.

Lists and Vectors

By now you must have read about vectors. A vector is a quantity which, unlike a scalar, can have many components. For example in Newton’s second law of motion

$$\vec{F} = m\frac{d^2\vec{r}}{dt^2} \tag{1}$$

the quantity m (mass) is a scalar. But the force $\vec{F}$ and the acceleration $\vec{a} = \frac{d^2\vec{r}}{dt^2}$ are vectors. As you can see in Eq. (1), and which is true in general, multiplying a vector $\vec{a}$ with a scalar m, gives a vector $\vec{F}$. A vector is described by its components in a chosen co-ordinate system. For example a vector $\vec{A}$ in cartesion co-ordinates is given by, $\vec{A} = (A_x, A_y, A_z)$.

In Mathematica vectors are represented in the same way. In Mathematica this object is called a *list*, because it can be used for more general objects such as matrices and tensors. In this worksheet we just work with vectors.

Type “A = {Ax, Ay, Az}”. This means Mathematica associates the object A with the list {Ax, Ay, Az}. Now type “B = {Bx, By, Bz}”. Type “Dot[A,B]”. This will give the dot product $\vec{A}\cdot\vec{B}$=AxBx+AyBy+AzBz which is the same as $|A||B|cos(\theta)$, where θ is the angle between the vectors $\vec{A}$ and $\vec{B}$.

Likewise, the cross product of two vectors ($\vec{A} \times \vec{B}$) yields another vector $\vec{C}$ = {AyBz-AzBy, AzBx-AxBz, AxBy-AyBx}. "Type Cross[A,B]" and verify that you indeed get the above expression in terms of the components of $\vec{A}$ and $\vec{B}$. Unit vectors can be easily written with lists as: $\hat{x}$ = {1,0,0}, $\hat{y}$ = {0,1,0}, $\hat{z}$ = {0,0,1}. Check with Mathematica that $\hat{x}\cdot\hat{y} = \hat{y}\cdot\hat{z} = \hat{z}\cdot\hat{x} = 0$.

You can see that the elements in the list {Ax, Ay, Az} of the vector $\vec{A} = \hat{x}A_x + \hat{y}A_y + \hat{z}A_z$ are its x, y, and z components. How do we access the individual components from A?.

Type "A[[2]]" and check that this gives Ay. How would you get *Mathematica* to print out the second element of the cross product "Cross[A,B]"?

Assignment 2. - Hand in by Monday Feb. 2nd

Problem 1. Consider two vectors $\vec{A} = (\frac{\sqrt{3}}{2}, \frac{1}{2}, 0)$, and $\vec{B} = (\frac{1}{2}, \frac{\sqrt{3}}{2}, 0)$. Using *Mathematica*:
(i) Check that they are both of unit magnitude.
(ii) Find $\vec{A} \cdot \vec{B}$.
(iii) Find the angle between these two vectors.
(iv) Find the cross pruduct of these two vectors.

Problem 2. Consider the unit vectors along x, y, and z directions: $\hat{x}$ = {1,0,0} $\hat{y}$ = {0,1,0} $\hat{z}$ = {0,0,1}. Verify: $\hat{x} \times \hat{y} = \hat{z}$, $\hat{y} \times \hat{z} = \hat{x}$, $\hat{z} \times \hat{x} = \hat{y}$.

Problem 3. Verify that for any three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ that $\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$.

Problem 4. The motion of a particle is given by $\vec{r}(t)$=a($\hat{x}$cos(ωt) + $\hat{y}$sin(ωt)) Find its velocity $\vec{v}$. Calculate $\vec{\Omega} \times \vec{r}$, where $\vec{\Omega} = (0, 0, \omega)$, and and verify $\vec{v} = \vec{\Omega} \times \vec{r}$. Do you recognise this motion? Plot the motion to confirm your intuition (use the help menu to look up how to use the command "ParametricPlot" for this problem - you will need to choose values for a and ω.).