Physics 492 homework V, due Fri Feb 20

Reading: Chapters 4, 5

Problems:

1. In the semiclassical limit, the Fermi energy of an ideal gas of $\mathcal{N}$ identical spin-1/2 particles with mass m in a volume V is

$$E_F(\mathcal{N}) = \frac{\hbar^2}{2m}\left(\frac{3\pi^2\mathcal{N}}{V}\right)^{2/3} .$$

Consider a nucleus with Z protons, $N = A - Z$ neutrons and radius $R = r_0\,A^{1/3}$, where $r_0 =$ 1.12 fm. In the ideal-gas model, the total internal kinetic energy of the nucleus, in terms of Fermi energies for protons and neutrons, is

$$E = \frac{3}{5}\,Z\,E_F(Z) + \frac{3}{5}\,N\,E_F(N)\,.$$

(a) Determine E_F and E for $^{16}_{8}$O.

(b) If $|N - Z| \ll A$, then

$$E \approx E_0 + a_A\,\frac{(N-Z)^2}{A}\,,$$

where $E_0 = \frac{3}{5}\,A\,E_F(A/2)$ is the energy of a symmetric nucleus with $N = Z = A/2$. Determine the value of a_A in the ideal-gas limit.

Hint: Write

$$N = \frac{A}{2} + \epsilon \qquad \text{and} \qquad Z = \frac{A}{2} - \epsilon\,,$$

where

$$N - Z = 2\epsilon\,,$$

and expand the ideal-gas energy in ϵ. Be careful in retaining the proper order of expansion. (This is a modified Problem 4.3 in Williams.)

2. Williams, Problem 5.1. Use the coefficient values given in class, i.e. $a_V = 15.85$ MeV, $a_S = 18.34$ MeV, $a_A = 23.22$ MeV and $a_C =$ 0.71 MeV. Note that to maintain the unit consistency, the mass formula in Williams should be actually written as

$$\begin{aligned} M'(Z,A)\,c^2 \;&=\; Z\,m_H\,c^2 + N\,m_n\,c^2 - a_V\,A \\ &\quad + \ldots - \delta\,. \end{aligned}$$

3. Williams, Problem 5.4.

4. Williams, Problem 5.5. Hint: Calculate a_C from the Q value of the β^+ decay of $^{35}_{18}$Ar. Estimate a_A using the fact that $^{135}_{56}$Ba is stable and thus the Q values of β^+ and β^- decays must be negative; $Q(\beta^+) < 0$ and $Q(\beta^-) <$ 0 imply upper and lower bounds on a_A, after substituting the value of a_C.