Problem Set #2

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PHY 854, Spring Semester, 2004

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These problems are due on Friday, April 23, 2003 5:00 p.m. to my mailbox on the first floor of BPS. Note, $\hbar = c = 1$.

Problem 11 *This was pretty dumb, since I left it in my starting point for this second problem set! Happy birthday.* Define left-handed and right-handed components and show that the Lagrangian density for free spin-$\frac{1}{2}$ fields

$$\mathcal{L}(x) = \bar{\psi}(x)\left(i\gamma^\mu\partial_\mu - m\right)\psi(x)$$

can be written

$$\mathcal{L}(x) = \bar{\psi}_L(x)i\gamma^\mu\partial_\mu\psi_L(x) + \bar{\psi}_R(x)i\gamma^\mu\partial_\mu\psi_R(x) - m\left(\bar{\psi}_R(x)\psi_L(x) + \bar{\psi}_L(x)\psi_R(x)\right).$$

Problem 12 The matrix element for the elastic scattering of an electron from a Coulomb potential is

$$\bar{u}(k')\frac{\gamma^0}{q^2}u(k)$$

where the k momentum is along the 3-axis and is the initial momentum of the electron and the k' momentum is the final momentum and is inclined at an angle of θ with respect to the 3-axis. q is the 4-momentum transfer, $q = k - k'$. Since the scattering is elastic, the particle *in* is the same electron as the particle *out*, so $E = E'$. Show that for

$$(+\text{helicity}) \to (-\text{helicity})$$

scattering that the amplitude is

$$A_{\uparrow\downarrow} \propto 2m\sin\frac{\theta}{2}.$$

Problem 13 Show that $\mathrm{Tr}(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu}$.

Problem 14 Show that $\overline{\not\epsilon\,'\not k\not\epsilon} = \not\epsilon\not k\not\epsilon\,'$.

Problem 15 The invariant amplitude for Compton scattering which we derived in class was the following:

$$T_{fi} = \bar{u}^{(f)}(p') \left[\not{\epsilon}' \frac{\not{p} + \not{k} + m}{2p \cdot k} \not{\epsilon} - \not{\epsilon} \frac{\not{p} - \not{k}' + m}{2p \cdot k'} \not{\epsilon}' \right] u^{(i)}(p)$$

where each term came from one of the two possible Feynman diagrams. Show that both diagrams are necessary in order to insure Gauge Invariance.