So,

$$* \quad S^{(2)}(e\gamma \to e\gamma) = \left(\frac{-ie}{2}\right)^2 \int d^4x_1 \int d^4x_2 \langle e\gamma | \gamma^\mu_{ij} \gamma^\nu_{\ell m} \{ \bar{\Psi}_i^-(x_1) \Psi_m^+(x_2) A_\mu(x_1) A_\nu(x_2)$$

$$* \quad \times \langle 0 | T[\Psi_j(x_1) \bar{\Psi}_\ell(x_2)] | 0 \rangle + \bar{\Psi}_\ell^-(x_2) \Psi_j^+(x_1) A_\mu(x_1) A_\nu(x_2)$$
$$\times \langle 0 | T[\bar{\Psi}_m(x_1) \Psi_i(x_2)] | 0 \rangle \} | e\gamma \rangle$$

We need to deal with the contracted terms

eg, what is $\langle 0 | T[\Psi_j(x_1) \bar{\Psi}_\ell(x_2)] | 0 \rangle$

We know it's

$$T[\Psi_j(x_1) \bar{\Psi}_\ell(x_2)] = \Psi_j(x_1) \bar{\Psi}_\ell(x_2)\, \theta(x_{10} - x_{20}) - \bar{\Psi}_\ell(x_2) \Psi_j(x_1)\, \theta(x_{20} - x_{10}) \qquad (3)$$

But, let's see it happen naturally. —

look at 1st term $x_{10} > x_{20}$

$$\langle 0 | T(\Psi_j(x_1) \bar{\Psi}_\ell(x_2)) | 0 \rangle = \langle 0 | T(\underbrace{\Psi_j^+ \bar{\Psi}_\ell^+ + \Psi_j^+ \bar{\Psi}_\ell^- + \Psi_j^- \bar{\Psi}_\ell^+ + \Psi_j^- \bar{\Psi}_\ell^-}_{\text{already time ordered}}) | 0 \rangle$$

$$\propto \langle 0 | \underset{=0}{ab} + aa^+ + \underset{=0}{b^+b} + \underset{=0}{b^+a^+} | 0 \rangle$$

$$(1) \qquad = \langle 0 | \Psi_j^+(x_1) \bar{\Psi}_\ell^-(x_2) | 0 \rangle \qquad x_{10} > x_{20}$$

$x_{20} > x_{10}$

$$\langle 0|T(\psi_j(x_1)\bar{\psi}_\ell(x_2))|0\rangle = \langle 0|T(\psi_j^+\bar{\psi}_\ell^+ + \psi_j^+\bar{\psi}_\ell^- + \psi_j^-\bar{\psi}_\ell^+ + \psi_j^-\bar{\psi}_\ell^-)|0\rangle$$

$$= - \langle 0|\bar{\psi}_\ell^+\psi_j^+ + \bar{\psi}_\ell^-\psi_j^+ + \bar{\psi}_\ell^+\psi_j^- + \bar{\psi}_\ell^-\psi_j^-|0\rangle$$

$$\propto \langle 0|\underset{0}{\underset{\shortparallel}{ba}} + \underset{0}{\underset{\shortparallel}{a^+a}} + \underset{0}{\underset{\nparallel}{bb^+}} + \underset{0}{\underset{\shortparallel}{a^+b^+}}|0\rangle$$

$$= -\langle 0|\bar{\psi}_\ell^+(x_2)\psi_j^-(x_1)|0\rangle \qquad x_{20} > x_{10} \quad ②$$

Look explicitly at the first one $\langle 0|\psi_j^+(x_1)\bar{\psi}_\ell^-(x_2)|0\rangle$ ①

$$\psi_j^{(+)}(x_1) = \sum_{i=1}^{2}\int dK\; a^{(i)}(k)u_j^{(i)}(k)e^{-ik\cdot x_1}$$

$$\bar{\psi}_\ell^-(x_2) = \sum_{m=1}^{2}\int dK'\; \bar{u}_\ell^{(m)}(k')a^{+(m)}(k')e^{ik'\cdot x_2}$$

$$\langle 0|\psi_j^+(x_1)\bar{\psi}_\ell^-(x_2)|0\rangle = \sum_i\sum_m\int\frac{d^3k}{(2\pi)^3 2E}\int\frac{d^3k'}{(2\pi)^3 2E'}\,u_j^{(i)}(k)\bar{u}_\ell^{(m)}(k')$$
$$\times e^{-i(k\cdot x_1 - k'\cdot x_2)}\langle 0|a^{(i)}(k)a^{+(m)}(k')|0\rangle$$

use anticommutator by adding

$\langle 0|a^+a|0\rangle$ $(=0)$

$(2\pi)^3 2E\,\delta(\vec{k}-\vec{k}')\,\delta_{im}$

do $\int d^3k'$ integrations

$$= \sum_i \int dK \quad u_j^{(i)}(k)\, \bar{u}_\ell^{(i)}(k)\, e^{-ik\cdot(x_1-x_2)}$$

use completeness.. to project out the + energy particles

$$= \int dK \; (\not{k}+m)_{j\ell}\, e^{-ik\cdot(x_1-x_2)} \qquad x_{10} > x_{20} \qquad ①$$

The second one

$$② \quad \langle 0|\, \bar{\Psi}_\ell^{+}(x_2)\, \Psi_j^{-}(x_1)\,|0\rangle = \sum_i \sum_m \int dK \int dK' \;\; \bar{v}_\ell^{(i)}(k')\, v_j^{(m)}(k)\, e^{-i(k\cdot x_2 - k'\cdot x_1)}$$

$$\times \langle 0|\, b^{(i)}(k)\, b^{+(m)}(k')\,|0\rangle$$

since
$$\Psi_j^{-}(x_1) = \sum_{i=1}^{2} \int dK'\; b^{+(i)}(k')\, v_j(k')\, e^{ik'\cdot x_1}$$

$$\bar{\Psi}_\ell^{+}(x_2) = \sum_{m=1}^{2} \int dK \;\; \bar{v}_\ell^{(m)}(k)\, b^{(m)}(k)\, e^{-ik\cdot x_2}$$

In the same fashion

$$\langle 0|\, \bar{\Psi}_\ell^{+}(x_2)\, \Psi_j^{-}(x_1)\,|0\rangle = \sum_i \int dK \;\; v_j^{(i)}(k)\, \bar{v}_\ell^{(i)}(k)\, e^{ik\cdot(x_1-x_2)}$$

$$= \int dK \; (\not{k}-m)_{j\ell}\, e^{ik\cdot(x_1-x_2)} \qquad ②$$

So, going all the way back.. remembering the negative sign,

$$\langle 0|T(\psi_j(x_1)\bar\psi_\ell(x_2))|0\rangle \quad ③$$

$$= \int dK\, (\not k+m)_{j\ell}\, e^{-ik\cdot(x_1-x_2)}\,\theta(x_{10}-x_{20})$$

$$- \int dK\, (\not k-m)_{j\ell}\, e^{ik\cdot(x_1-x_2)}\,\theta(x_{20}-x_{10})$$

Look at the second term...

$$-\int dK\,(\gamma^0 k_0-\vec\gamma\cdot\vec k-m)_{j\ell}\, e^{ik\cdot(x_1-x_2)}$$

note $\quad ik\cdot(x_1-x_2) = iE(x_{10}-x_{20}) - \vec k\cdot(\vec x_1-\vec x_2)$

$$= (-i)(-E)(x_{10}-x_{20}) - \vec k\cdot(\vec x_1-\vec x_2)$$

change variables $\vec k \to -\vec k \equiv \vec p$

$$-\int dP\,\left[(-\gamma^0)(-E)+\vec\gamma\cdot\vec p-m\right]_{j\ell}\, e^{-i(-E)(x_{10}-x_{20})}\, e^{i\vec p\cdot(\vec x_1-\vec x_2)}$$

if $\quad p^\mu \equiv (-E,\vec p)\quad$ then

$$\uparrow\; + \int dP\,[\underbrace{\gamma^0 p_0-\vec\gamma\cdot\vec p+m}_{(\not p+m)}]_{j\ell}\, \underbrace{e^{-ip^0\cdot(x_{10}-x_{20})}\, e^{i\vec p\cdot(\vec x_1-\vec x_2)}}_{e^{-ip\cdot(x_1-x_2)}}$$

Of course p^μ is a dummy, so can call it k^μ with understanding that in 2nd term $k^0=-E$

$$\langle 0|T(\psi_j(x_1)\bar\psi_\ell(x_2))|0\rangle = \int dK\,(\not k+m)_{j\ell}\, e^{-ik\cdot(x_1-x_2)}$$

for $\quad x_{10}>x_{20}$ then $k^0=E$; for $x_{20}>x_{10}$ then $k^0=-E$

where we are...

I showed the non-zero terms in the Wick expansion for $\bar{\Psi}(x)\gamma^\mu\Psi(x)A_\mu(x)$ to second order to include a variety of stand-alone, familiar processes (Compton scattering, Möller scattering, and Bhabha scattering) as well as some less familiar, "add-on" processes (... (electron/positron self energy, photon vacuum polarization) and some non-interesting, unmeasurable processes (non-scattering & vacuum fluctuation).

We are working specifically on $e\gamma \to e\gamma$

$$* \quad J^{(2)}(e\gamma \to e\gamma) = \frac{(-ie)^2}{2} \int d^4x_1 \int d^4x_2 \, \langle e\gamma | \gamma^\mu_{ij} \gamma^\nu_{lm}$$

$$\Big\{ \bar{\Psi}_i(x_1) \Psi^+_m(x_2) A_\mu(x_1) A_\nu(x_2) \langle 0 | T[\Psi_j(x_1) \bar{\Psi}_l(x_2)] | 0 \rangle$$

$$* \quad - \bar{\Psi}_l(x_2) \Psi^+_j(x_1) A_\mu(x_1) A_\nu(x_2) \langle 0 | T[\bar{\Psi}_i(x_1) \Psi_m(x_2)] | 0 \rangle \Big\} | e\gamma \rangle$$

we're concentrating on the contracted term, which I showed to be

$$\langle 0 | T(\bar{\Psi}(x_1)\Psi(x_2)) | 0 \rangle = \int dK \, (\not{k} + m) \, e^{-ik\cdot(x_1 - x_2)}$$

with $\quad k^0 = E \quad x_{10} > x_{20}$

$\qquad\quad k^0 = -E \quad x_{20} > x_{10}$

look at the special integral

$$I = \frac{1}{(2\pi)^4} \int_{-\infty}^{\infty} d^4k \left(\frac{i}{\not{k}-m} \right)_{jl} e^{-ik(x_1-x_2)}$$

no constraint on k^μ

multiply by $\dfrac{\not{k}+m}{\not{k}+m}$

denominator —

$$(\not{k}+m)(\not{k}-m) = \gamma^\nu k_\nu \gamma^\mu k_\mu - \gamma^\nu k_\nu m + m\gamma^\mu k_\mu - m^2$$
$$= \gamma^\nu k_\nu \gamma^\mu k_\mu - m^2$$

note $\quad A_\nu B_\mu \gamma^\mu \gamma^\nu = B_\mu A_\nu \left(-\gamma^\mu\gamma^\nu + 2g^{\mu\nu}\right)$
$$= -B_\mu A_\nu \gamma^\mu \gamma^\nu + 2A\cdot B$$

so

$$\not{A}\not{B} = -\not{B}\not{A} + 2A\cdot B$$
$$\{\not{A}, \not{B}\} = 2A\cdot B$$

so

$$\{\not{k}, \not{k}\} = 2k\cdot k$$
$$\not{k}\not{k} = k\cdot k = k^2$$

and denominator becomes.

$$k^2 - m^2 = k_0 k_0 - \vec{k}\cdot\vec{k} - m^2$$
$$= \left(k_0 - \sqrt{\vec{k}\cdot\vec{k}+m^2}\right)\left(k_0 + \sqrt{\vec{k}\cdot\vec{k}+m^2}\right)$$

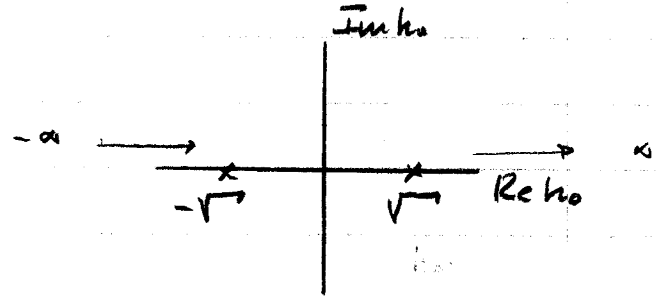
and I get

$$I = \frac{i}{(2\pi)^4} \int_{-\infty}^{\infty} d^3k \int_{-\infty}^{\infty} dk^0 \left[\frac{(\not{k}+m)\, e^{-ik\cdot(x_1-x_2)}}{\Big(k_0-\underbrace{\sqrt{\vec{k}\cdot\vec{k}+m^2}}_{E}\Big)\Big(k_0+\underbrace{\sqrt{\vec{k}\cdot\vec{k}+m^2}}_{-E}\Big)} \right]$$

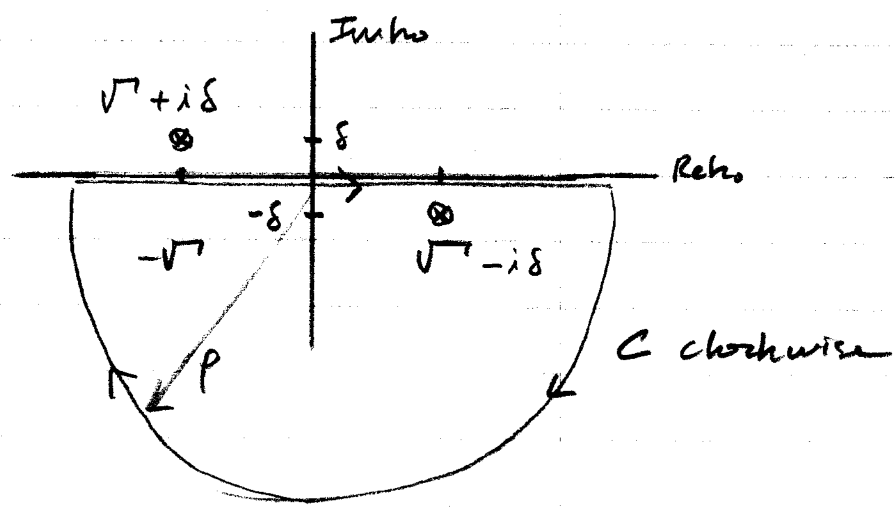
which has simple poles at $k_0 = \pm\sqrt{\vec{k}\cdot\vec{k}+m^2}$

So, $$I_0 = \int_{-\infty}^{\infty} dk_0 \, \frac{(\not{k}+m)\, e^{-ik^0(x_{10}-x_{20})}}{(k_0-\sqrt{})(k_0+\sqrt{})}$$

Map this onto the complex k_0 plane.



avoid the poles in the standard way by shifting them and use Cauchy's Theorem



As $\rho \to$ large and $h^0 = -i|h^0|$, the exponential becomes,

$$e^{-ih^0(x_{10}-x_{20})} \longrightarrow e^{-i(-i|h^0|)(x_{10}-x_{20})}$$

$$= e^{-|h^0|\underbrace{(x_{10}-x_{20})}_{>0}}$$

so as $\rho \to \infty$ $|h^0| \to \infty$ and exponential damps.

Then,

$$\oint_C dh_0 (\quad) = -2\pi i \,(\overbrace{\Sigma \text{residues of enclosed poles}}^{R})$$

$$= \int_{-\infty}^{\infty} dh_0 \,(\text{real, straight line})$$

$$+ \cancel{\int_{-\infty}^{\infty} (\text{curve})}^{\to 0}$$

$$-2\pi i\, R = \int_{-\infty}^{\infty} (\,)\, dh_0 = I_0$$

The pole is simple, so the residue is

$$\frac{(\not{k}+m)\, e^{-ih^0(x_{10}-x_{20})}}{\cancel{(h^0-\sqrt{}+i\delta)}(h^0+\sqrt{}-i\delta)} \cancel{(h^0-k_p^0)} \Bigg|_{h^0=k_p=\sqrt{}-i\delta}$$

$$I_0 = -2\pi i \frac{(\not{k}_p + m)\, e^{-i[\sqrt{\ } - i\delta](x_{10}-x_{20})}}{(\sqrt{\ } - i\delta + \sqrt{\ } - i\delta)}$$

$\delta \to 0$

$$I_0 = -2\pi i \frac{(\not{k} + m)\, e^{-i\sqrt{\vec{k}\cdot\vec{k}+m^2}\,(x_{10}-x_{20})}}{2\sqrt{\vec{k}\cdot\vec{k}+m^2}}$$

and

$$I = \frac{1}{\cancel{(2\pi)^4}\,(2\pi)^3} \cancel{(-2\pi i)}\, \cancel{i} \int d^3k \, \frac{(\not{k}+m)\, e^{i\vec{k}\cdot(\vec{x}_1-\vec{x}_2)}\, e^{-i\sqrt{\vec{k}\cdot\vec{k}+m^2}\,(x_{10}-x_{20})}}{2\sqrt{\vec{k}\cdot\vec{k}+m^2}}$$

$$= \int \frac{d^3k}{(2\pi)^3\, 2E} (\not{k}+m)\, e^{-ik\cdot(x_1-x_2)} \qquad E = +\sqrt{\vec{k}\cdot\vec{k}+m^2}$$

$$= \int dK\, (\not{k}+m)\, e^{-ik\cdot(x_1-x_2)} \qquad ①$$

which is what we had for

$$\langle 0 | T(\psi(x_1)\bar{\psi}(x_2)) | 0 \rangle \qquad \text{for } x_{10} > x_{20}$$

Note if we had had <u>scalar</u> fields, one can go all the way back to the original T product and would find

$$\int dP\, e^{-ip\cdot(x_1-x_2)} = \langle 0 | T(\phi(x_1)\phi^{\dagger}(x_2)) | 0 \rangle \equiv i\Delta^{(+)}(x_1-x_2)$$

$x_{10} > x_{20}$

called the Positive Frequency propagator — one of a family

Likewise, $\langle 0 | T(\varphi^+(x_2)\,\phi(x_1)) | 0 \rangle \equiv -i\Delta^{(-)}(x_1 - x_2)$

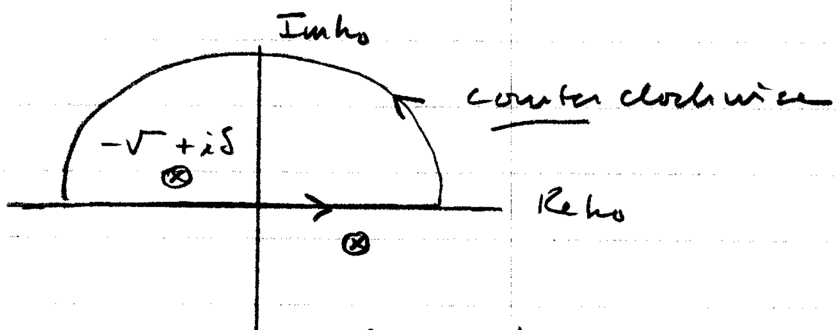
$$= \int dP\, e^{+ip\cdot(x_1 - x_2)} \quad \text{for} \quad x_{20} > x_{10}$$

In general the Feynman scalar Propagator:

$$i\Delta_F(x_1 - x_2) = i\Delta^{(+)}(x_1 - x_2)\,\theta(x_{10} - x_{20}) - i\Delta^{(-)}(x_1 - x_2)\,\theta(x_{20} - x_{10})$$

We've just seen that the Dirac propagator for $x_{20} > x_{20}$ looks like that of the scalar, but for the projection operator (as I hinted at before)

Back to Dirac fields -- what about $x_{20} > x_{10}$?



so $\quad I_0 = +2\pi i \left(\sum R \right) \Big|_{k^0 = -\sqrt{} + i\delta}$

so $$I = (2\pi i)\frac{i}{(2\pi)^4} \int \frac{d^3k}{2\left(-\sqrt{\vec{k}\cdot\vec{k} + m^2}\right)}\, e^{i\vec{k}\cdot(\vec{x}_1 - \vec{x}_2)} (\not{k} + m)\, e^{-i\left(-\sqrt{\vec{k}\cdot\vec{k} + m^2}\right)(x_{10} - x_{20})}$$

Call $\quad k_0 = -\sqrt{\vec{k}\cdot\vec{k} + m^2} \equiv -E$

and $$I = -\int \frac{d^3k}{(2\pi)^3(-2E)} (\not{k} + m)\, e^{-ik\cdot(x_1 - x_2)}$$

which is what we need for $\langle 0|T(\psi(x_1)\bar{\psi}(x_2))|0\rangle$
when $x_{20} > x_{10}$ ②

So, we have a single way of writing $\psi(x_1)\bar{\psi}(x_2)$

* regardless of the time ordering. *

$\langle 0|T(\psi_j(x_1)\bar{\psi}_\ell(x_2))|0\rangle$

$$= \frac{i}{(2\pi)^4}\int \frac{d^4k \quad (\not{k}+m)_{j\ell}\, e^{-ik\cdot(x_1-x_2)}}{\left[k_0-\sqrt{\vec{k}\cdot\vec{k}+m^2}+i\delta\right]\left[k_0+\sqrt{\vec{k}\cdot\vec{k}+m^2}-i\delta\right]}$$

$$\text{denominator} = k_0^2-(\vec{k}\cdot\vec{k}+m^2)+\underbrace{2i\delta\sqrt{\vec{k}\cdot\vec{k}+m^2}}_{\text{call this } i\eta}+\delta^2$$

$$= k\cdot k+m^2+i\eta$$

and

$$\langle 0|T(\psi_j(x_1)\bar{\psi}_\ell(x_2))|0\rangle = \int \frac{d^4k}{(2\pi)^4}\,\frac{(\not{k}+m)_{j\ell}}{(k^2+m^2+i\eta)}\,e^{-ik\cdot(x_1-x_2)}$$

(usually written forgetting the η)

$$= \int \frac{d^4k}{(2\pi)^4}\left(\frac{i}{\not{k}-m}\right)e^{-ik\cdot(x_1-x_2)}$$

$$\equiv iS_F(x_1-x_2)$$

the Feynman fermion propagator

Notice

$$S_{F\,j\ell}(x_1-x_2) = (i\not{\partial}+m)_{j\ell}\,\Delta^{(+)}(x_1-x_2)\,\theta(x_{10}-x_{20})$$

$$- (i\not{\partial}+m)_{j\ell}\,\Delta^{(-)}(x_1-x_2)\,\theta(x_{20}-x_{10})$$

Remember what these are...

look at $\quad S_F(x_1-x_2) = \frac{-i}{(2\pi)^4}\int d^4k\,\frac{(\not{k}\ \ )\,e^{-ik\cdot(x_1-x_2)}}{\not{k}-m}$

operate with the Dirac Equation operator.

$$(i\not{\partial}_1-m)\,S_F(x_1-x_2) = \frac{-i\cdot i}{(2\pi)^4}\int d^4k\,\frac{(i(-i\not{k})-m)\,e^{-ik\cdot(x_1-x_2)}}{\not{k}-m}$$

$$= \frac{1}{(2\pi)^4}\int d^4k\,\frac{\not{k}-m}{\not{k}-m}\,e^{-ik\cdot(x_1-x_2)}$$

$$= \frac{1}{(2\pi)^4}\int d^4k\,e^{-ik\cdot(x_1-x_2)}$$

$$= \delta(x_1-x_2)$$

proving that $S_F(x_1-x_2)$ is the Green's Function for the Dirac Equation.

We will need the same thing for the photon. Again

$$\langle 0|T(A_\mu(x_1)A_\nu(x_2)|0\rangle =$$

$$\langle 0|A^+_\mu(x_1)A^-_\nu(x_2)|0\rangle\,\theta(x_{10}-x_{20}) + \langle 0|A^-_\nu(x_2)A^+_\mu(x_1)|0\rangle\,\theta(x_{20}-x_{10})$$

proceed the same way..

first, for $x_{10} > x_{20}$

$$\langle 0|T(\quad)|0\rangle = \int dK \int dK' \sum_{\lambda=0}^{3}\sum_{\lambda'=0}^{3} \epsilon_{\mu(\lambda)}(k)\,\epsilon_{\nu(\lambda')}(k')\cdot$$
$$e^{-ik\cdot x_1}\, e^{ik'\cdot x_2} \langle 0|a_{(\lambda)}(k)a^+_{(\lambda')}(k')|0\rangle$$

Remember, for photons the commutation relations are a little odd

$$[a_{(\lambda)}(k), a^+_{(\lambda')}(k')] = -g_{\lambda\lambda'}\, 2k_0\,(2\pi)^3\,\delta(\vec{k}-\vec{k}')$$

$$iD_{F\mu\nu}(x_1-x_2) = \int dK \sum_{\lambda=0}^{3}(-g_{\lambda\lambda'})\,\epsilon_{\mu(\lambda)}(k)\,\epsilon_{\nu(\lambda)}(k)\,e^{-ik\cdot(x_1-x_2)}$$

Remember the completeness relation

$$\sum_{\lambda=0}^{3} g_{\lambda\lambda}\,\epsilon_{\mu(\lambda)}(k)\,\epsilon_{\nu(\lambda)}(k) = g_{\mu\nu} \qquad (\text{I wrote } \textstyle\sum \rho_\lambda \epsilon_\lambda \epsilon_\lambda = -g_{\mu\nu})$$

same.

With the same logic as before, we get a similar expression for $x_{20} > x_{10}$.

For the photon, we consider an integral like

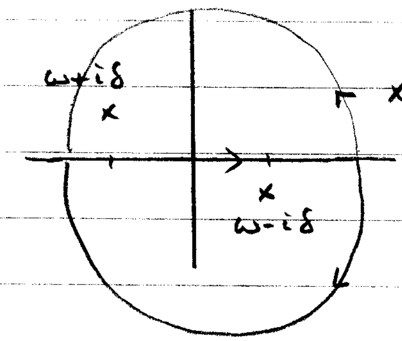
$$\frac{i}{(2\pi)^4}\int \frac{d^4k}{k^\mu k_\mu}\, e^{-ik\cdot(x_1 - x_2)}$$

$$k^2 = k_0^2 - \vec{k}\cdot\vec{k} = (k_0 - \sqrt{\vec{k}\cdot\vec{k}})(k_0 + \sqrt{\vec{k}\cdot\vec{k}})$$

which leads to $\displaystyle I_0 = \int_{-\infty}^{\infty} dk_0 \, \frac{e^{-ik_0(x_{10}-x_{20})}}{(k_0+\omega)(k_0-\omega)}$

where $\omega \equiv \sqrt{\vec{k}\cdot\vec{k}}$

Again,



and we get

$$D_{F\mu\nu}(x_1-x_2) = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x_1-x_2)} D_{F\mu\nu}(k)$$

where $\displaystyle D_{F\mu\nu}(k) = \frac{-g_{\mu\nu}}{k^2+i\epsilon}$ like before

This is the momentum space Feynman Propagator.

Note, this is in a particular gauge – The Feynman Gauge. We can generalize as before with the gauge fixing parameter, in which we would have

$$D_{F\mu\nu}(k) = \frac{-g_{\mu\nu}}{k^2+i\epsilon} + \frac{\xi-1}{\xi}\frac{k_\mu k_\nu}{(k^2+i\eta)^2}$$

Feynman $\xi = 1$

Landau $\xi \to \infty$ (transverse in 4d, ie $k^\mu D_{F\mu\nu} = 0$)

This is all covariant -- we could have done it for only the physical, transverse states -- that's often done. However, it is instructive to break down the components of the polarization sum.

Define the unit vector:

$$\eta_\mu \equiv (1, \vec{0}) \qquad \text{timelike and } \perp \text{ to } \epsilon^\mu_{(1,2)}$$

$$= \epsilon_\mu^{(0)} \quad \text{scalar polarization}$$

and define another unit vector:

$$\pi^\mu \equiv \frac{k^\mu - (k\cdot\eta)\eta^\mu}{\sqrt{(k\cdot\eta)^2 - k^2}} \qquad \text{spacelike and orthogonal: } \pi\cdot\epsilon = 0, \quad \pi\cdot\eta = 0$$

So, this provides a basis $(\eta_\mu, \pi^\mu, \epsilon^\mu_{(1,2)})$ for which we can write a completeness relation

$$\eta^\mu\eta^\nu - \sum_{i=1}^{2} \epsilon^\mu_{(i)}\epsilon^\nu_{(i)} - \pi^\mu\pi^\nu = g^{\mu\nu}$$

$\vdots$

so,

$$iD_F^{\mu\nu}(k) = \frac{\sum_{i=1}^{2} \epsilon^\mu_{(i)}\epsilon^\nu_{(i)}}{k^2 + i\epsilon} \qquad \text{transverse piece}$$

$$+ \frac{1}{k^2+i\epsilon}\,\frac{k^2\eta^\mu\eta^\nu}{\sqrt{(k\cdot\eta)^2 - k^2}} \qquad \text{Coulomb piece}$$

$$+ \frac{1}{k^2+i\epsilon}\,\frac{k^\mu k^\nu - (k^\mu\eta^\nu + k^\nu\eta^\mu)(k\cdot\eta)}{\sqrt{(k\cdot\eta)^2 - k^2}} \qquad \text{"residual piece"}$$

so,

$$i D_F^{\mu\nu}(k) = \frac{1}{k^2 + i\epsilon} \Big\{ \sum_{i=1}^{2} \epsilon^{\mu}_{(i)} \epsilon^{\nu}_{(i)} \qquad \text{transverse}$$

$$+ \frac{[k^\mu - (k\cdot\eta)\eta^\mu][k^\nu - (k\cdot\eta)\eta^\nu]}{(k\cdot\eta)^2 - k^2} \qquad \text{longitudinal}$$

$$+ (-1)\eta^\mu \eta^\nu \Big\} \qquad \text{scalar}$$

Consider the first term -- the regular transverse field

Combine the 2nd two --

$$i D_F^{\mu\nu}(k) = i D_F^{\mu\nu}(k)_T + i D_F^{\mu\nu}(k)_C + i D_F^{\mu\nu}(k)_{\text{"Residual"}}$$

$$\cancel{\frac{1}{(k^2 + i\epsilon)}} \; \frac{\cancel{k^2}\, \eta^\mu \eta^\nu}{\sqrt{(k\cdot\eta)^2 - k^2}}$$

$$\frac{1}{(k^2 - i\epsilon)} \; \frac{k^\mu k^\nu - (k^\mu \eta^\nu + k^\nu \eta^\mu)(k\cdot\eta)}{\sqrt{(k\cdot\eta)^2 - k^2}}$$

In configuration space, the C term is

$$D_F^{\mu\nu}(x) = \frac{1}{(2\pi)^4} \int d^4k \; D_F^{\mu\nu}(k)_c \; e^{-ik\cdot x}$$

$\vdots$

$$= \frac{g^{\mu 0} g^{\nu 0}}{(2\pi)^4} \int d^3k \, \frac{e^{i\vec{k}\cdot\vec{x}}}{|\vec{k}|^2} \int dk^0 \, e^{-ik^0 x^0}$$

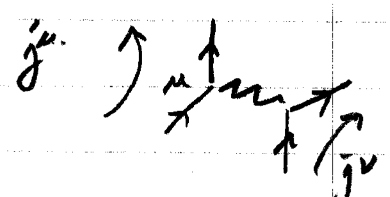
$$= g^{\mu 0} g^{\nu 0} \frac{1}{4\pi|\vec{x}|} \delta(x^0)$$

like an instantaneous Coulomb interaction.

Coulomb — mixture of long and scalar.

So, $\sum_{\lambda=0}^{3}$ becomes <u>physical</u> ..

The "Residual piece?" well, $D_F^{\mu\nu}$ always sits between conserved currents. That is, in momentum space



$k^\mu j_\mu = 0$

Residual piece is proportional to k^μ and k^ν

so, not <u>physical</u>