

# Experiment 1

## Introduction to 191 Lab

### 1. Introduction

In Physics 191 we will make extensive use of *Kaleidagraph* [*Kgraph*], a software package for graphing and data analysis, and *Excel*, for calculations. *Kgraph* is a substantial program but is nonetheless intuitive and extremely easy to use. Three major things that *Kgraph* does for us are:

1. Making high quality, easily customized graphs of your data.
2. Curve fitting, i.e., finding the parameters of a theoretical function that best describe your data. *Kgraph* provides estimates of the **statistical uncertainties of the fit parameters**, which are difficult to obtain with other commercial software (such as *Excel*).
3. Algebraic manipulation of numbers (though *Excel* is even more powerful)

### 2. Goals:

1. Familiarize yourself with basic calculations in *Excel*
2. Familiarize yourself with *Kaleidagraph's* plotting, calculation, and fitting options
3. Get a first impression of the style of labs and analysis for the class
4. Have a first look at histograms and standard deviation
5. Have a first look at fitting and residuals
6. Have a first try at searching for flaws (“systematic errors”) in data.

### 3. Background

3.1 In this first lab, we will learn how to use *Excel* and *Kgraph* on a PC equipped with the Windows XP operating system. Your lab report will contain printouts of the graphs that you generate. **The appendices to this lab write-up, and your notes made while performing this lab, will also serve as your guide for doing basic analysis operations.** You may write your report outside of class by hand or with Word. But you must **use your time carefully** in class to plot and analyze your data. Since you will be changing lab partners regularly, **take turns, and be sure each of you understands the software.** *KEEP this write-up for reference in future labs.*

3.2 For this laboratory exercise, you will analyze information from an actual mileage log kept for a Prius automobile. The log usually contains an entry for each tank of gas purchased (missing data were occasionally estimated). The Prius uses a hybrid engine system, with both a gasoline engine and electric motors. The gasoline engine ultimately provides all the power, but a large battery stores energy when the engine is not heavily loaded, or when the car is braking. The electric motor uses energy stored in the battery to provide extra power after a stop, when passing, when going in reverse, or even to allow the engine to be turned off temporarily. The battery and electric motor allows the gas engine to run nearer its optimum efficiency, part of why the Prius gets more miles per gallon than most cars.

The odometer measures the distance (in miles) a car has traveled by counting the number of turns of the tires. The amount of gas purchased is calculated by the gas pump. The Prius has a dashboard meter that displays the miles per gallon (mpg) being obtained at the moment, and since the last reset (usually in this mileage log, that means since the last tank of gas). These readouts use the same distance measurement as the odometer, but the gas consumption estimate is based on the time (measured in microseconds) that the computer has opened the fuel injectors. This assumes that the fuel pressure is constant and therefore the amount of fuel consumed is proportional to the time the injector was open.

#### 4 Excel Data Calculations Goal: print 1<sup>st</sup> page of modified spreadsheet

4.1 Use the desktop shortcut to the R drive to open R:\exp1\_ref\Prius.xls, an *Excel* file containing the mileage log and some date information. **Now SAVE this file to the U drive:** File|Save As|U: (in Filename box)|Enter, then pick your *section* folder and the *exp1* folder inside it. **Careful: saving on R will hang!** The Days column calculates number of days since the last fill-up. The *month* column calculates the months, starting from the beginning of 2002, so **month 7.5 is the middle of July, 2002**, and month 20 is the beginning of August, 2003. The Month of year column gives the month, independent of year, so month 9.3 would be early September of 2002, or 2003, or 2004. You can click on a cell and the formula used will be displayed.

4.2 Now you will calculate some other quantities from the original data using *Excel* formulas. See the Appendix on *Excel* if you need help. Calculate Miles (the mileage since the last tank; but use 456 for the miles on the first tank of gas); MPG Calc = miles / gallons; miles/day; and calc/read = MPG Calc/mpg readout. For miles per day, you will get an error for a long day of freeway driving; instead of dividing by the days, divide by the larger of 1, and the number of days: see Help | max function. When finished, print the 1<sup>st</sup> page (File|Print|Print Page(s) From: 1 To: 1) then **save and close your file**: *Kgraph* won't open the file if *Excel* still has it open.

#### 5. Kaleidagraph: Histograms and Statistics Goal: print 2 histograms and record statistics

##### 5.1 Navigating *Kgraph*

5.1.1. See the Appendix to this lab for a terse introduction to basic *Kgraph*; use as needed.

5.1.2. You can modify an existing graph by double-clicking or right-clicking objects to bring up a menu for that object: for example, the title of a graph, or an axis label.

5.1.3 *Kgraph* has a good Help system; the >> spins you through successive related entries.

5.1.4. If you want to print out the "Statistics" screen, or anything else you can't figure out how to print from a Windows program, hold down ALT and hit the Print Screen key. A copy of the active window is now on the clipboard. Then open Word, and paste with Ctrl-V, and print the Word document. The statistics screen or other clipboard contents can also be pasted into a *Kgraph* Plot, or a *Kgraph* Layout window.

5.2 Begin your analysis of the mileage data by opening *Kgraph*, and using *Kgraph* to open the Prius.xls file you just created. *Kgraph* properly opens an *Excel* spreadsheet with column headings as long as the headings are in the first row.

5.3 Make a histogram plot of the MPG Calc column. The plot will consist of bars rising from the x axis. In a histogram, data points are considered as being grouped into "bins" (ranges of values, say x between 20 and 30, and so on). The extent of the bar along the x axis will represent the

range of values of the variable in each “bin”, and the height of the bar (along the y axis) will represent how many data points fell into each bin. For your first plot, make the histogram with a bin size of 5 miles per gallon. You can find more about histograms by looking up the term in the index of Taylor.

**5.4 For every plot in this entire class** you should: Give the plot a title which explains the reason for this plot, such as Temperature Dependence of Velocity (metric units); no two plots should have the same title. Include your name, section, experiment, and date in the upper right hand corner of the graph (Help|Find|Text Tool). Make sure that the graph labels, legends, etc. do not overwrite any significant part of the graph. Label the axes of every graph with names and units, e.g., for a histogram Number of Log Entries and Calculated MPG (mi/gal); or for a scatter plot, say Velocity (m/s) and Time (months). **Print your histogram and save it.**

5.5 Now make and print the histogram with a bin size of 2 miles per gallon. Does the new histogram look the same as the previous graph? Why or why not? Did you learn anything new by changing the bin size? **As you write in your lab notebook, or lab report, always include the number of the part of the write-up (e.g. 5.5).**

5.6 Now you will use one of the advanced functions of Kgraph to analyze the data. Click on the frame of the data sheet to highlight it; then click on the MPG Calc column, and then click Function|**Statistics** as described in the Appendix. Note the entries for Mean and Std Deviation. The Mean is just the average value; Std Deviation is an abbreviation for Standard Deviation, a measure of the scatter of the values around the average. Both are discussed in Taylor. Write these two values down in your notebook or paste the statistics block onto one of your histograms. On the histogram with 5 mpg bins, indicate the location of the mean, and draw a horizontal line stretching from (mean – standard deviation) to (mean + standard deviation).

**6. Kaleidagraph: Scatter Plots and Fits** Goal: print mpg plot, fuzzy parabola fits and residuals.

6.1 **Scatter Plot** Now you will seek a better understanding of the mpg data by plotting it as a function of time. Make a scatter plot of calculated mpg as a function of *month*, that is, mpg vs. time. **Whenever we say “y vs. x”** we mean **the first variable should be on the y (vertical) axis** and the second should be on the x (horizontal) axis. Do not use the Line Plot, as it is a “connect the dots” plot not well suited to our uses: by default it doesn’t display much of the data, and the lines both over-lead the eye and get in the way of fits we will apply to the data. Make the data 18-point hollow diamonds. (e.g. Plot|Variable Settings|Marker size|18) Label the axes of the graph with names and units, e.g., Velocity (m/s) and Temperature (K). Does this graph tell you anything about the reason for the structure you saw in the mpg histogram? How would you explain it? *Hint*: look at the definition of *month* in section 4.1.

6.2 Next you will “**fit**” a mathematical model to the mileage data. There isn’t a precise theory of how mileage should vary with time, so the models will be approximate. The “fitting” consists of writing down a mathematical expression (“curve”) with some unknown parameters and letting Kgraph automatically try values of the parameters to find those that best “fit” the data. The mathematical method Kgraph uses to find the parameters is called “least squares”. Least squares fitting minimizes the sum of the square of the difference (“residual”) between the measured data and the values predicted by the curve. This will be discussed in detail in Chapter 8 of Taylor.

6.3 To get a feel for fitting, you will now work through an example where the model is perfectly known, and see how well *Kgraph* reconstructs the true model. Open in *Kgraph* the file `r:\expl_ref\fuzzy_parabola.xls`. Make a scatter plot of  $y=f(x)$  vs.  $x$ . Now fit the model  $a + bx + cx^2$  to the data using the directions in the Appendix. Write down the values you obtain for the parameters  $a$ ,  $b$  and  $c$ . The curve should pass perfectly through the data and *Kgraph* should report essentially zero uncertainty (“error”) in the parameters. Check that you got the correct values by opening the `fuzzy_parabola.xls` file with *Excel*. Look at the equation used for the  $f(x)$  column and write down the correct values of the parameters in your notebook.

6.4 Now make a new plot of “fuzzy  $f$ ” vs.  $x$ . The “fuzzy” value was created by adding “noise” to the value of  $f$  (here, a random number between  $-1$  and  $+1$ ). The last column gives the difference between the fuzzy and real value. Fit this fuzzy data to the parabola. Print the plot with the fit, the values of the parameters and their uncertainties. Are they close to the correct values?

6.5 Now look at how the fit differs from the data by examining the “residuals”, defined as the difference:  $\text{residual} = \text{data} - \text{fit}$ , at each data point. *Kgraph* uses  $\text{fit} - \text{data}$ , but that isn’t a big problem. Make and print a histogram of the residuals (see Appendix). You should see values roughly between  $-1$  and  $1$ : the differences between the fit and the data are about the same as the “fuzz” we put in. If we’d looked at the residuals of the fit to the un-fuzzed  $f(x)$ , the values would have been tiny. *Extra Credit*: plot the residuals for the un-fuzzed fit.

7.1 **Data Analysis.** Goal: print 2 fits to mpg vs. time; % difference between calc and readout

Now return to the mpg vs. month data. You will make two fits to the data, one based on a cosine curve and the other on a 6<sup>th</sup> order polynomial. **Before doing so, predict** (write down before you do the fits!) which you think will do the better job, and why.

7.2 Make a fit with the **cosine** curve as described in the Appendix. You will need reasonable starting values for this fit to work properly. Print the plot with the fit curve and the fit parameters as labels for the figure. Also, histogram the residuals from this fit. Use the statistics function to calculate the standard deviation of the residuals: this, as you just saw, is a measure of how well the fit matches the data.

7.3 Perform another fit to the data: a 7-parameter fit, to a **6<sup>th</sup> order polynomial**. You may need to make two plots in order to show both fits. Print this fit as well. Find the standard deviation of the residuals of this fit. For your report, discuss which fit actually looks “better” to your eye as a representation of the data. Does that correspond to the plot with the smaller standard deviation of residuals? *Extra Credit*: You could also make a scatterplot of the residuals vs. time. Ideally, there would be no longer any obvious time dependence of the residuals if the fit had captured the main time dependence. Is that the case here?

7.4 Now **cross check** of the two methods for calculating miles per gallon to see if they’re roughly equivalent and how close they are to each other. You could approach this question in several ways: plotting or examining statistics for the column mpg calc/readout, or making a scatter plot of the calculated vs. readout miles per gallon and performing a fit. Choose one of these methods and attempt to say quantitatively within how many percent % the two methods give the same answer, and how much (in %) this answer varies.

7.5 *Extra Credit Questions*. How might you study the mileage for highway as compared to city driving? The data between Jan 1 and Aug 1 2003 correspond to a time in New Mexico rather than Michigan; are there noticeable differences in the data due to the location? What might cause these differences? What do you think is the capacity of the gas tank?

7.6 **Finishing up**: As you **exit Kaleidagraph**, select "**None**" in the dialogue box when you are asked whether you want to save changes. **It is advisable to back up files from the U Drive to a USB flash drive** if you will need your data for next week (**the U drive is not backed up!**):

- 1) Insert USB flash drive in front of the computer, or on USB extender
- 2) Open the U drive by double-clicking on the shortcut
- 3) Select in the Address box of this window your flash drive (E: or F:)
- 4) Again open the U drive by double-clicking on the shortcut (gives a second window)
- 5) Drag the files or folders from the U: window to the flash drive window
- 6) Click Safely Remove Hardware icon in system tray (bottom right)
- 7) click Remove E; after it says it's OK to do so, actually remove your flash drive.

Finally, **Log Out** from Windows (this will erase all changes you made except for the files you have saved on the U drive or your USB flash drive).

**8. Summary Tables** In each lab report you should present your critical results in summary tables which organize your results for you, and for your readers. In many experiments you will also need to organize your data recording in tables. At the beginning, we will give you explicit tables to use; as time goes on, you should be able to think through for yourself what tables you will need. For this experiment, your summary tables should look like the following:

Quantity	Mean	Standard Deviation
MPG Calculated		
MPG Cosine Fit Residuals		
MPG Polynomial Fit Residuals		
MPG Calculated / MPG Readout (%)		

f(x) fits	a	b	c
true parabola parameters			
f(x) fit parameters			
fuzzy f(x) fit parameters			
uncertainty of fuzzy fit parameters			

### 9. Questions to be discussed in your report:

9.1 Why should one look at histograms with different bin widths?

9.2 How well do the two numbers Mean and Standard Deviation summarize your MGP histogram data? What features do they describe, and what do they miss?

9.3 What do you think causes the structure you see in the finer-binned mpg histogram?

9.4 Does the Prius go farther on a gallon of gas than the vehicle you are most familiar with? If possible, give a quantitative comparison, and state how reliable your conclusion is.

9.5 Are the parameter “errors” in the parabola fits about the size of the difference between the “fuzzed” parameters and the original ones? Is this what you would expect, roughly?

9.6 Which fit to the mileage data was better, the cosine fit or the polynomial fit? On what (numerical) basis did you decide? Was that consistent with your prediction?

9.7 Which fit, the cosine fit or the polynomial fit, would be more likely to correctly describe the data for the next year? *Hint*: look at the behavior at the edge(s) of the plot.

**10. Searching for Flaws:** Where could the measurements be misleading? Any experimental procedure is subject to errors of execution, mistaken assumptions, or biases. Part of our job as experimentalists is to think about them. At the beginning of the term, we will guide your thinking about these, but you should strive to be able to do this more independently as the term progresses. Here we will suggest some sources of uncertainty in the data and procedures. For items in ***bold italics***, hints are given below: but try by yourself first!

**10.1** What would happen to the miles per gallon if the tires were under-inflated? By about how many percent might under-inflation affect the estimated distance traveled? Would under-inflation cause a measurement to be too low, or too high?

10.2 Would tire under-inflation affect the ratio of mpg calculated divided by mpg readout?

**10.3** The calculated mpg depends on the precision of the measurement of gasoline volume by the gas station pump. What do you estimate for the fractional % of uncertainty in the number gallons?

**10.4** The calculated mpg in effect assumes that the gas tank is filled back up to the same level after each filling. This is usually determined by the automatic shutoff. How accurately (in gallons) do you think the shutoff valve at different gas stations measures “tank full”? What would this translate to for a typical tank of gas for this car?

**10.5** How accurately (in %) would you guess that a manufacturer would want to have calibrated the fuel consumption? How accurate would it have to be, to be small compared to the uncertainties due to tire inflation?

10.6 From these considerations, and the data, to what % accuracy do you think the mpg is known? For a measurement of 45mpg, how many mpg would this uncertainty be?

**10.7** Is the scatter in the data about the fits larger or smaller than your estimate from 10.6? If larger, what other effects might be causing such variations from one tank to the next?

10.8 Is the mpg according to the readout systematically different from the calculated value? By how many % If so, which measurement would you tend to believe more? Why?

**11. Improvements:** What was the muddiest point of the lab? Where could the write-up be improved? Be specific—state exactly where and how should it be improved.

**12. Reading and homework: Turn these exercises in with your lab report. It's due next week even though class isn't meeting to do a lab.** Your instructor will discuss how to turn it in.

**Read** Taylor chapter 1, 2 (you may skip sections 2.6 and 2.9), 4.2, and 5.1. Read section 1.3 and chapter 2 especially carefully—they give the *purpose of uncertainty calculations*, to be able to make *quantitative comparisons* between measured values and between measurements and predictions. See also the handout on the important points on uncertainties.

**Homework 1:** Work through the *Excel Tutorial* in the Appendix and print out both the resulting spreadsheet results, and the formulas. Do **Taylor problems** 2.4, 2.6, 4.1, and 5.4. In problem 4.1, use your calculator, *Excel*, or *Kaleidagraph* to check your values.

### Hints for section 10:

10.1 Speedometers are often off by 2 - 5 mph at 65 mph, so a 3 - 8% error in the speedometer is not out of the question, though that's the speedometer, which is perhaps more complicated than the odometer. The odometer works by counting the revolutions of the tires and converting turns into distance traveled. This assumes the tires are of constant circumference. The accuracy implied by our uncertainty was  $\approx 1\text{mi} / 456\text{mi} = 0.2\%$  which seems awfully strict when you think of the change of shape in the tires when they are fully inflated vs. under-inflated. You might estimate the tire height (diameter) might change 1/2 inch out of about 2 feet, or about 2 %, due to changes in tire pressure, from being warm or cold, or over or under-inflated. (This alone might give a 1 mpg apparent improvement for highway driving, which keeps the tires warm).

10.3 For a first estimate use the recording error of .1 gal; for 11.3 gal that's 0.9% (though the pump records more decimal places, that's all that was recorded in the log). Assume that all pumps are calibrated by the state to measure the same within well less than the 0.9 % measurement error on a tank (at least we hope so). What about the definition of a full tank? Is that repeatable from pump to pump? What is its uncertainty? Hard to guess again but it might be another 0.1 gallon, or perhaps even more. This latter might not matter so much in the long run: if the pump cut off early for one tank, you'd miss some of the gas you actually used for those miles. But the next tank, you'd probably record some gas for miles which really were recorded for the previous tank. Over time it would average out, but this uncertainty would contribute to the uncertainty of any single tank.

10.5 See 10.1 above

10.7 If the scatter of measurements is large compared to your estimated error (as seen by either the calculated vs. readout data, or the size of the fit residuals), then the measurement was dominated by sources of uncertainty other than those you have thought about. These are sometimes called uncontrolled variables. They might not have anything to do with the measurement process itself, but rather changes in what is actually being measured. A driver might be more interested in speed than good mileage on a particular trip. Weather, traffic, road conditions, or the load carried might vary.

## Appendix: Kaleidagraph Essentials

**Histograms** Gallery|Stat|Histogram|*pick data*|New Plot where Xxx | Yyy means: find menu Xxx then select item Yyy; *Italics* indicates something you must type, or select. To **rebin**, Plot|Plot Options|Specifying the Bin Size|OK , then Plot|Axis Options|*enter new bin size*

**Statistics** Highlight data column, then Function|Statistics and either write down results and |OK, or |Clipboard|OK and then paste where you want them (Plot, Layout, or Word).

**y vs. x plot** (*x is horizontal!*): Gallery|Linear|Scatter|*select x and y variables*|NewPlot

**Curve Fitting** CurveFit|General|EditGeneral|Add|NewFit|*give new fitname in box*|Edit|  
Entering equation: say want to fit a parabola  $y = \text{parabola } a + b x + c x^2$ . Kgraph's convention for the first fit parameter is m1 (and so on). So you could enter in the Edit box:

$m1 + m2*x + m3*x^2; m1=1; m2=1; m3=1$

where m1 stands for your parameter a, m2 for b, m3 for c. The extra equations give **nonzero** guesses for the parameters which the fit routine **must** have to start from. Now, having entered the expression you are fitting, OK|OK|CurveFit|General|*fitname*|*check the box*|OK  
If you'd wanted an equation of the form  $C + a \cos(k x + b)$ , you'd use  $m1+m2*\cos(m3*x + m4)$ .

To **display the fit equation**, Plot|DisplayEquation. This shows you the fit parameters, and their uncertainties (labeled "errors"). The value of m1 is the best fit value of a, etc.

**Residuals:** To make a new column in the data window containing fit residuals, **immediately** after you've performed the fit (or redone the fit): CurveFit|General|*fitname*|View|CopyResiduals ; you may want to edit the data window column heading by clicking on it.

**If the function fit doesn't look much like the data**, you probably need to give **better initial values** than 1. Do this by CurveFit|General|*fitname*|Define| then edit the initial values in the box:

$m1 + m2*\cos(m3*x + m4); m1=45; m2=10; m3=30; m4=180$

Why might these be better values? The C constant (m1) should be near the average y value; the m2 should be the amount above or below the average; m3 should be such that the range of one cycle of  $x * m3$  is 360 degrees (**if Degrees is checked**) or  $2\pi$  if Radians is checked; and 180 (Degrees) would be appropriate for a maximum the middle of the range.

**Remove a previous fit:** CurveFit|General|*fitname*|Deselect

**Multiple Plots per page:** Windows|ShowLayout|*click on layout*| then a Layout menu appears. Layout|SelectPlot|*click on plot*| for the plots you want on the page (2 or 3 will be big enough to read). Then Layout|ArrangeLayout| and select say 2 or 3 rows and one column to arrange your plots. You can drag, resize, or delete the plots; add text; or paste a clipboard.

It's really best to just jump in, but the Kgraph tutorial offers more detail. Particularly helpful is example 2. You don't have to perform all the tutorial steps, but it will give you a feel for what's in each menu. Optional: Example 4 on Laying out multiple plots on a page.



## Appendix: Introduction to *EXCEL* (and Tutorial)

This introduction will show you the basics you need for this lab. Open *Excel* by double-clicking the icon with the mouse. Then open a new spreadsheet by File | New (and possibly Blank Workbook). Files that end in “.xls,” are Excel spreadsheets. **You can copy and paste data from the *Excel* spreadsheet to the *Kgraph* spreadsheet.** *Kgraph* can open a spreadsheet too, and if it's just data and single-cell column headers in row 1, it will label the columns.

The *Excel* spreadsheet is made up of rectangles called “cells.”

To enter text into a cell, click on the cell with the mouse, type the desired text, and press Return or Enter.

To execute a mathematical **formula** in a cell, you must **always first type “=”** (an equals sign), followed by the desired formula. For example, to compute  $5+6$ , type “**=5+6**” into an empty cell and press “Enter.” The answer, “11,” appears in the cell where you entered the formula. The formula you entered in the cell appears at the top of the screen under the menu bar. This works for any cell. If you want to see the formula for any cell, just click on the cell and the formula for that cell appears at the top of the spreadsheet.

Now we will fill a **column A** labeled X with numbers ranging from 0 to 1.4 in steps of 0.2. Do not take the time to do this all by hand! You can have *Excel* do it for you. In cell **A1** enter X and type Enter. In the next cell of the column, **A2**, enter 0, our first value. Then in **A3**, we can give *Excel* a formula we want it to follow in order to fill in the rest of the column. In each cell, we want it to add 0.2 to the value in the cell just above. That will fill in the column in steps of 0.2. Therefore in cell **A3** enter the formula “**=A2+0.2**” and press return. Highlight 7 cells of the column, starting with the cell that has the formula in it, A3. Then go to Edit | Fill | Down and then let go of the mouse button. Your column should be filled in increments of 0.2.

In the next **column B** (Label it Sin(x) in **B1**) you are going to compute the sine of X using a formula. In **B2** type “**=Sin(A2)**” and hit Return. By typing **A2** in the parentheses we are telling *Excel* to take the sine of the value (in radians) which it finds in **A2**. Then you can use the same process as above to “fill-down” the entire column. If you do this just as described above, *Excel* knows to start with **A2**, take the sine of each successive value in the A-column, and place the new value in each successive cell of the B-column. Now repeat this same procedure, only this time for cosine X in the **C-column**.

### More hints:

If in doubt, use parentheses to make sure things get calculated in the right order. For example,  $3 + 5/2$  results in 5.5, but  $(3 + 5)/2$  results in 4. It would have been better to use  $3 + (5/2)$  in Excel for the first case.

To **check your spreadsheet formulas**, type Ctrl-` (that is, hold down the Ctrl key and type the ` key (which has a ~ above it). If you now print your spreadsheet, you can see all the formulas. Typing Ctrl-` again toggles back to showing the calculated numbers.

Instead of typing a cell number (say **O27**) you can click on the cell while entering the formula.

You can change formatting with the % and .00 → .0 and .0 → .00 buttons.

The formula `=A17` **copies the contents** of cell A17 into the cell in which you typed the formula.

**Copying formulas:** The menu command Edit | Fill | Down is equivalent to:  
 clicking the cell with the formula cell will be highlighted, with dark borders  
 hovering the mouse over the lower right corner mouse pointer turns from a hollow to a solid cross  
 dragging the mouse down over more cells the formula is “copied”

When the formula is copied, the cell references change automatically! So if your formula in A3 was `A2+.2`, when it is filled or dragged down to cell A4, it becomes `A3 + .2`; if you are doing the same set of calculations on a whole column of numbers, you can set up the formulas in the first row, then highlight and drag down the formulas in several columns at once.

A similar process allows you to copy formulas sideways (across columns), or copy and paste into another column. Put 100 in D2. Next, enter `=D2` in D3 to copy D2's contents. Now highlight A3:A8, hit copy (Ctrl-c) and then move the mouse to D4 and paste (Ctrl-v). This fills the **D column** with numbers spaced by .2, but starting from 100.

Sometimes this automatic renumbering isn't what you want to do, so you need to be able to stop Excel from adjusting cell numbers. One way to always refer to single cell is to give it a name. Type 10 into cell E1. Highlight E1, then click your mouse in the name box (it will say E1) left of where the formula appears. Type *fred* there and when you use the name *fred* in your formulas and it will always refer to that specific cell. Enter in E3 the formula `=fred^A3` and copy it down through E4:E9. You should see numbers ranging from 1.58 through 25.12 in the **E column**.

When your spreadsheet is complete, make sure you save and print!

### Common operations in Excel

you can substitute cells for numbers with these operations

What?	Mathematics	Excel Equation	Value
Addition	$11 + 12$	<code>= 11 + 12</code>	23
Subtraction	$29 - 21$	<code>= 29 - 21</code>	8
Multiplication	$30 \times 15$	<code>= 30 * 15</code>	450
Division	$44 / 12$	<code>= 44/12</code>	3.66666667
Example	$3 + \frac{4}{5 \times 2} - 3 \times 7$	<code>= 3 + (4/(5*2) - (3*7))</code>	17.6
Power	$6^3$	<code>= 6^(.3)</code>	1.7117
Exponential	$e^{6.1}$	<code>= exp(6.1)</code>	445.86
Square Root	$\sqrt{3^2 + 4^2}$	<code>= sqrt ( 3^2 + 4^2)</code>	5
Sine or Cosine	Sin(x) or Cos(x)	<code>= sin(x) or = cos(x)</code>	x = .5 (radians): sin = .479
Pi	Cos (2 $\pi$ x)	<code>= cos(2*pi()*x)</code>	.809 for x = .1

**Operations on groups of cells:** use as an example cells A3...A9 containing .2, .4, .... 1.4  
 The group can be denoted by A3:A9, or you can highlight a group of cells while entering the formula

Sum	$\sum a_i$	<code>= sum(A3:A9)</code>	5.6
Mean		<code>= average(A3:A9)</code>	0.8
Standard Deviation		<code>= stdev(A3:A9)</code>	.316