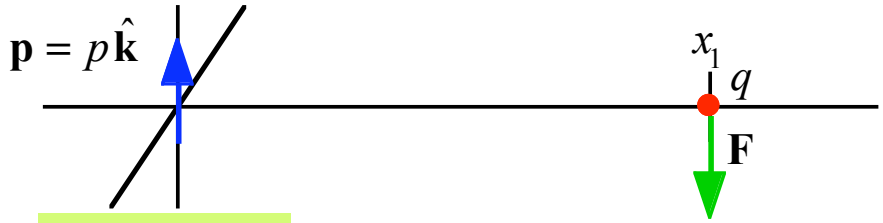
PHY481: Electromagnetism

Dipoles and torque

Charges near conductors

Problem 3.32 - Dipoles and torque

a) Point dipole at the origin. Force on a displaced charge



$$\mathbf{F}_p = \frac{+qp\hat{\mathbf{k}}}{4\pi\varepsilon_0 x_1^3}$$

$$\mathbf{F} = q\mathbf{E} = \frac{-qp\hat{\mathbf{k}}}{4\pi\varepsilon_0 x_1^3}$$

F on charge

Forces are "balanced", but what about torque?
Will a bar between the point charge and the dipole rotate if released?

Violates angular momentum conservation: No external torques -> **L** is constant
The bar will not rotate when released.

The torque on the charge: $\boldsymbol{\tau}_q = \mathbf{r} \times \mathbf{F} = \hat{\mathbf{j}}\, qp / 4\pi\varepsilon_0 x_1^2$

The torque on the dipole: $\boldsymbol{\tau}_p = \mathbf{p} \times \mathbf{E} = -\hat{\mathbf{j}}\, qp / 4\pi\varepsilon_0 x_1^2$

Torques also "balance" !

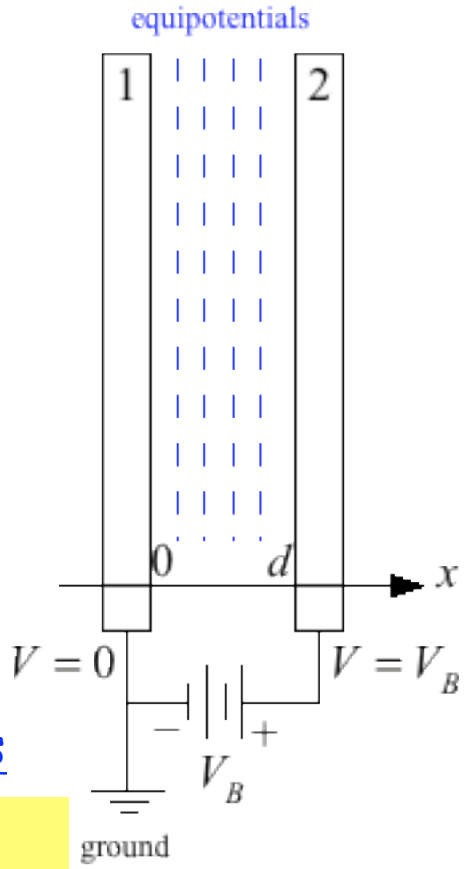
Boundary value problems-1: parallel plates

Parallel plates (find potential, field & surface charge)

The ground is a constant potential, so let $V_1 = 0$.
Battery draws electrons from plate 2 and puts them on plate 1. Process "continues" until $V_2 = V_B$.

Potential between the plates

Between the plates the potential depends only on x.



Laplace's equation: $\nabla^2 V(x) = 0$

Boundary conditions: $V(0) = 0; \quad V(d) = V_B$

General solution (ODE)

$$\nabla^2 V = \frac{d}{dx}\left(\frac{dV}{dx}\right) = 0; \frac{dV}{dx} = c_1$$

$$V(x) = \underline{c_1 x + c_2}$$

Apply boundary conditions

$$V(0) = \underline{c_2 = 0}$$

$$V(d) = c_1 d = V_B; \underline{c_1 = \frac{V_B}{d}}$$

Potential

$$V(x) = \underline{V_B \frac{x}{d}}$$

Parallel plates (cont'd)

Electric field

Electric field

$$\mathbf{E} = -\nabla V = -\nabla\left(V_B \frac{x}{d}\right) \qquad -\frac{V_B}{d}\hat{\mathbf{i}}$$

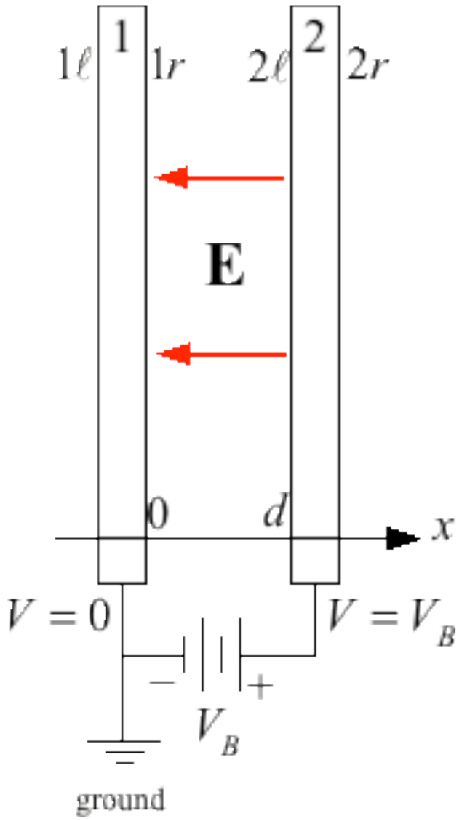
E_n -> Surface charges

Surface 1r charge density

$$E_{1n} = \frac{\sigma_{1r}}{\varepsilon_0} = -\frac{V_B}{d} \qquad \sigma_{1r} = -\frac{\varepsilon_0 V_B}{d}$$

Surface 2ℓ charge density

$$E_{2n} = \frac{\sigma_{2\ell}}{\varepsilon_0} = \frac{V_B}{d} \qquad \sigma_{2\ell} = +\frac{\varepsilon_0 V_B}{d}$$



Surface charge

$$Q_{2\ell} = -Q_{1r} = \sigma A = \frac{\varepsilon_0 V_B A}{d}$$

Force on each plate

$$F_2 = Q_2 (E_{2n})_{non\text{-}q_2} = \frac{1}{2}\frac{\sigma_2^2}{\varepsilon_0} A$$

Parallel plates (cont'd)

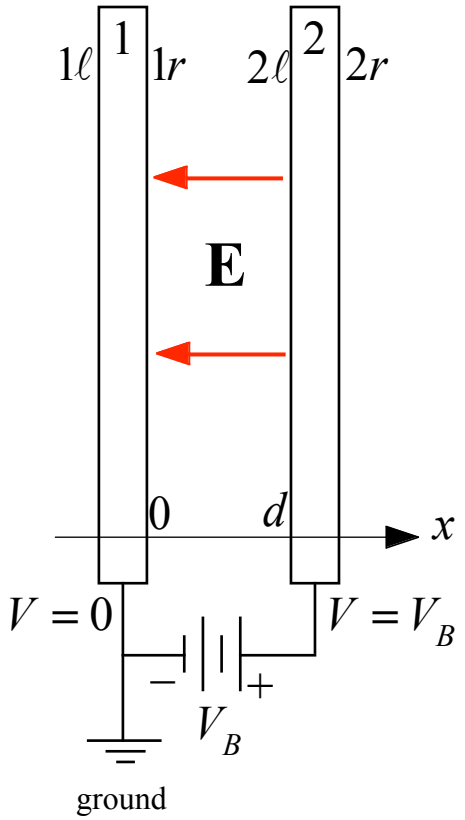
Capacitance

Two ways to know the electric field

$$E = \frac{\sigma}{\varepsilon_0}; \quad E = \frac{V_B}{d}$$

Capacitance is a geometrical factor relating the charge on a conductor and its potential.

$$Q = \sigma A = \frac{\varepsilon_0 A}{d} V_B = C V_B \qquad C = \frac{\varepsilon_0 A}{d}$$

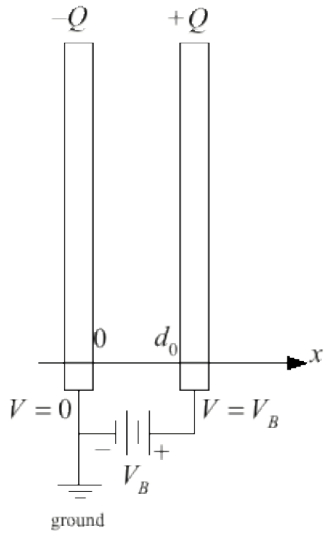


Energy storage

$$U = \frac{\varepsilon_0}{2} \int E^2 d^3x' = \frac{\varepsilon_0}{2} \frac{V_B^2}{d^2} Ad = \frac{1}{2} \frac{\varepsilon_0 A}{d} V_B^2 \qquad U = \frac{1}{2} C V_B^2$$

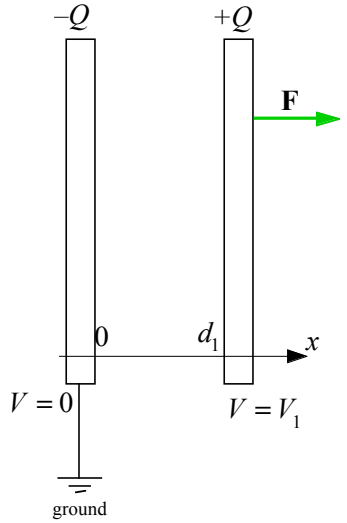
Changing separation of the plates

Plates charged



$$Q = CV_B = \varepsilon_0 A V_B / d_0$$

Disconnect battery and separate plates

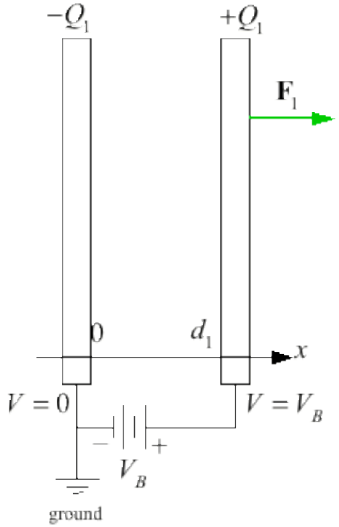


Charge remains at Q
Voltage changes to V_1

$$Q = C_1 V_1 = \varepsilon_0 A V_1 / d_1$$

$$V_1 = V_B d_1 / d_0$$

Leave battery connected and separate plates



Voltage remains at V_B
Charge changes to Q_1

$$Q_1 = C_1 V_B = \varepsilon_0 A V_B / d_1$$

$$Q_1 = Q d_0 / d_1$$

Stored energy

$$\mathcal{E} = \frac{1}{2} C V_B^2 = \frac{1}{2} \frac{\varepsilon_0 A}{d_0} V_B^2$$

$$\mathcal{E}_1 = \frac{1}{2} C_1 V_1^2 = \frac{d_1}{d_0} \mathcal{E}$$ ↑

$$\mathcal{E}_1' = \frac{1}{2} C_1 V_B^2 = \frac{d_0}{d_1} \mathcal{E}$$ ↓ ?

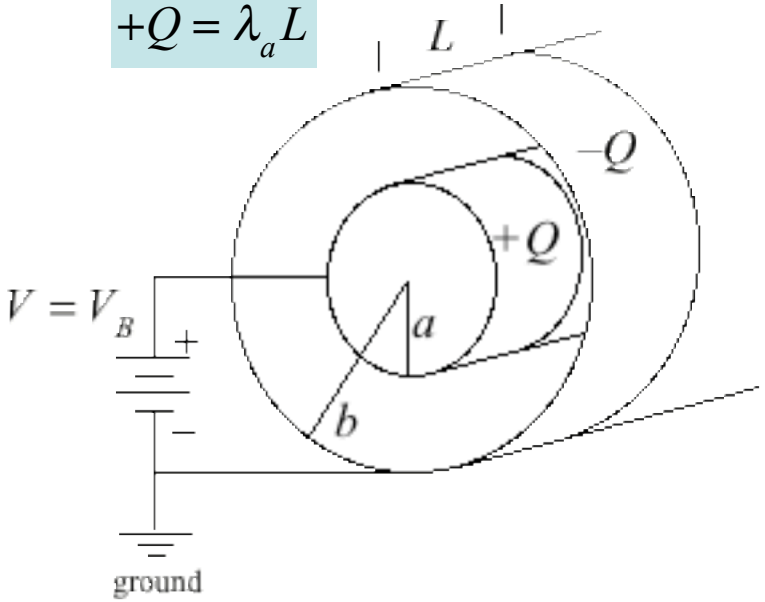
Cylindrical capacitor

Capacitance of nested long cylinders

Find charge Q for the given $V = V_B$

$$\lambda_a = \sigma_a (2\pi a)$$

$$+Q = \lambda_a L$$



Gauss's law between the cylinders

$$\oint_S \mathbf{E}\cdot d\mathbf{A} = 2\pi r L E(r) = \frac{\lambda_a L}{\varepsilon_0}$$

Field between the cylinders

$$E(r) = \frac{\lambda_a}{2\pi\varepsilon_0 r}$$

Potential between the cylinders

$$V(r) = -\int_b^r E dr = -\frac{\lambda_a}{2\pi\varepsilon_0}\int_b^r \frac{dr}{r} = \frac{\lambda_a}{2\pi\varepsilon_0}\ln(b/r)$$

Charge and potential relationship

$$V_B = \frac{\lambda_a}{2\pi\varepsilon_0}\ln(b/a); \quad \lambda_a = \frac{2\pi\varepsilon_0}{\ln(b/a)}V_B$$

$$Q = \lambda_a L = \frac{2\pi\varepsilon_0 L}{\ln(b/a)}V_B = CV_B$$

Capacitance per unit length

$$C/L = \frac{2\pi\varepsilon_0}{\ln(b/a)}$$