RC and RL Circuits

RC Circuits

In this lab we study a simple circuit with a resistor and a capacitor from two points of view, one in time and the other in frequency. The viewpoint in time is based on a differential equation. The equation shows that the RC circuit is an approximate integrator or approximate differentiator. The viewpoint in frequency sees the RC circuit as a filter, either low-pass or high-pass.

Experiment 1, A capacitor stores charge:

Set up the circuit below to charge the capacitor to 5 volts. Disconnect the power supply and watch the trace decay on the 'scope screen. Estimate the decay time. It will be shown that this decay time, $\tau = RC$, where R is the resistance in ohms and C is the capacitance in farads. From this estimate calculate an approximate value for the effective resistance in parallel with the capacitor. (This resistance is the parallel combination of the intrinsic leakage resistance within the capacitor and the input impedance of the 'scope.) [Ans.: about 1 s]
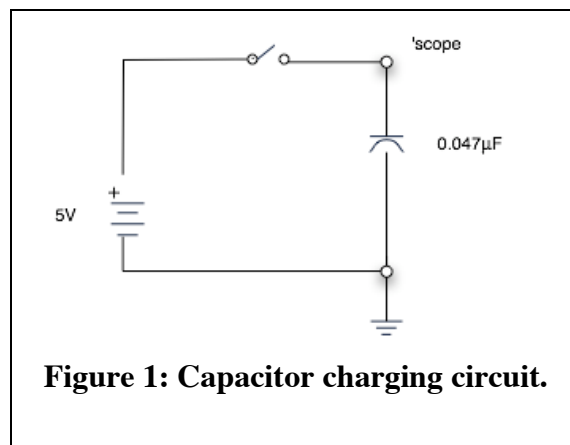

Figure 1: Capacitor charging circuit.

Next, replace the 0.047 μF capacitor by a 1000μF electrolytic capacitor [Pay attention to the capacitor polarity!] and watch the voltage across it after you disconnect the power supply. While you are waiting for something to happen, calculate the expected decay time. Come to a decision about whether you want to wait for something to happen. Act according to that decision.

Experiment 2, The RC integrator in time:

Consider the RC circuit in Figure 2 below:

Figure 2: RC Circuit.
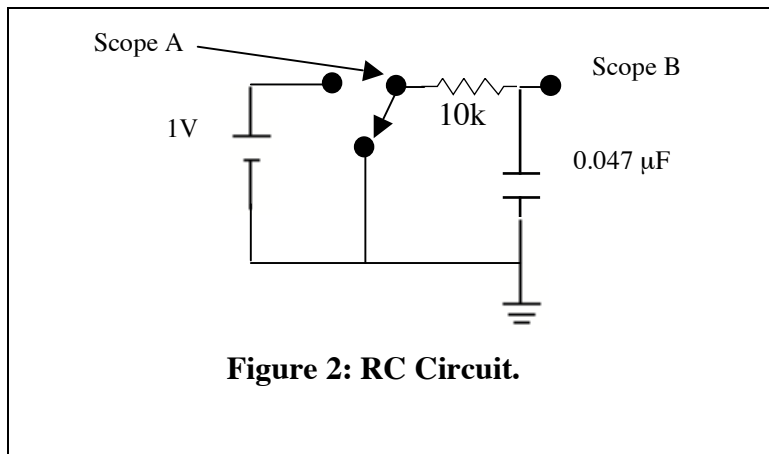
In lecture you will learn that this circuit can be described by a differential equation for q(t), the charge on the capacitor as a function of time. If you have time, you may wish to write down the equation and show that a solution for the voltage on the capacitor, $V_C = q(t)/C$, consistent with no initial charge on the capacitor, is:

$$V_C = V_0(1 - e^{-t/\tau}) \text{ where } \tau = RC \text{ and } V_0 \text{ is the initial voltge.}$$

Now build the circuit, replacing the battery and switch by a square wave generator. (Note: The square wave generator has positive and negative outputs, but this is the same as switching the battery with an added constant offset and a scale factor.)

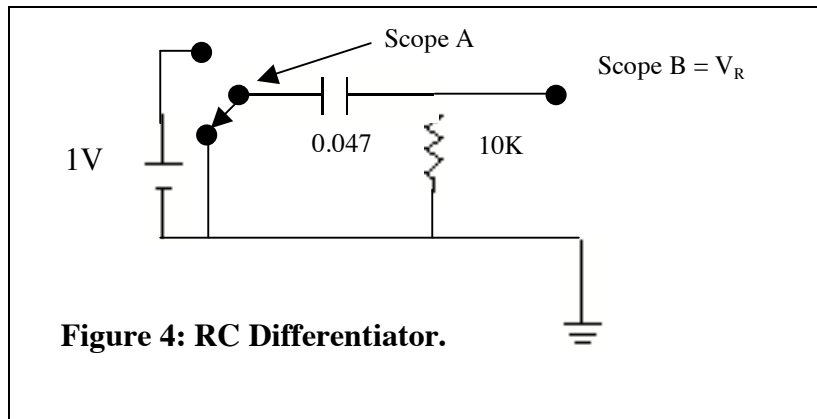Set the square wave frequency to 200 Hz, and observe the capacitor voltage.

Figure 3: Square Wave and Integrator Output.
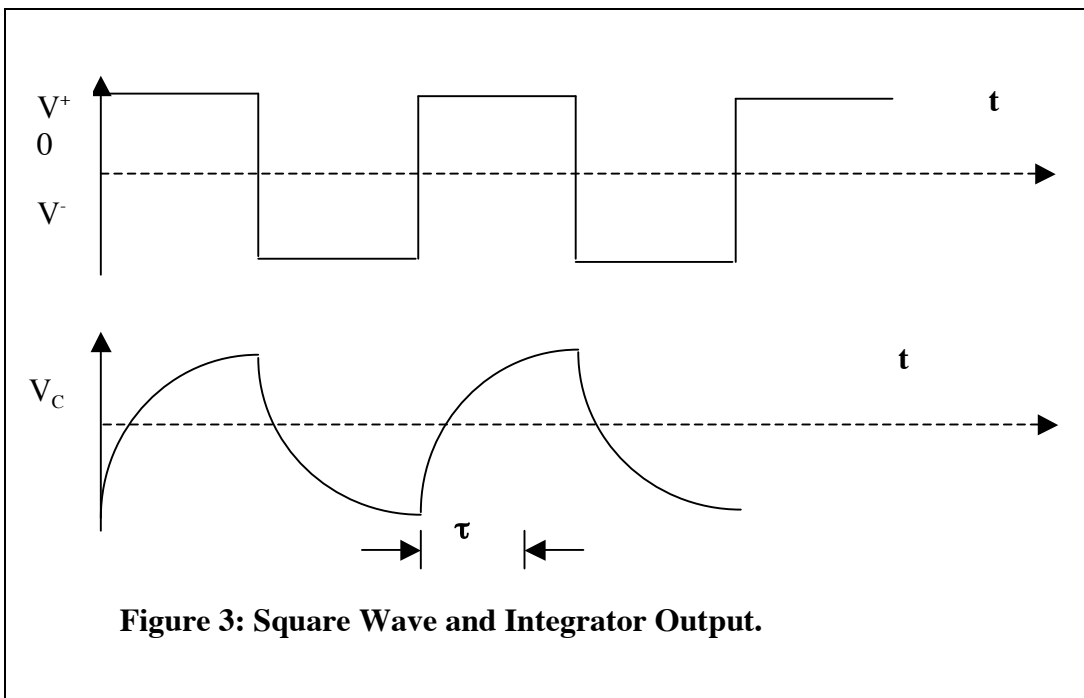
Use the 'scope to measure the time required to rise to a value of $(V_+ - V_-)(1-e^{-1})$. Accuracy in this measurement is improved if the pattern nearly fills the screen. This rise time must be equal to τ. Compare with the calculated value of τ.
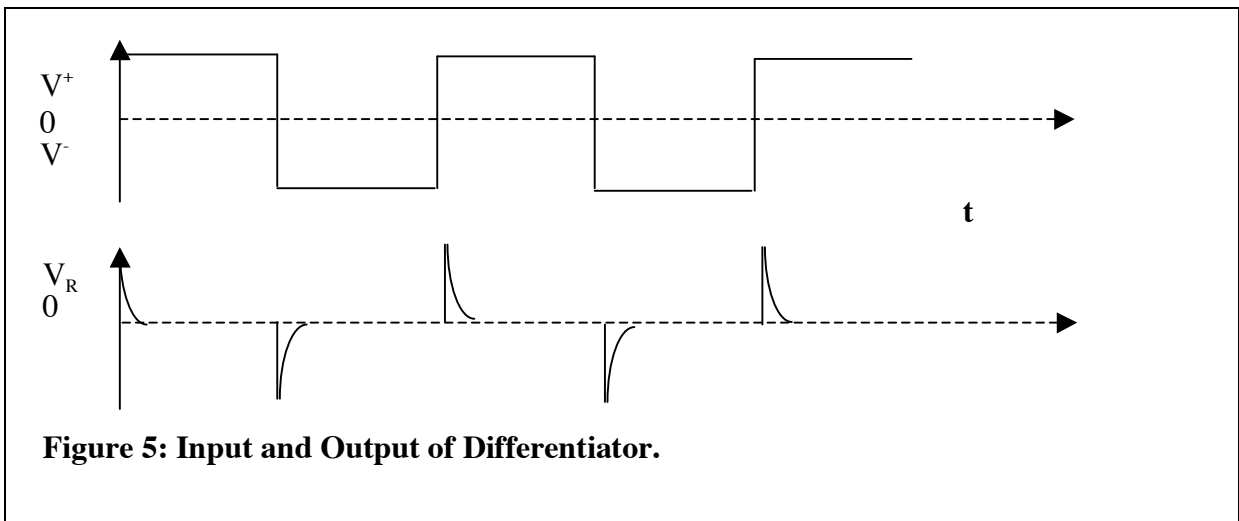
Increase the square wave frequency to 900 Hz. Is the RC circuit a better approximation to a true integrator at this frequency? Sketch the response of a true integrator to a square-wave input.

Experiment 3, The RC differentiator in time:

Consider the RC circuit in Figure 4 below:

Figure 4: RC Differentiator.

The output is the voltage across the resistor, which is the current, or *dq/dt* multiplied by the resistance R. If you have time, show that the solution for this voltage, consistent with no initial charge on the capacitor, is $V_R = e^{-t/\tau}$, where $\tau = RC$.

Figure 5: Input and Output of Differentiator.

Now build the differentiator circuit, replacing the battery and switch by a square wave generator. Set the square wave frequency to 200 Hz, and observe the resistor voltage.

Use the 'scope to measure the time required to fall (or rise) by a factor of e^{-1}. Accuracy in this measurement is improved if the pattern nearly fills the screen. This rise time must be equal to τ. Compare with the calculated value of τ. Sketch the derivative of a square wave. How does the output of the differentiator circuit compare?

Experiment 4, The RC low-pass filter:

The low-pass filter is simply the integrator circuit above, but we replace the source by a sine oscillator so that we can measure the response at a single frequency. (The sine wave is the only waveform that has only a single frequency.) We define the transfer function for a filter as the ratio $H(\omega) = V_{out}/V_{in}$, the ratio of output to input voltages, where ω is the angular frequency of the input sinusoidal voltage.

The transfer function in this case is given by:
$$H(\omega)=1/(1+j\omega\tau).$$
where, j is the imaginary number $\sqrt{-1}$.

Calculate the frequency [$f = \omega/(2\pi)$] of the so called "half-power point." This is simply the frequency where the output voltage amplitude is equal to the input voltage amplitude divided by $\sqrt{2}$.

Calculate the phase shift at this frequency $\phi = (\tan^{-1}(Im(H(\omega))/Re(H(\omega)))$.

Build the circuit and find the frequency for half power. Use the 'scope to find the phase shift at that frequency and compare with calculations. (Note: To find the phase shift, find the time delay, Δt, between equivalent zero crossings of the input and output. Then use the idea that
$$\phi = 2\,\pi\,f\,\Delta\,t,$$
or
$$\phi = 360\,f\,\Delta\,t,$$
for radians or degrees respectively. The phase shift is positive if the output leads the input. Does the output lead the input for this filter?

Now, instead of the half-power point we consider the half-voltage point, where the output amplitude is half the input amplitude. Show that this point occurs when $\omega\tau = \sqrt{3}$. Calculate the frequency for the low-pass filter above. Show that the phase shift at this point is -60 degrees.

Change the oscillator frequency to find the half-voltage point. Compare frequencies and phase shifts with your calculations.

Experiment 5, The RC high-pass filter:

The high-pass filter is simply the differentiator circuit above, but we replace the source by a sine oscillator so that we can measure the response at a single frequency.

The transfer function is
$$H(\omega) = j\omega\tau/(1+j\omega\tau).$$
Show that in the limit of high frequency $H = 1$.

Calculate the frequency [$f =\omega/(2\pi)$] of the "half-power point."

Calculate the phase shift at this frequency.

Build the circuit and find the frequency for half power. Use the 'scope to find the phase shift at that frequency and compare with calculations.

The phase shift is positive if the output leads the input. Does the output lead the input for this filter?

If you have time, do the "half-voltage" calculations and measurements, as for the RC low-pass filter.

RL Circuits

This part of the lab uses a 27 mH inductor and resistors.

Experiment 6, Real inductors – the ugly truth:

Use an ohmmeter to measure the DC resistance of the inductor. Remember the answer.

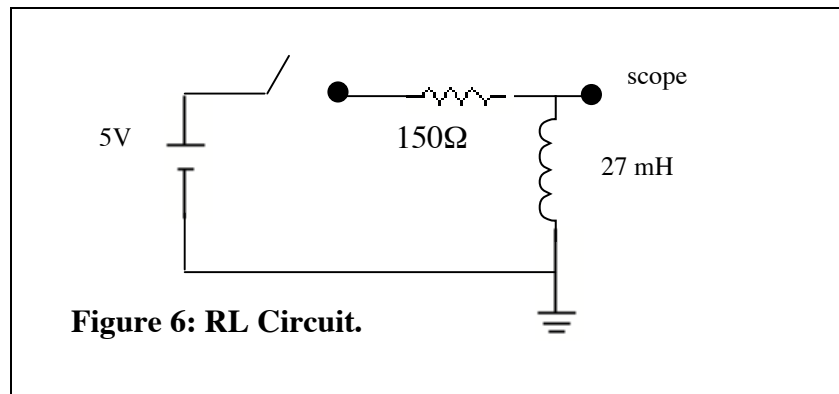Experiment 7, Real inductors – arcs and sparks:

Set up the circuit below.

Figure 6: RL Circuit.

Once equilibrium is established, after the switch is closed, there remains a voltage across the inductor. Why should this be?

Disconnect the power supply abruptly and carefully watch the voltage across the inductor. Connect, disconnect, connect, disconnect … You should see spikes that exceed the original supply voltage. How can this be? How can you get more voltage from the inductor than the power supply voltage? Is there a violation of a Kirchoff law? Is there a violation of conservation energy?

Experiment 8, The RL differentiatior:

Replace the power supply and switch above with a square wave generator.

Figure 7: RL Differentiator.
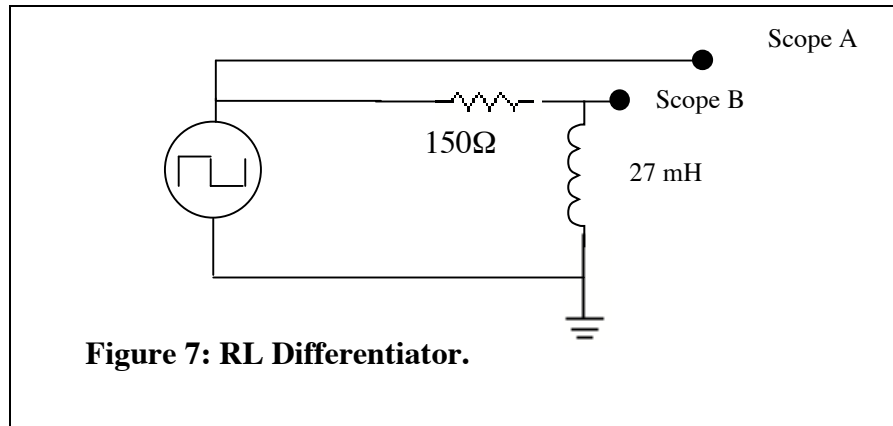
Calculate time constant $\tau = L/R$. Remember to include the resistance intrinsic to the inductor in R. Measure the time constant on the 'scope.

Experiment 9, The RL integrator:

Design an RL integrator and verify its operation on the 'scope.