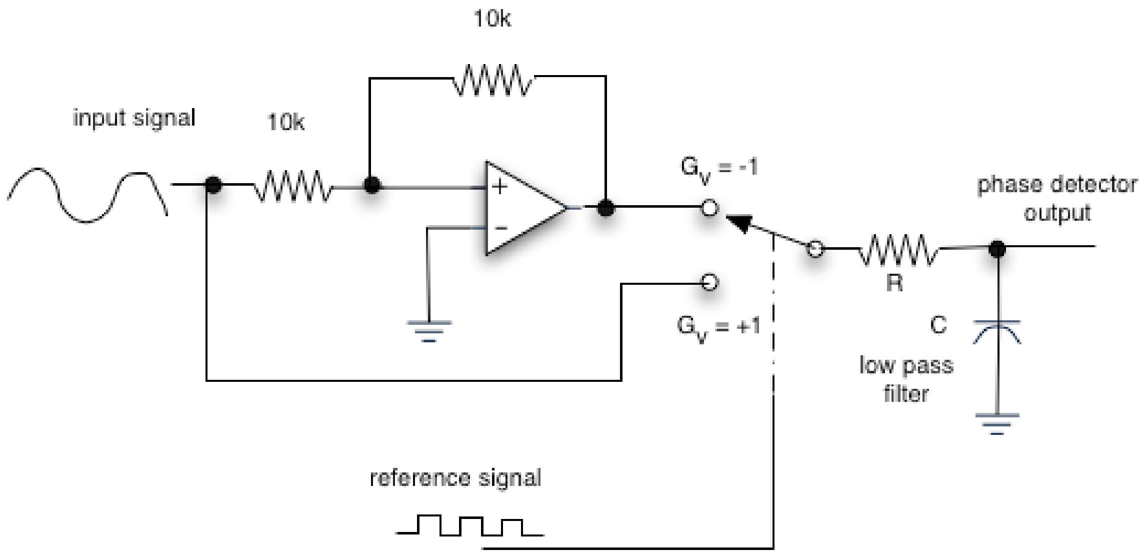
Problem HH15.2

The figure below shows a basic phase detector.



An analog signal passes through a linear amplifier whose gain is reversed by a square-wave “reference” signal controlling an FET switch. The output signal passes through a low-pass filter, RC.

Let’s assume we apply a signal $E_s \cos(\omega t + \phi)$ to the phase detector with transitions at the zeros of $\sin\omega t$, i.e. at $t = 0, \pi/\omega, 2\pi/\omega$ etc. Let us further assume that we average the output, V_{out}, by passing it through a low-pass filter whose time constant is longer than one period:

$$\tau = RC >> T = 2\pi/\omega$$

Then the low pass filter output is

$$\langle E_s \cos(\omega t + \phi)\rangle \Big|_0^{\pi/\omega} - \langle E_s \cos(\omega t + \phi)\rangle \Big|_{\pi/\omega}^{2\pi/\omega}$$

where the brackets represent averages, and the minus sign comes from the gain reversal over alternate half cycles of V_{ref}.

Show that

$$\langle V_{out}\rangle = -(2E_s/\pi)\sin\phi$$

by doing the integrals and assuming unity gain for the amplifier