Physics for Scientists & Engineers 2

Spring Semester 2005

Lecture 27

Review

- Lenz's law states that a current is induced in the loop that tends to oppose the change in magnetic flux
- The induced emf due to a changing magnetic field is given by

$$\oint \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt}$$

- The unit of inductance is the henry (H)

$$[L] = \frac{[\Phi_B]}{[i]} \Rightarrow 1\ \mathrm{H} = \frac{1\ \mathrm{Tm}^2}{1\ \mathrm{A}}$$

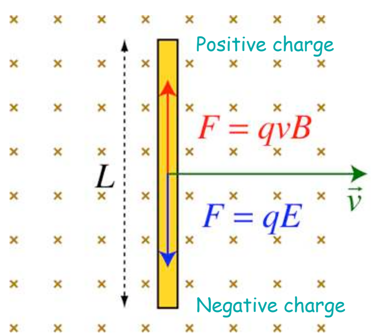
- The inductance of a solenoid of length l and area A with n turns per unit length an is given by

$$L = \mu_0 n^2 l A$$

Induced Voltage on a Moving Wire in a Magnetic Field

- Consider a conducting wire of length *L* moving with constant speed *v* perpendicular to a constant magnetic field *B* as shown below

Magnetic field is directed into the screen



Induced Voltage on a Moving Wire in a Magnetic Field (2)

- The magnetic field will exert a force on the electrons the wire, causing them to move
- A negative charge will build up at one end of the wire and a positive charge will build up at the opposite end of wire, producing an electric force that cancels the magnetic force, leaving the electrons at equilibrium

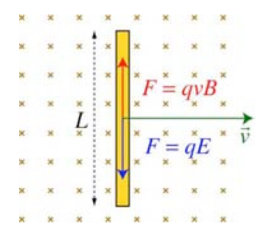
$$F_B = qvB = F_E = qE$$

- Thus we have an electric field with magnitude

$$E = vB$$

- Which produces a voltage between the ends of the wire given by

$$\frac{V}{L} = vB \Rightarrow V = vLB$$



Self Inductance and Mutual Induction

- Consider the situation in which two coils, or inductors, are close to each other
- A current in the first coil produces magnetic flux in the second coil
- Changing the current in the first coil will induce an emf in the second coil
- However, the changing current in the first coil also induces an emf in itself
- This phenomenon is called self-induction
- The resulting emf is termed the self-induced emf.

Self Induction

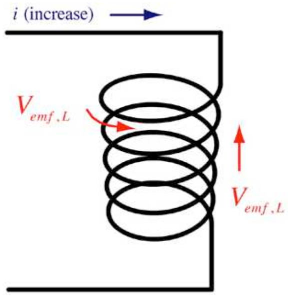
- Faraday's Law of Induction tells us that the self-induced emf for any inductor is given by

$$V_{emf,L} = -\frac{d\left(N\Phi_B\right)}{dt} = -\frac{d\left(Li\right)}{dt} = -L\frac{di}{dt}$$

- Thus in any inductor, a self-induced emf appears when the current changes with time
- This self-induced emf depends on the time rate change of the current and the inductance of the device
- Lenz's Law provides the direction of the self-induced emf
- The minus sign in provides the clue that the induced emf always opposes any change in current

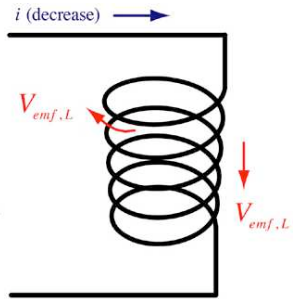
Self Inductance (2)

- In the figure below, the current flowing through an inductor is increasing with time
- Thus a self-induced emf arises to oppose the increase in current



Self Inductance (3)

- In the figure below, the current flowing through an inductor is decreasing with time
- Thus a self-induced emf arises to oppose the decrease in current



RL Circuits

- We have assumed that our inductors have no resistance
- Now let's treat inductors that have resistance
- We know that if we place a source of external voltage, V_{emf}, into a single loop circuit containing a resistor *R* and a capacitor *C*, the charge *q* on the capacitor builds up over time as

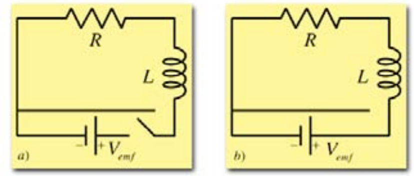
$$q = CV_{emf}\left(1 - e^{-t/\tau_C}\right)$$

- where the time constant of the circuit is given by $\tau_C = RC$
- The same time constant governs the decrease of the initial charge *q* in the circuit if the emf is suddenly removed

$$q = q_0 e^{-t/\tau_C}$$

RL Circuits (2)

- If we place an emf in a single loop circuit containing a resistance *R* and an inductor *L*, a similar phenomenon occurs



- If we had connected only the resistor and not the inductor, the current would instantaneously rise to the value given by Ohm's Law as soon as we closed the switch
- However, in the circuit with both the resistor and the inductor, the increasing current flowing through the inductor creates a self-induced emf that tends to oppose the increase in current
- As time passes, the change in current decreases and the opposing self-induced emf decreases and after a long time, the current is steady

RL Circuits (3)

- We can use Kirchhof's loop rule to analyze this circuit assuming that the current *i* at any given time is flowing through the circuit in a counterclockwise direction
- The emf source represents a gain in potential, $+V_{emf}$, and the resistor represents a drop in potential, $-iR$
- The self-inductance of the inductor represents a drop in potential because it is opposing the increase in current
- The drop in potential due to the inductor is proportional to the time rate change of the current and is given by

$$V_{emf,L} = -L\frac{di}{dt}$$

RL Circuits (4)

- Thus we can write the sum of the potential drops around the circuit as

$$V_{emf} - iR - L\frac{di}{dt} = 0$$

- We can rewrite this equation as

$$L\frac{di}{dt} + iR = V_{emf}$$

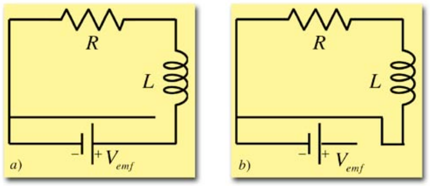
- The solution to this differential equation is

$$i(t) = \frac{V_{emf}}{R}\left(1 - e^{-t/(L/R)}\right)$$

- We can see that the time constant of this circuit is $\tau_L = L/R$

RL Circuits (5)

- Now consider the case in which an emf source had been connected to the circuit and is suddenly removed



- We can use our previous equation with V_{emf} = 0 to describe the time dependence of this circuit

$$L\frac{di}{dt}+iR=0$$

RL Circuits (6)

- The solution to this differential equation is

$$i(t)=i_0 e^{-t/\tau_L}$$

- where the initial conditions when the emf was connected can be used to determine the initial current, $i_0 = V_{emf}/R$
- This equation describes a single loop circuit with a resistor and an inductor that initially has a current i_0
- The current drops with time exponentially with a time constant $\tau_L = L/R$ and after a long time the current in the circuit is zero

Energy of a Magnetic Field

- We can think of an inductor as a device that can store energy in a magnetic field in the manner similar to the way we think of a capacitor as a device that can store energy in an electric field
- The energy stored in the electric field of a capacitor is given by

$$U_E=\frac{1}{2}\frac{q^2}{C}$$

- Consider the situation in which an inductor is connected to a source of emf
- The current begins to flow through the inductor producing a self-induced emf opposing the increase in current

Energy of a Magnetic Field (2)

- The instantaneous power provided by the emf source is the product of the current and voltage in the circuit

$$P=V_{emf}i=\left(L\frac{di}{dt}\right)i$$

- Integrating this power over the time it takes to reach a final current yields the energy stored in the magnetic field of the inductor

$$U_B=\int_0^t Pdt=\int_0^i Li'di'=\frac{1}{2}Li^2$$

Applications

- Our computers and many of our consumer electronics use magnetization and induction to store and retrieve information
- Examples are computer hard drives, videotapes, audio tapes, and the magnetic strips on credit cards
- Storage of the information is accomplished by using an electromagnet in the "write-head"
- A current that varies in time is sent to the electromagnet and creates a magnetic field that magnetizes the ferromagnetic coding of the storage medium as it passes by the magnet

Applications (2)

- Retrieval of the information basically reverses the process of information storage
- As the storage medium passes by the "read head", which is another coil, the magnetization causes a change of the magnetic field inside the coil
- This change of the magnetic field induces a current in the read head which is then processed by the information technology or consumer electronics device