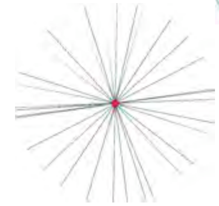
Physics for Scientists & Engineers 2

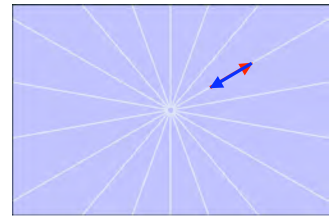
Spring Semester 2005

Lecture 5

Review -Field Lines from a Point Charge

- The electric field lines from a point charge extend out radially
- For a positive point charge, the field lines point outward
  - Terminate at infinity
- For a negative charge, the field lines point inward
  - Originate at infinity





Electric Field from a Point Charge

- Suppose we have two charges, q and q_0, separated by a distance r, the electric force between the two charges is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}$$

- We can consider q_0 to be a test charge, and we can define the electric field from charge q as

$$E = \frac{F}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

- The electric field is a vector and to add electric fields we must add the components separately

Electric Field from Three Point Charges

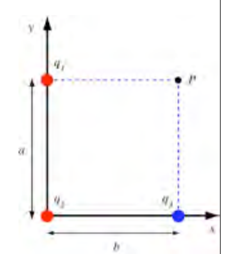
- Consider three charges

$$q_1 = 1.50\ \mu\text{C}$$
$$q_2 = 2.50\ \mu\text{C}$$
$$q_3 = -3.50\ \mu\text{C}$$

- The three charges are placed at

$$q_1 : (a,0)$$
$$q_2 : (0,0)$$
$$q_3 : (0,b)$$

- What is the electric field at point *P*?



Electric Field from Three Point Charges (2)

- The electric field at P due to q_1 is

$$\vec{E}_1 = k\frac{q_1}{b^2}\vec{e}_x$$

- The electric field at P due to q_3 is

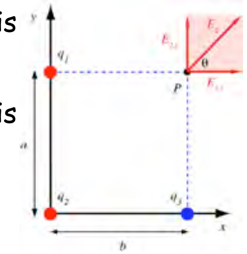
$$\vec{E}_3 = k\frac{q_3}{a^2}\vec{e}_y$$

- The electric field at P due to q_2 is

$$\vec{E}_2 = \left(\frac{kq_2\cos\theta}{a^2+b^2}\right)\vec{e}_x + \left(\frac{kq_2\sin\theta}{a^2+b^2}\right)\vec{e}_y$$

a^2+b^2 is the distance from q_2 to point P squared

θ is given by $\tan^{-1}(a/b)$



Electric Field from Three Point Charges (3)

- Now we add the components

$$E_x = k\frac{q_1}{b^2} + \left(\frac{kq_2\cos\theta}{a^2+b^2}\right) = k\left(\frac{q_1}{b^2} + \frac{q_2\cos\theta}{a^2+b^2}\right)$$

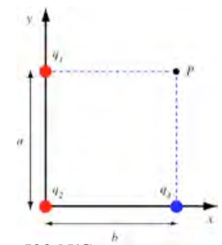
$$E_y = \left(\frac{kq_2\sin\theta}{a^2+b^2}\right) + k\frac{q_3}{a^2} = k\left(\frac{q_2\sin\theta}{a^2+b^2} + \frac{q_3}{a^2}\right)$$

$$\theta = \tan^{-1}\left(\frac{a}{b}\right) = \tan^{-1}\left(\frac{8.00\text{ m}}{6.00\text{ m}}\right) = 53.1°$$

$$a^2+b^2 = (8.00\text{ m})^2 + (6.00\text{ m})^2 = 100\text{ m}^2$$

$$E_x = 8.99\cdot10^9\left(\frac{1.50\ \mu\text{C}}{(6.00\text{ m})^2} + \frac{(2.50\ \mu\text{C})\cos(53.1°)}{100\text{ m}^2}\right) = 509\text{ N/C}$$

$$E_y = 8.99\cdot10^9\left(\frac{(2.50\ \mu\text{C})\sin(53.1°)}{100\text{ m}^2} + \frac{-3.50\ \mu\text{C}}{(8.00\text{ m})^2}\right) = -311\text{ N/C}$$



Electric Field from Three Point Charges (4)

- Now we use the components to get the magnitude

$$E = \sqrt{E_x^2 + E_y^2}$$

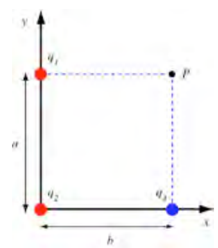
$$E = \sqrt{(509\text{ N/C})^2 + (-311\text{ N/C})^2} = 597\text{ N/C}$$

- Now the direction

$$\theta = \tan^{-1}\left(\frac{E_y}{E_x}\right) = \tan^{-1}\left(\frac{-311\text{ N/C}}{509\text{ N/C}}\right)$$

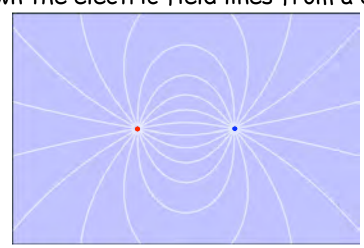
$$\theta = -31.5°$$

electric field points to the right and down



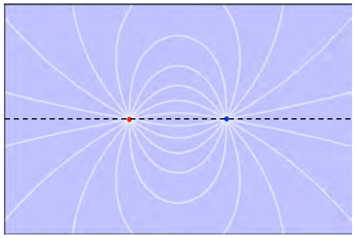
Electric Field from an Electric Dipole

- A system of two oppositely charged point particles is called an electric dipole
- The vector sum of the electric field from the two charges gives the electric field of the dipole
- We have shown the electric field lines from a dipole



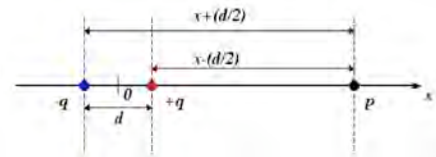
Electric Field from an Electric Dipole (2)

- Let's get an expression for the electric field of a dipole along a line including both charges
- We will derive a general expression good anywhere along the dashed line and then get an expression for the electric field a long distance from the dipole



Electric Field from an Electric Dipole (3)

- Let's put both charges on the x-axis a distance d apart
  - Put $-q$ at $x = -d/2$
  - Put $+q$ at $x = +d/2$
- Calculate the electric field at a point P a distance x from the origin



Electric Field from an Electric Dipole (4)

- The principle of superposition tells us that the electric field at any point x is the sum of the electric field from $+q$ and $-q$

$$E = E_+ + E_- = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_+^2} + \frac{1}{4\pi\varepsilon_0}\frac{-q}{r_-^2}$$

- Replacing r_+ and r_- we get

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(x-\frac{1}{2}d\right)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(x+\frac{1}{2}d\right)^2}$$

Electric Field from an Electric Dipole (5)

- Rearranging we get

$$E = \frac{q}{4\pi\varepsilon_0 x^2}\left(\left(1-\frac{d}{2x}\right)^{-2} - \left(1+\frac{d}{2x}\right)^{-2}\right)$$

- This equation gives the electric field everywhere on the x-axis except for $x = \pm d$
- Let's look at this equation far away ($x \gg d$)

$$E \approx \frac{q}{4\pi\varepsilon_0 x^2}\left(\left(1+\frac{d}{x}-\ldots\right)-\left(1-\frac{d}{x}+\ldots\right)\right)$$

or

$$E \approx \frac{q}{4\pi\varepsilon_0 x^2}\left(\frac{2d}{x}\right) = \frac{qd}{2\pi\varepsilon_0 x^3}$$

Definition of Electric Dipole

- We can define the vector electric dipole moment as a vector that points from the negative charge to the positive charge

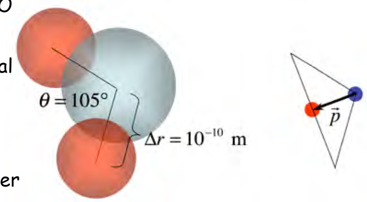
$$\vec{p} = q\vec{d}$$

  - p is the magnitude of the dipole moment
  - q is the magnitude of one of the opposite charges
  - d is the distance between the charges
- Using this definition we can write the electric field far away from an electric dipole as

$$E = \frac{p}{2\pi\varepsilon_0 x^3}$$

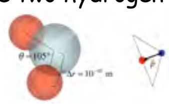
Example - Electric Dipole Moment of Water

- Everyone knows that water is H_2O
- The charge distributions of the atoms are approximately spherical and the hydrogen atoms are arranged such that they make an angle of 105° with each other
- Suppose we approximate the water molecules as two positive charges located at the center of the hydrogen atoms and two negative charges located at the center of the oxygen atom. What is the electric dipole moment of a water molecule?



Electric Dipole Moment of Water (2)

- We assume that the center of mass of the two hydrogen atoms is halfway between the two atoms and that the two positive charges are effectively located there



- The distance between the these two positive charges and the two negative charges assumed at the center of the oxygen atom is

$$d = (10^{-10}\ \text{m})\cos 52.5° = 0.6 \cdot 10^{-10}\ \text{m}$$

- Our result for the electric dipole moment of water is then

$$p = 2ed = (3.2 \cdot 10^{-19}\ \text{C})(0.6 \cdot 10^{-10}\ \text{m}) = 2 \cdot 10^{-29}\ \text{C m}$$

- This oversimplified result is close to the known value of $6.2 \cdot 10^{-30}$ C · m
  - Assumed charge distribution not what actually happens