Physics for Scientists & Engineers 2

Spring Semester 2005

Lecture 6

Force from Electric Fields

- We defined the electric field in terms of the electric force

$$\vec{E} = \frac{\vec{F}}{q_0}$$

- Now let's look at the electric force produced by an electric field on a charge q

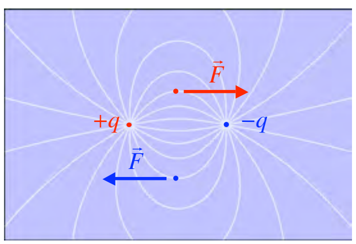
$$\vec{F} = q\vec{E}$$

- The force exerted on a positive charge is in the same direction as the electric field and has the magnitude

$$F = qE$$

Force from Electric Fields (2)

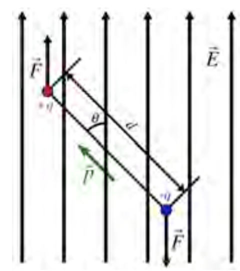
- Electric field lines represent the direction and magnitude of the electric field
- The electric force on a positive charge will always be tangent to the electric field lines and point in the direction of the electric field



- For negative charges, the electric force is on the opposite direction

Torque on an Electric Dipole

- We know that an electric field will exert a force on a point charge in the direction of the field
- Now let's put an electric dipole in an electric field
- The dipole is composed of two charges, $+q$ and $-q$ located a distance d apart
- The electric field is constant and points in the upward direction
- The electric dipole moment makes an angle θ with the electric field



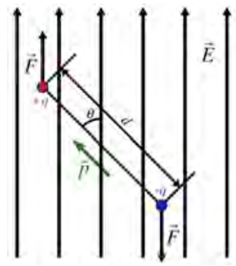
Torque on an Electric Dipole (2)

- The electric field will exert a force upward on $+q$ and downward on $-q$, each with magnitude qE
- These forces balance so that there is no net force on the dipole
- However, there is a net torque around the center of mass of the dipole given by

$$\tau = (\text{Force}_+)(\text{Moment Arm}_+) + (\text{Force}_-)(\text{Moment Arm}_-)$$

- Each moment arm has the value

$$\tfrac{1}{2} d \sin\theta$$



Torque on an Electric Dipole (3)

- We can then rewrite our expression for the torque as

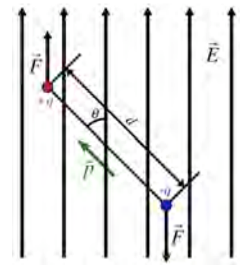
$$\tau = qE\left(\frac{d}{2}\sin\theta\right) + qE\left(\frac{d}{2}\sin\theta\right)$$

- Which we can simplify to

$$\tau = qdE\sin\theta$$

- Remembering our definition for the electric dipole moment,
  $p = qd$
  we can write

$$\tau = pE\sin\theta$$

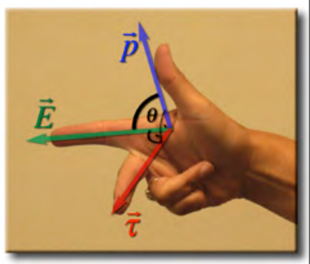


Torque on an Electric Dipole (4)

- We can then rewrite our expression for the torque as as a vector cross product

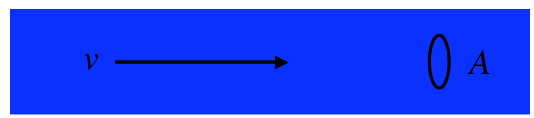
$$\vec{\tau} = \vec{p} \times \vec{E}$$

- To get the direction
  - Use your right hand!
  - Put your thumb in the direction of the electric dipole
  - Put your index finger in the direction of the electric field
  - Your middle finger will point in the direction of the torque
    - The torque will be perpendicular to both the electric dipole and the electric field



Electric Flux

- Let's imagine that we put a ring with area A perpendicular to a stream of water flowing with velocity v

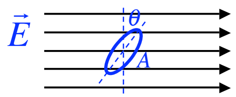


- The product of the area A and the velocity v, Av, gives the volume of water passing through the ring per unit time
  - The units are m^3/s
- If we tilt the ring at an angle θ, the resulting area is $A\cos\theta$, and the volume of water per unit time flowing through the ring is $Av\cos\theta$

Electric Flux (2)

- We call the amount of water flowing through the ring the flux of water

  $\Phi = Av\cos\theta = \vec{A}\cdot\vec{v}$
- We can make an analogy with electric field lines from a constant electric field and flowing water



- We call the density of electric field lines through an area *A* the electric flux given by

  $\Phi = EA\cos\theta$

Electric Flux (3)

- The previous result applies only to constant electric fields
- In the more general case where the electric field is not constant everywhere

  $\vec{E}(\vec{r})$
- We define the electric flux through a closed surface in terms of an integral over the closed surface

  $\Phi = \oint \vec{E}\cdot d\vec{A}$

Gauss' Law

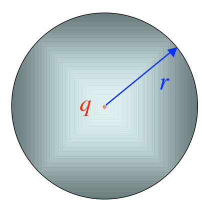
- Now that we have a definition for the electric flux, we can formulate Gauss' Law (named for German mathematician and scientist Johann Carl Friedrich Gauss, 1777 - 1855) as

  $\varepsilon_0\Phi = q$
- If we add the definition of the electric flux we get another expression for Gauss' Law

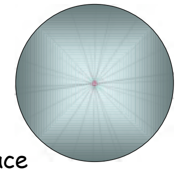
  $\varepsilon_0\oint \vec{E}\cdot d\vec{A} = q$
- Gauss' Law tells us that the integral of the electric field times the area is proportional to the net charge inside the closed surface

Gauss' Law and Coulomb's Law

- We can derive Coulomb's Law from Gauss' Law
- We start with a point charge *q*
- We assume a spherical surface with radius *r* surrounding this charge
  - We call this surface a Gaussian surface



Gauss' Law and Coulomb's Law (2)



- We know that the electric field from a point charge is radial, and thus is perpendicular to the Gaussian surface everywhere
- Thus the electric field is parallel to the surface normal vector for the entire surface

$$\vec{E} \cdot d\vec{A} = EdA\cos 0° = EdA$$

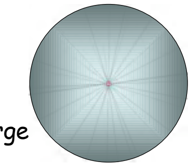
- So we can write Gauss' law as

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 \oint EdA = q$$

- Because the magnitude of the electric field is the same at every point on the Gaussian surface we can write

$$\varepsilon_0 E \oint dA = q$$

Gauss' Law and Coulomb's Law (3)



- Now we are left with a simple integral over a spherical surface

$$\oint dA = 4\pi r^2$$

- So for Guass's law related to a point charge we get

$$\varepsilon_0 E\left(4\pi r^2\right) = q$$

- Which gives

$$E = \frac{q}{4\pi\varepsilon_0 r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = k\frac{q}{r^2}$$

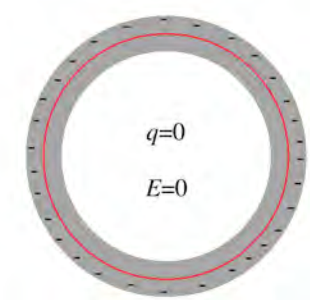
- Electric field from Coulomb's law for a point charge!

Shielding

- The most important application of Gauss' Law
  - The electric field inside a closed conductor is zero.
- We can understand this fact if we think of a closed conductor
  - The conduction electrons will repel each other
  - The conduction electrons will all move to the surface of the conductor
  - We can then draw a Gaussian surface inside the conductor that encloses no charge and thus the electric field is zero

Shielding Illustration

- Start with a hollow conductor
- Add charge to the conductor
- The charge will move to the surface
- We can define a Gaussian surface that encloses zero charge
  - No electric field!



Shielding Demonstration

- We will demonstrate shielding in two ways
- We will place Styrofoam peanuts in a container on a Van de Graaff generator
  - In a plastic cup
  - In a metal cup
- We will place a professor in a wire cage and try to fry him with large sparks from a Van de Graaff generator
  - Note that the shielding effect does not require a solid conductor
    - *A wire mesh will also work, as long as you don't get too close to the open areas*