Physics for Scientists & Engineers 2

Spring Semester 2005

Lecture 12

Capacitors

- Capacitors are devices that can store electrical energy
- Capacitors are used in many every-day applications
  - Heart defibrillators
  - Camera flash units
- Capacitors are an integral part of modern day electronics
  - Capacitors and be micro-sized on computer chips or super-sized for high power circuits such as FM radio transmitters

Capacitance

- Capacitors come in a variety of sizes and shapes
- However, in general, a capacitor consists of two separated conductors, usually called plates, even if these conductors are not simple planes

- We will start our study of capacitors by defining a convenient geometry and generalize from there
- We will start with a capacitor consisting of two parallel conducting planes, each with area *A* separated by a distance *d*
- We assume that these plates are in a vacuum (air is very close to a vacuum)
- We call this device a parallel plate capacitor

Parallel Plate Capacitor

- We charge the capacitor by placing
  - a charge $+q$ on the top plate
  - a charge $-q$ on the bottom plate
- Because the plates are conductors, the charge will distribute itself evenly over the surface of the conducting plates
- The electric potential, V, is proportional to the amount of charge on the plates

Parallel Plate Capacitor (2)

- The proportionality constant between the charge *q* and the electric potential difference *V* is the capacitance *C*

  $q = CV$

- We will call the electric potential difference *V* the potential or the voltage between the plates
- The capacitance of a device depends on the area of the plates and the distance between the plates, but does not depend on the voltage across the plates or the charge on the plates
- The capacitance of a device tells us how much charge is required to produce a given voltage across the plates

Definition of Capacitance

- The definition of capacitance is

  $$C = \frac{q}{V}$$

- The units of capacitance are coulombs per volt
- The unit of capacitance has been given the name farad (abbreviated F) named after British physicist Michael Faraday (1791 - 1867)

  $$1\ \mathrm{F} = \frac{1\ \mathrm{C}}{1\ \mathrm{V}}$$

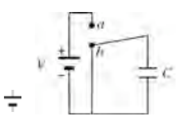
- A farad is a very large capacitance
  - Typically we deal with μF (10^{-6} F), nF (10^{-9} F), or pF (10^{-12} F)

Charging/Discharging a Capacitor

- We can charge a capacitor by connecting the capacitor to a battery or to a power supply
- A battery or power supply is designed to supply charge at a given voltage
- When we connect a capacitor to a battery, charge flows from the battery until the capacitor is fully charged
- If we then disconnect the battery or power supply, the capacitor will retain its charge and voltage
- A real-life capacitor will leak charge, but here we will assume ideal capacitors that hold their charge and voltage indefinitely

Charging/Discharging a Capacitor (2)

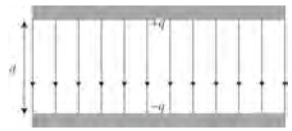
- Let's illustrate the charging processing using a circuit diagram
- In the circuit diagram
  - Lines represent conductors
  - The battery or power supply is represented by
  - The capacitor is represented by the symbol



- This circuit has a switch
  - When the switch is between positions *a* and *b*, the circuit is open (not connected)
  - When the switch is in position *a*, the battery is connected across the capacitor
  - When the switch is in position *b*, the two plates of the capacitor are connected
    - Charge will flow between the plates and the capacitor will discharge

Parallel Plate Capacitor

- Consider two parallel conducting plates separated by a distance *d*



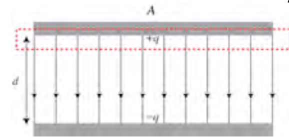
- This arrangement is called a parallel plate capacitor
- The upper plate has +*q* and the lower plate has -*q*
- The electric field between the plates points from the positively charge plate to the negatively charged plate
- We will assume ideal parallel plate capacitors in which the electric field is constant between the plates and zero elsewhere
- Real-life capacitors have fringe field near the edges

Parallel Plate Capacitor (2)

- We can calculate the electric field between the plates using Gauss' Law

$$\varepsilon_0 \oint \vec{E} \bullet d\vec{A} = q$$

- We take a Gaussian surface shown by the red dashed line



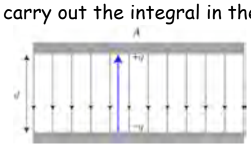
- The electric field is zero inside the conductor so the top part of the Gaussian surface does not contribute to the integral
- The vertical parts of the Gaussian surface do not contribute because the electric field is zero outside the capacitor
- The only contribution to the integral comes from the Gaussian surface inside the constant electric field of the capacitor

Parallel Plate Capacitor (3)

- We then get the electric field E inside the parallel plate capacitor to be $\varepsilon_0 EA = q$
- Now we calculate the electric potential across the plates of the capacitor in terms of the electric field

$$\Delta V = -\int_i^f \vec{E} \bullet d\vec{s}$$

- We define the electric potential across the capacitor to be *V* and we carry out the integral in the direction of the blue arrow



- The integral gives us $V = Ed$

Parallel Plate Capacitor (4)

- Remembering the definition of capacitance

$$C = \frac{q}{V}$$

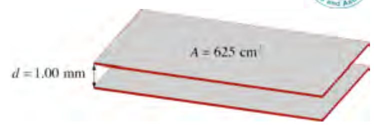
- We get the result for the capacitance of a parallel plate capacitor

$$C = \frac{\varepsilon_0 A}{d}$$

- Where
  - *A* is the area of each plate
  - *d* is the distance between the plates
- Note that this result for the capacitance of a parallel plate capacitor depends only on the geometry of the capacitor and not on the amount of charge or the voltage across the capacitor

Example - Capacitance of a Parallel Plate Capacitor

- We have a parallel plate capacitor constructed of two parallel plates, each with area 625 cm² separated by a distance of 1.00 mm.
- What is the capacitance of this parallel plate capacitor?

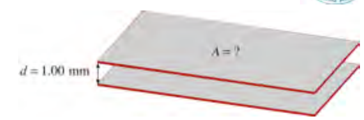


$$C = \frac{\varepsilon_0 A}{d}$$
$$A = 625\ \text{cm}^2 = 0.0625\ \text{m}^2$$
$$d = 1.00\ \text{mm} = 1.00\cdot 10^{-3}\ \text{m}$$
$$C = \frac{\left(8.85\cdot 10^{-12}\ \text{F/m}\right)\left(0.0625\ \text{m}^2\right)}{1.00\cdot 10^{-3}\ \text{m}} = 5.53\cdot 10^{-10}\ \text{F}$$
$$C = 0.553\ \text{nF}$$

A parallel plate capacitor constructed out of square conducting plates 25 cm x 25 cm separated by 1 mm produces a capacitor with a capacitance of about 0.5 nF

Example 2 - Capacitance of a Parallel Plate Capacitor

- We have a parallel plate capacitor constructed of two parallel plates separated by a distance of 1.00 mm.
- What area is required to produce a capacitance of 1.00 F?

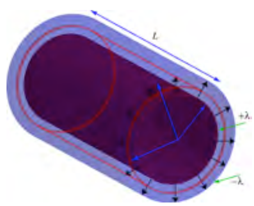


$$C = \frac{\varepsilon_0 A}{d}$$
$$d = 1.00\ \text{mm} = 1.00\cdot 10^{-3}\ \text{m}$$
$$A = \frac{dC}{\varepsilon_0} = \frac{\left(1.00\cdot 10^{-3}\ \text{m}\right)\left(1.00\ \text{F}\right)}{\left(8.85\cdot 10^{-12}\ \text{F/m}\right)} = 1.13\cdot 10^{8}\ \text{m}^2$$

A parallel plate capacitor constructed out of square conducting plates 10.6 km x 10.6 km (6 miles x 6 miles) separated by 1 mm produces a capacitor with a capacitance of 1 F

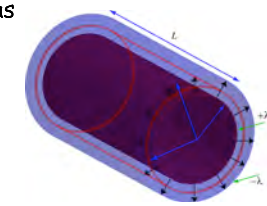
Cylindrical Capacitor

- Consider a capacitor constructed of two collinear conducting cylinders of length L
- The inner cylinder has radius r_1 and the outer cylinder has radius r_2
- Both cylinders have charge per unit length λ with the inner cylinder having positive charge and the outer cylinder having negative charge
- We will assume an ideal cylindrical capacitor
  - The electric field points radially from the inner cylinder to the outer cylinder
  - The electric field is zero outside the collinear cylinders



Cylindrical Capacitor (2)

- We can apply Gauss' Law to get the electric field between the two cylinder using a Gaussian surface in the for of a cylinder with radius r and length L as illustrated by the red lines



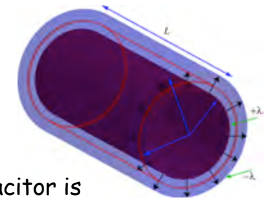
$$\varepsilon_0 \oint \vec{E}\bullet d\vec{A} = \varepsilon_0 EA = \varepsilon_0 E\left(2\pi r\right)L = q = \lambda L$$

- Which we can rewrite to get an expression for the electric field between the two cylinders

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Cylindrical Capacitor (3)

- As we did for the parallel plate capacitor, we define the voltage difference across the two cylinders to be *V*, which we obtain by integrating from the negatively charged cylinder to the positively charged cylinder



$$V = \int_{r_2}^{r_1} E dr = \int_{r_2}^{r_1} \frac{\lambda}{2\pi\varepsilon_0 r} dr = \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{r_2}{r_1}\right)$$

- The capacitance of a cylindrical capacitor is

$$C = \frac{q}{V} = \frac{\lambda L}{\frac{\lambda}{2\pi\varepsilon_0}\ln(r_2/r_1)} = \frac{2\pi\varepsilon_0 L}{\ln(r_2/r_1)}$$

Example - Capacitance of a Coaxial Cable

- Assume that we have a 20.0 m long coaxial cable composed of a conductor and a coaxial shield around the conductor. The radius of the conductor is 0.250 mm and the radius of the shield is 2.00 mm.
- What is the capacitance of the coaxial cable?

We can think of the conductor of the cable as a cylinder because all the charge will reside on the surface of the conductor

The capacitance of the cable is

$$C = \frac{2\pi\varepsilon_0 L}{\ln(r_2/r_1)}$$

$L = 20.0 \text{ m}$

$r_1 = 0.250 \text{ mm} = 2.50 \cdot 10^{-4} \text{ m}$

$r_2 = 2.00 \text{ mm} = 2.00 \cdot 10^{-3} \text{ m}$

$$C = \frac{2\pi\varepsilon_0 (20.0 \text{ m})}{\ln(2.00 \cdot 10^{-3} \text{ m} / 2.5 \cdot 10^{-4} \text{ m})} = 5.35 \cdot 10^{-10} \text{ F} = 0.535 \text{ nF}$$