This week we will study charges in motion
- Electric charge moving from one region to another is called electric current
- Current is all around us
- Current is flowing through light bulbs, iPods, and lightning strikes
- Current consists of mobile electrons traveling in conducting materials
- Direct current is defined as a current that flows in one direction.

Direct Current

Electric Current

- We define the electric current $i$ as the net charge passing a given point in a given time
- Random motion of electrons in conductors or the flowing of electrically neutral atoms are not current in spite of the fact that large amounts of charge are moving past a given point
- If net charge $dq$ passes a point in time $dt$ we define the current $i$ to be

$$i = \frac{dq}{dt}$$

Electric Current (2)

- The amount of charge $q$ passing a given point in time $t$ is the integral of the current with respect to time given by

$$q = \int dq = \int_i idt$$

- We will use charge conservation, implying that charge flowing in a conductor is never lost
- Therefore the same amount of charge must flow through one end of the conductor that exits from the other end of the conductor.
The Ampere

- The unit of current is coulombs per second, which has been given the unit ampere, named after French physicist André Ampère, (1775-1836)
- The ampere is abbreviated as A and is given by
  \[ 1 \text{ A} = \frac{1 \text{ C}}{1 \text{ s}} \]
- Some typical currents are
  - Flashlight - 1 A
  - The starter in your car - 200 A
  - iPod - 50 mA
  - In a lightning strike (for a short time) - 100,000 A

Batteries

- In the following days we will make extensive use of batteries as devices that provide direct currents in circuits
- If you examine a battery, you will find its voltage written on it
- This voltage is the potential difference it can provide to a circuit
- You will also find their ratings in units of mAh
- This rating provides information on the total charge that they can deliver when fully charged
- The quantity mAh is another unit of charge:
  \[ 1 \text{ mAh} = (10^{-3} \text{ A})(3600 \text{ s}) = 3.6 \text{ As} = 3.6 \text{ C} \]

Current

- Current is a scalar
- Current has a sign but not a direction
- This week we will represent the direction of the current flowing in a conductor using an arrow
- This arrow represents whether the net current is positive or negative in a conductor at a given point but does not represent a direction in three dimensions
- Physically the charge carriers in a conductor are electrons that are negatively charged
- However, as is conventionally done, we define positive current as the net flow of positive charge carriers past a given point per unit time.

Current Density

- Let’s consider current flowing in a conductor
- Taking a plane through the conductor, we can define the current per unit area flowing through the conductor at that point as the current density \( \vec{j} \)
- We take the direction of \( \vec{j} \) as the direction of the velocity of the charges crossing the plane
- If the cross sectional area is small, the magnitude of \( \vec{j} \) will be large
- If the cross section area is large, the magnitude of \( \vec{j} \) will be small.
Current Density (2)

- The current flowing through the surface is
  \[ i = \int J \cdot d\mathbf{A} \]
- where \( d\mathbf{A} \) is the differential area element perpendicular to the surface.
- If the current is constant and perpendicular to the surface, then and we can write an expression for the magnitude of the current density
  \[ J = \frac{i}{A} \]

Electron Drift Velocity

- In a conductor that is not carrying current, the conduction electrons move randomly
- When current flows through the conductor, the electrons still move randomly but with an added drift velocity, \( v_d \)
- The magnitude of the velocity of random motion is on the order of \( 10^6 \text{ m/s} \) while the magnitude of the drift velocity is on the order of \( 10^{-4} \text{ m/s} \)
- We can relate the current density \( J \) to the drift velocity \( v_d \) of the moving electrons

Electron Drift Velocity (2)

- Consider a conductor with cross sectional area \( A \) and electric field \( E \)
- Suppose that there are \( n \) electrons per unit volume.
- The negatively charged electrons will drift in a direction opposite to the electric field by definition
- We assume that all the electrons have the same drift velocity \( v_d \) and that the current density \( J \) is uniform
- In a time interval \( dt \) each electron moves a distance \( v_d t \)
- The volume is then \( A v_d t \) and the number of electrons is \( n A v_d t \)

Electron Drift Velocity (3)

- Each electron has charge \( e \) so that the charge \( dq \) that flows through the differential area in time \( dt \) is
  \[ dq = nev_d dt \]
- And the current is
  \[ i = \frac{dq}{dt} = nev_d A \]
- The current density is
  \[ J = \frac{i}{A} = nev_d \]
- The current density and the drift velocity are parallel vectors, pointing in the same direction, and we can write
  \[ J = (ne)v_d \]
Electron Drift Velocity (4)

- Consider a wire carrying a current
- The physical current carriers are negatively charged electrons
- These electrons are moving to the left in this drawing
- However, the electric field, current density, drift velocity, and current are all to the right because of the convention that these quantities refer to positive charges

Resistance and Resistivity

- Some materials conduct electricity better than others
- If we apply a given voltage across a conductor, we get a large current
- If we apply the same voltage across an insulator, we get little current
- The property of a material that describes its ability to conduct electric currents is called the resistivity, $\rho$
- The property of a particular device or object that describes its ability to conduct electric currents is called the resistance, $R$

Resistance and Resistivity (2)

- If we apply an electric potential difference $V$ across a conductor and measure the resulting current $i$ in the conductor, we can define the resistance $R$ of that conductor as
  $R = \frac{V}{i}$
- The unit of resistance is volt per ampere
- In honor of German physicist George Simon Ohm (1789-1854) resistance has been given the unit ohm, $\Omega$
  $1 \Omega = \frac{1 V}{1 A}$

Resistance and Resistivity (3)

- The resistance of a conductor can depend on the direction the current flows in the conductor
  - For example, semiconductors
- We will assume that the resistance of the device is uniform for all directions of the current.
- The resistance of a conductor depends on the material from which the conductor is constructed as well as the geometry of the conductor
- First we discuss the effects of the material of the conductor and then we will discuss the effects of geometry on resistance.
Resistivity

- The conducting properties of a material are characterized in terms of its resistivity.
- We define the resistivity, \( \rho \), of a material in terms of the magnitude of the applied electric field, \( E \), and the magnitude of the resulting current density, \( J \), as

\[
\rho = \frac{E}{J}
\]

- The units of resistivity are

\[
\left( \frac{\text{V}}{\text{m}} \right) = \frac{\text{V}}{\text{m}} = \Omega \text{m}
\]

Typical Resistivities

- The resistivities of some representative conductors at 20° C are listed in the table below.

<table>
<thead>
<tr>
<th>Material</th>
<th>Resistivity ( \rho ) (( \Omega \text{m} ))</th>
<th>Resistivity ( \rho ) (( \mu \Omega \text{cm} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver</td>
<td>1.59\times10^{-8}</td>
<td>1.59</td>
</tr>
<tr>
<td>Copper</td>
<td>1.72\times10^{-8}</td>
<td>1.72</td>
</tr>
<tr>
<td>Gold</td>
<td>2.44\times10^{-8}</td>
<td>2.44</td>
</tr>
<tr>
<td>Aluminum</td>
<td>2.82\times10^{-8}</td>
<td>2.82</td>
</tr>
<tr>
<td>Nickel</td>
<td>6.84\times10^{-8}</td>
<td>6.84</td>
</tr>
<tr>
<td>Mercury</td>
<td>95.8\times10^{-8}</td>
<td>95.8</td>
</tr>
</tbody>
</table>

- As you can see, typical values for the resistivity of conductors used in wires are on the order of \( 10^{-8} \, \Omega \).