Physics for Scientists & Engineers 2

Spring Semester 2005

Lecture 18

Review

- The resistance R of a device is given by

$$R = \rho \frac{L}{A}$$

  - ρ is resistivity of the material from which the device is constructed
  - L is the length of the device
  - A is the cross sectional area of the device
- The temperature dependence of the resistivity of metals is given by

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0)$$

  - ρ is the resistivity at temperature T
  - ρ_0 is the resistivity at temperature T_0
  - α is the temperature coefficient of electric resistivity for the material under consideration

Review (2)

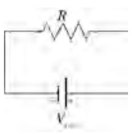
- The temperature dependence of the resistance of metals is given by

$$R - R_0 = R_0 \alpha (T - T_0)$$

  - R is the resistance at temperature T
  - R is the resistance at temperature T_0
  - α is the temperature coefficient of electric resistivity for the material under consideration
- Ohm's Law for a circuit consisting of a resistor and a battery is given by

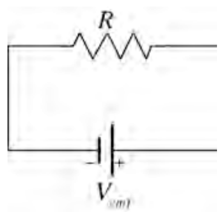
$$V_{emf} = iR$$

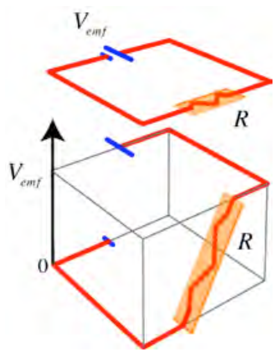
  - V_{emf} is the emf or voltage produced by the battery
  - i is the current
  - R is the resistance of the resistor



Review (3)

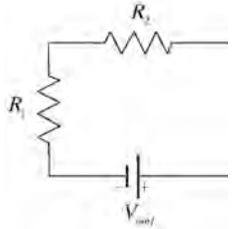
- We can visualize a circuit with a battery and a resistor in three dimensions





Resistances in Series

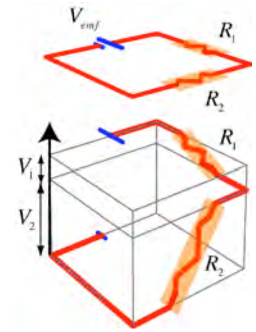
- Resistors connected such that all the current in a circuit must flow through each of the resistors are connected in series
- If we connect two resistors R_1 and R_2 in series with one source of emf with voltage V_{emf}, we have the circuit shown below



Two Resistors in 3D

- To illustrate the voltage drops in this circuit we can represent the same circuit in three dimensions
- The voltage drop across resistor R_1 is V_1
- The voltage drop across resistor R_2 is V_2
- The sum of the two voltage drops must equal the voltage supplied by the battery

$$V_{emf} = V_1 + V_2$$



Resistors in Series

- The current must flow through all the elements of the circuit so the current flowing through each element of the circuit is the same
- For each resistor we can apply Ohm's Law

$$V_{emf} = iR_1 + iR_2 = iR_{eq}$$

- Where

$$R_{eq} = R_1 + R_2$$

- We can generalize this result for a circuit with two resistors in series to a circuit with n resistors in series

$$R_{eq} = \sum_{i=1}^{n} R_i$$

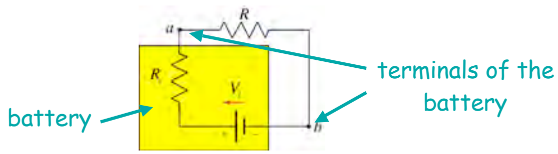
Example: Internal Resistance of a Battery

- When a battery is not connected in a circuit, the voltage across its terminals is V_t
- When the battery is connected in series with a resistor with resistance R, current i flows through the circuit
- When current is flowing, the voltage, V, across the terminals of the battery is lower than V_t
- This drop occurs because the battery has an internal resistance, R_i, that can be thought of as being series with the external resistor
- We can express this relationship as

$$V_t = iR_{eq} = i\left(R + R_i\right)$$

Example: Internal Resistance of a Battery (2)

- We can represent the battery, its internal resistance and the external resistance in this circuit diagram



- Consider a battery that has a voltage of 12.0 V when it is not connected to a circuit
- When we connect a 10.0 Ω resistor across the terminals, the voltage across the battery drops to 10.9 V
- What is the internal resistance of the battery?

Example: Internal Resistance of a Battery (3)

- The current flowing through the external resistor is

$$i = \frac{V}{R} = \frac{10.9\ \mathrm{V}}{10.0\ \Omega} = 1.09\ \mathrm{A}$$

- The current flowing in the complete circuit must be the same so

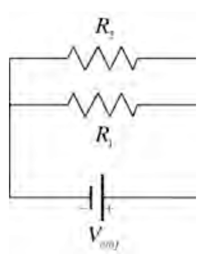
$$V_t = iR_{eq} = i\left(R + R_i\right)$$

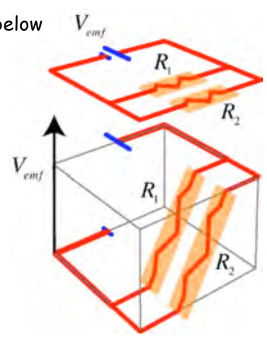
$$\left(R + R_i\right) = \frac{V}{i}$$

$$R_i = \frac{V}{i} - R = \frac{12.0\ \mathrm{V}}{1.09\ \mathrm{A}} - 10.0\ \Omega = 1.0\ \Omega$$

Resistances in Parallel

- Instead of connecting resistors in series so that all the current must pass through both resistors, we can connect the resistors in parallel such that the current is divided between the two resistors
- This type of circuit is shown is below





Resistance in Parallel (2)

- In this case the voltage drop across each resistor is equal to the voltage provides by the source of emf
- Using Ohm's Law we can write the current in each resistor

$$i_1 = \frac{V_{emf}}{R_1} \qquad i_2 = \frac{V_{emf}}{R_2}$$

- The total current in the circuit must equal the sum of these currents

$$i = i_1 + i_2$$

- Which we can rewrite as

$$i = i_1 + i_2 = \frac{V_{emf}}{R_1} + \frac{V_{emf}}{R_2} = V_{emf}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Resistance in Parallel (3)

- We can then rewrite Ohm's Law as

$$i = V_{emf}\frac{1}{R_{eq}}$$

- Where

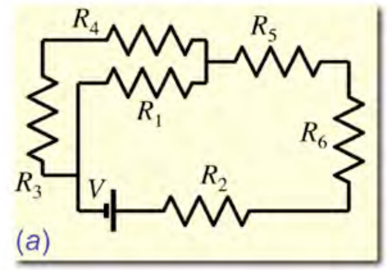
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

- We can generalize this result for two parallel resistors to *n* parallel resistors

$$\frac{1}{R_{eq}} = \sum_{i=1}^{n}\frac{1}{R_i}$$

Example: Network of Resistors

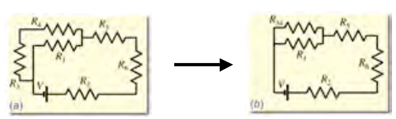
- Consider a network of resistors as shown below



- What is the current flowing in this circuit?

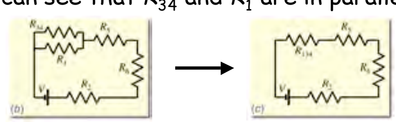
Example: Network of Resistors (2)

- We see that R_3 and R_4 are in series



$$R_{34} = R_3 + R_4$$

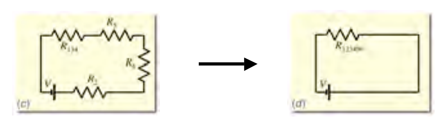
- We can see that R_{34} and R_1 are in parallel



$$\frac{1}{R_{134}} = \frac{1}{R_1} + \frac{1}{R_{34}} \quad \text{or} \quad R_{134} = \frac{R_1 R_{34}}{R_1 + R_{34}}$$

Example: Network of Resistors (3)

- We see that R_2, R_5, R_6, and R_{134} are in series



$$R_{123456} = R_2 + R_5 + R_6 + R_{134}$$

$$R_{123456} = R_2 + R_5 + R_6 + \frac{R_1 R_{34}}{R_1 + R_{34}}$$

$$R_{123456} = R_2 + R_5 + R_6 + \frac{R_1\left(R_3 + R_4\right)}{R_1 + R_3 + R_4}$$

$$i = \frac{V_{emf}}{R_{123456}}$$

Energy and Power in Electric Circuits

- Consider a simple circuit in which a source of emf with voltage *V* causes a current *i* to flow in a circuit
- The work required to move a differential amount of charge *dq* is equal to the differential electric potential energy *dU* given by
  $dU = dqV$
- The definition of current is
  $i = dq / dt$
- So we can rewrite the differential electric potential energy as
  $dU = idtV$
- The definition of power *P* is
  $P = dU / dt$
- Which gives us

$$P = \frac{dU}{dt} = \frac{idtV}{dt} = iV$$

Energy and Power

- The power dissipated in a circuit or circuit element is given by the product of the current times the voltage
- Using Ohm's Law we can write equivalent formulations of the power

$$P = iV = i^2R = \frac{V^2}{R}$$

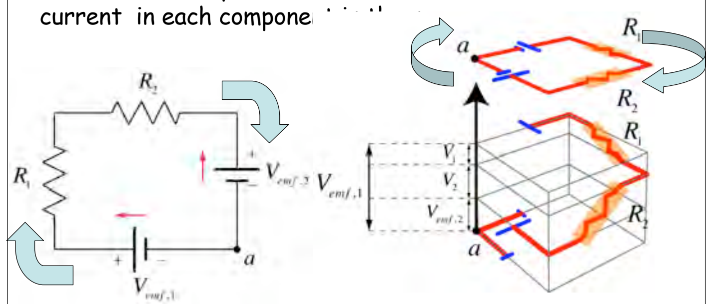
- The unit of power is the watt (W)
- Electrical devices are rated by the amount of power they consume in watts
- Your electricity bill is based on how many kilowatt-hours of electrical energy you consume
- The energy is converted to heat, motion, light, …

Single Loop Circuits

- We have been studying circuits with various networks of resistors but only one source of emf
- Circuits can contain multiple sources of emf as well as multiple resistors
- We begin our study of more complicated circuits by analyzing a circuit with two sources of emf
  - $V_{emf,1}$ and $V_{emf,2}$
- And two resistors
  - R_1 and R_2
- connected in series in a single loop
- We will assume that the two sources of emf have opposite polarity

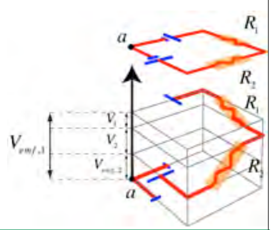
Single Loop Circuits (2)

- Starting at point *a* with *V*=0, we proceed around the circuit in a clockwise direction
- Because the components of the circuit are in series, the current in each compone



Single Loop Circuits (3)

- The first circuit component is a source of emf $V_{emf,1}$, which produces a positive voltage gain of $V_{emf,1}$
- Next we find resistor R_1, which produces a voltage drop V_1 given by iR_1
- Continuing around the circuit we find resistor R_2, which produces a voltage drop V_2 given by iR_2
- Next we meet a second source of emf, $V_{emf,2}$
- This source of emf is wired into the circuit with a polarity opposite that of $V_{emf,1}$
- We treat this component as a voltage drop with magnitude of $V_{emf,2}$ rather than a voltage gain
- We now have completed the circuit and we are back at point *a*



Single Loop Circuits (4)

- We can write the analysis of the voltages in this circuit as

$$V_{emf,1} - V_1 - V_2 - V_{emf,2} = V_{emf,1} - iR_1 - iR_2 - V_{emf,2} = 0$$

- We can generalize this result to state that the voltage drops across components in a single loop circuit must sum to zero
- This statement must be qualified with conventions for assigning the sign of the voltage drops around the circuit
- We must define the direction with which we move around the loop and we must define the direction of the current
- If we move around the circuit in the same direction as the current, the voltage drops across resistors will be negative
- If we move around the circuit in the opposite direction from the current, the voltage drops across resistors will be positive

Single Loop Circuits (5)

- If we move around the circuit and encounter a source of emf pointing in the same direction, we assume that this component contributes a positive voltage
- If we encounter a source of emf pointing in the opposite direction, we consider that component to contribute a negative voltage
- Thus we will get the same information from the analysis of a simple circuit independent of the direction we choose to analyze the circuit.