Physics for Scientists & Engineers 2

Spring Semester 2005

Lecture 19

Review

- n resistors in series can be replaced by an equivalent resistance given by the sum of the resistances of the resistors in series

$$R_{eq} = \sum_{i=1}^{n} R_i$$

- n resistors in parallel can be replaced by an equivalent resistance given by the sum of the resistances of the resistors in parallel

$$\frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i}$$

- The power dissipated in a circuit or circuit element is given by

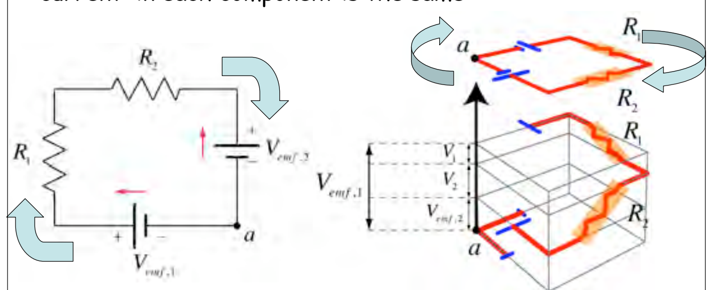
$$P = iV = i^2R = \frac{V^2}{R}$$

Single Loop Circuits

- We have been studying circuits with various networks of resistors but only one source of emf
- Circuits can contain multiple sources of emf as well as multiple resistors
- We begin our study of more complicated circuits by analyzing a circuit with two sources of emf
  - $V_{emf,1}$ and $V_{emf,2}$
- And two resistors
  - R_1 and R_2
- connected in series in a single loop
- We will assume that the two sources of emf have opposite polarity

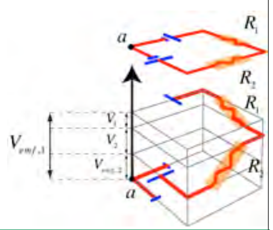
Single Loop Circuits (2)

- Starting at point a with V=0, we proceed around the circuit in a clockwise direction
- Because the components of the circuit are in series, the current in each component is the same



Single Loop Circuits (3)

- The first circuit component is a source of emf $V_{emf,1}$, which produces a positive voltage gain of $V_{emf,1}$
- Next we find resistor R_1, which produces a voltage drop V_1 given by iR_1
- Continuing around the circuit we find resistor R_2, which produces a voltage drop V_2 given by iR_2
- Next we meet a second source of emf, $V_{emf,2}$
- This source of emf is wired into the circuit with a polarity opposite that of $V_{emf,1}$
- We treat this component as a voltage drop with magnitude of $V_{emf,2}$ rather than a voltage gain
- We now have completed the circuit and we are back at point *a*



Single Loop Circuits (4)

- We can write the analysis of the voltages in this circuit as

$$V_{emf,1} - V_1 - V_2 - V_{emf,2} = V_{emf,1} - iR_1 - iR_2 - V_{emf,2} = 0$$

- We can generalize this result to state that the voltage drops across components in a single loop circuit must sum to zero
- This statement must be qualified with conventions for assigning the sign of the voltage drops around the circuit
- We must define the direction with which we move around the loop and we must define the direction of the current
- If we move around the circuit in the same direction as the current, the voltage drops across resistors will be negative
- If we move around the circuit in the opposite direction from the current, the voltage drops across resistors will be positive

Single Loop Circuits (5)

- If we move around the circuit and encounter a source of emf pointing in the same direction, we assume that this component contributes a positive voltage
- If we encounter a source of emf pointing in the opposite direction, we consider that component to contribute a negative voltage
- Thus we will get the same information from the analysis of a simple circuit independent of the direction we choose to analyze the circuit.

Kirchhoff's Law, Multi-loop Circuits

- One can create multi-loop circuits that cannot be resolved into simple circuits containing parallel or series resistors
- To handle these types of circuits, we must apply Kirchhoff's Rules
- Kirchhoff's Rules can be stated as
  - Kirchhoff's Junction Rule
    - The sum of the currents entering a junction must equal the sum of the current leaving a junction
  - Kirchhoff's Loop Rule
    - The sum of voltage drops around a complete circuit loop must sum to zero.

Kirchhoff's Junction Rule

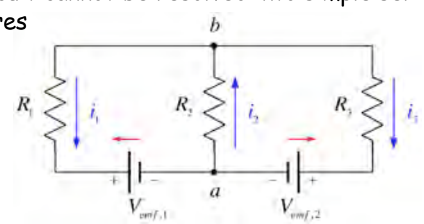
- Kirchhoff's Junction Rule is a direct consequence of the conservation of charge
- In a conductor, charge cannot accumulate at one point
- The amount of the charge residing in the junction must remain constant at all times because of charge conservation
  - all charges streaming into a junction must also leave the junction, which is exactly what Kirchhoff's Junction Rule states.

Kirchhoff's Loop Rule

- Kirchhoff's Loop Rule is a direct consequence of the conservation of electric potential energy
- Suppose that this rule was not valid
  - we could construct a way around a loop in such a way that each turn would increase the potential of a charge traveling around the loop
  - we would always increase the energy of this charge, in obvious contradiction to energy conservation
- Kirchhoff's Loop Rule is equivalent to the law of energy conservation.

Example - Kirchhoff's Laws

- Consider a circuit that has three resistors, R_1, R_2, and R_3 and two sources of emf, $V_{emf,1}$ and $V_{emf,2}$
- This circuit cannot be resolved into simple series or parallel structures



- To analyze this circuit, we need to assign currents flowing through the resistors
- We can choose these currents arbitrarily

Example - Kirchhoff's Laws (2)

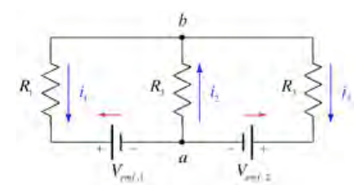
- Consider junction *b* where the incoming current must equal the outgoing current

$$i_2 = i_1 + i_3$$

- At junction *a* we again equate the incoming current and the outgoing current

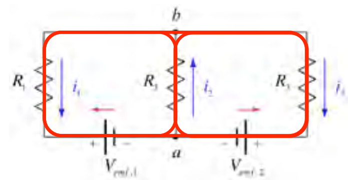
$$i_1 + i_3 = i_2$$

- This equation gives us the same information as the previous equation
- We need more information to determine the three currents



Example - Kirchhoff's Laws (3)

- To get these equations we can apply Kirchhoff's Loop Rule
- We can identify three loops in the circuit
  - Left
    - R_1, R_2, $V_{emf,1}$
  - Right
    - R_2, R_3, $V_{emf,2}$
  - Outer
    - R_1, R_3, $V_{emf,1}$, $V_{emf,2}$



Example - Kirchhoff's Laws (4)

- Going around the left loop counterclockwise starting at point *b* we get

$$-i_1R_1 - V_{emf,1} - i_2R_2 = 0 \;\Rightarrow\; i_1R_1 + V_{emf,1} + i_2R_2 = 0$$

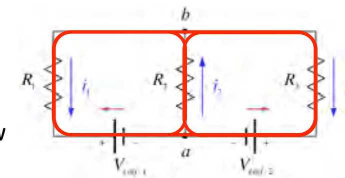
- Going around the right loop counterclockwise starting at point *b* we get

$$-i_3R_3 - V_{emf,2} - i_2R_2 = 0 \;\Rightarrow\; i_3R_3 + V_{emf,2} + i_2R_2 = 0$$

- Going around the outer loop counterclockwise starting at point *b* we get

$$-i_3R_3 - V_{emf,2} + V_{emf,1} + i_1R_1 = 0$$

- This equation gives us no new information



Example - Kirchhoff's Laws (5)

- We now have three equations

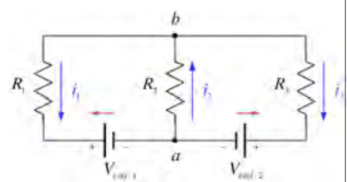
$$i_1 + i_3 = i_2 \qquad i_1R_1 + V_{emf,1} + i_2R_2 = 0 \qquad i_3R_3 + V_{emf,2} + i_2R_2 = 0$$

- And we have three unknowns i_1, i_2, and i_3
- We can solve these three equations in a variety of ways

$$i_1 = -\frac{(R_2+R_3)V_{emf,1} - R_2V_{emf,2}}{R_1R_2 + R_1R_3 + R_2R_3}$$

$$i_2 = -\frac{R_3V_{emf,1} + R_1V_{emf,2}}{R_1R_2 + R_1R_3 + R_2R_3}$$

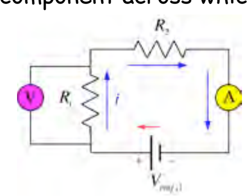
$$i_3 = -\frac{-R_2V_{emf,1} + (R_1+R_2)V_{emf,2}}{R_1R_2 + R_1R_3 + R_2R_3}$$



Ammeter and Voltmeters

- A device used to measure current is called an ammeter
- A device used to measure voltage is called a voltmeter
- To measure the current, the ammeter must be placed in the circuit in series
- To measure the voltage, the voltmeter must be wired in parallel with the component across which the voltage is to be measured

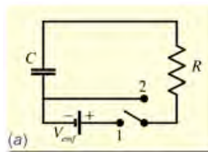
Voltmeter in parallel
High resistance



Ammeter in series
Low resistance

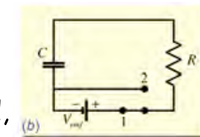
RC Circuits

- So far we have dealt with circuits containing sources of emf and resistors
- The currents in these circuits did not vary in time
- Today we will study circuits that contain capacitors as well as sources of emf and resistors
- These circuits have currents that vary with time
- Consider a circuit with
  - a source of emf, V_{emf},
  - a resistor R,
  - a capacitor C



RC Circuits (2)

- We then close the switch, and current begins to flow in the circuit, charging the capacitor
- The current is provided by the source of emf, which maintains a constant voltage
- When the capacitor is fully charged, no more current flows in the circuit

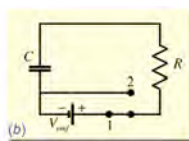


- When the capacitor is fully charged, the voltage across the plates will be equal to the voltage provided by the source of emf and the total charge q_{tot} on the capacitor will be $q_{tot} = CV_{emf}$

Capacitor Charging

- Going around the circuit in a counterclockwise direction we can write

$$V_{emf} - V_R - V_C = V_{emf} - iR - \frac{q}{C} = 0$$



- We can rewrite this equation remembering that $i = dq/dt$

$$R\frac{dq}{dt} + \frac{q}{C} = V_{emf} \quad \Rightarrow \quad \frac{dq}{dt} + \frac{q}{RC} = \frac{V_{emf}}{R}$$

- The solution of this differential equation is

$$q(t) = q_0\left(1 - e^{-\frac{t}{\tau}}\right)$$

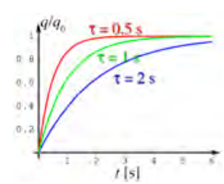
- Where $q_0 = CV_{emf}$ and $\tau = RC$

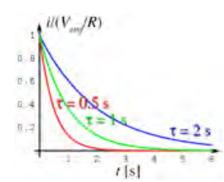
Capacitor Charging (2)

- We can get the current flowing in the circuit by differentiating the charge with respect to time

$$i = \frac{dq}{dt} = \left(\frac{V_{emf}}{R}\right)e^{\left(-\frac{t}{RC}\right)}$$

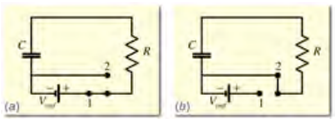
- The charge and current as a function of time are shown here ($\tau = RC$)





Capacitor Discharging

- Now let's take a resistor *R* and a fully charged capacitor *C* with charge q_0 and connect them together by moving the switch from position 1 to position 2



- In this case current will flow in the circuit until the capacitor is completely discharged
- While the capacitor is discharging we can apply the Loop Rule around the circuit and obtain

$$-iR - V_C = -iR - \frac{q}{C} = 0 \quad \Rightarrow \quad R\frac{dq}{dt} + \frac{q}{C} = 0$$

Capacitor Discharging (2)

- The solution of this differential equation for the charge is

$$q = q_0 e^{\left(-\frac{t}{RC}\right)}$$

- Differentiating charge we get the current

$$i = \frac{dq}{dt} = \left(\frac{q_0}{RC}\right) e^{\left(-\frac{t}{RC}\right)}$$

- The equations describing the time dependence of the charging and discharging of capacitors all involve the exponential factor $e^{-t/RC}$
- The product of the resistance times the capacitance is defined as the time constant τ of a RC circuit
- We can characterize an RC circuit by specifying the time constant of the circuit.