Physics for Scientists & Engineers 2

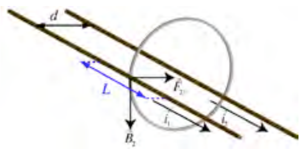
Spring Semester 2005

Lecture 24

Review

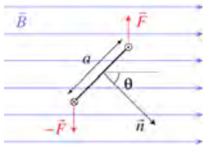
- The force between two current-carrying wires is given by

$$F_{12} = \frac{\mu_0 i_1 i_2 L}{2\pi d}$$



- The torque exerted by a magnetic field on a current-carrying loop is given by

$$\tau = iAB\sin\theta$$



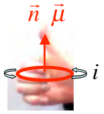
Review (2)

- We define the magnitude of the magnetic dipole moment of a coil to be

$$\mu = NiA$$

- We can express the torque on a coil in a magnetic field as

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$



- The magnetic potential energy of a magnetic dipole in a magnetic field is given by

$$U = -\vec{\mu} \cdot \vec{B} = -\mu B\cos\theta$$

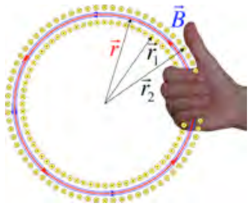
Review (3)

- The magnetic field inside an ideal solenoid is given by

$$B = \mu_0 in$$

- The magnetic field inside an ideal toroidal magnet is given by

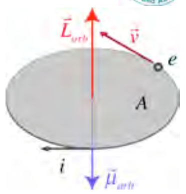
$$B = \frac{\mu_0 Ni}{2\pi r}$$



Atoms as Magnets

- The atoms that make up all matter contain moving electrons that form current loops that produce magnetic fields
- In most materials, these current loops are randomly oriented and produce no net magnetic field
- Some materials naturally have some fraction of these current loops aligned and produce a net magnetic field and are called magnetic
- Other materials can have these current loops aligned by an external magnetic field and become magnetized.
- Let’s construct a very much-simplified model of the atom

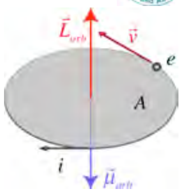
Atoms as Magnets (2)



- Consider an electron moving at a constant speed *v* in a circular orbit with radius *r* as illustrated to the right
- We can think of the moving charge of the electron as a current *i*
- Current is defined as the charge per unit time passing a particular point
- For this case the charge is the charge of the electron *e* and the time is related to the period of the orbit

$$i = \frac{e}{T} = \frac{e}{(2\pi r)/v} = \frac{ve}{2\pi r}$$

Atoms as Magnets (3)



- The magnetic moment of the orbiting electron is given by

$$\mu_{orb} = iA = \frac{ve}{2\pi r}\pi r^2 = \frac{ver}{2}$$

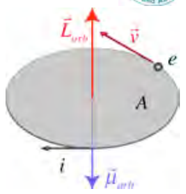
- We can define the orbital angular momentum of the electron to be

$$L_{orb} = rp = rmv$$

  - where *m* is the mass of the electron
- Solving and substituting gives us

$$L_{orb} = rm\left(\frac{2\mu_{orb}}{er}\right) = \frac{2m\mu_{orb}}{e}$$

Atoms as Magnets (4)



- Rewriting and remembering that the magnetic dipole moment and the angular momentum are vector quantities we can write

$$\vec{\mu}_{orb} = -\frac{e}{2m}\vec{L}_{orb}$$

  - The negative sign arises because of the definition of current as the flow of positive charge
- This result can be applied to the hydrogen atom, and the correct result is obtained
- However, other predictions of the properties of atoms based on the idea that electrons exist in circular orbits in atoms disagree with experimental observations

Ferromagnetism

- The elements iron, nickel, cobalt, gadolinium, and dysprosium and alloys containing these elements exhibit ferromagnetism
- Ferromagnetic materials show long-range ordering at the atomic level, which causes the dipole moments of atoms to line up with each other in a limited region called a domain
- Within this domain, the magnetic field can be strong
- However, in the bulk these domains are randomly oriented leaving no net magnetic field
- An external magnetic field can align these domains and produce magnetic fields

Ferromagnetism (2)

- A ferromagnetic material will retain all or some of this induced magnetism when the external magnetic field is removed
- In addition, the magnetic field produced by a current in a device like a solenoid or toroid will be larger if a ferromagnetic
- Demo
  - Insert a ferromagnetic material in the core of a solenoid and see how much the magnetic field is increased

Diamagnetism

- Most materials exhibit diamagnetism
- However diamagnetism is weak compared with the other two types of magnetism and is thus masked by those forms if they are present in the material
- In diamagnetic materials, a weak magnetic dipole moment is induced by an external magnetic field in a direction opposite the direction of the external field
- The induced magnetic field disappears when the external field is removed
- If the external field is non-uniform, in interaction of the induced dipole moment of the diamagnetic material with the external field creates a force directed from a region of greater magnetic field to a region of lower magnetic field

Diamagnetism (2)

- An example of a live frog exhibiting diamagnetism is shown below

A live frog being levitated by a strong magnetic field at the High Field Magnet Laboratory, Radboud University Nijmegen, The Netherlands.

- In this picture diamagnetic forces induced by a non-uniform external magnetic field of 16 T are levitating a live frog
- The normally negligible diamagnetic force is large enough in this case to overcome gravity

Paramagnetism

- Materials containing certain transition elements, actinides, and rare earths exhibit paramagnetism
- Each atom of these elements has a permanent magnetic dipole, but these dipole moments are randomly oriented and produce no net magnetic field
- In the presence of an external magnetic field, some of these magnetic dipole moments align in the same direction as the external field
- When the external field is removed, the induced magnetic dipole moment disappears
- If the external field is non-uniform, this induced magnetic dipole moment interacts with the external field to produce a force directed from a region of lower magnetic field to a region of higher magnetic field.

Nuclear Magnetic Resonance

- Elementary particles such as protons have an intrinsic magnetic dipole moment
- Consider the case in which we place protons in a strong magnetic field
- Because of quantum mechanical reasons, the magnetic dipole moment of a proton can only have two directions, parallel or anti-parallel with the external field
- The difference in energy between the two states is given by the difference in magnetic potential energy, which is $2\mu B$ where μ is the component of the proton's magnetic moment along the direction of the external field B

Nuclear Magnetic Resonance (2)

- If we now introduce a time-varying electric field at the proper frequency, we can induce some of the protons to flip the direction of their magnetic dipole moments from anti-parallel to parallel to the external field, gaining energy
- Because the magnetic potential energy can only have two possible values, the energy required to flip the direction is a discrete value depending on the magnitude of the external field
- Thus only one given oscillation frequency will cause the dipole moment to flip
- If the time-varying electric field is switched off, the protons in the higher energy that are aligned with the field will flip back to their original state, emitting electromagnetic energy that can be detected

Nuclear Magnetic Resonance (3)

- A magnetic resonance imaging device uses the physical principle of nuclear magnetic resonance just described
- This technique can image the location of the protons in a human body by introducing a time varying electric field, and then varying the magnetic field in a known, precise manner to produce a three dimensional picture of the distribution of tissue containing hydrogen
- The quality of this imaging depends on the strength of the external magnetic field

Superconductivity

- Magnets for industrial applications and scientific research can be constructed using ordinary resistive wire with current flowing through the wires
- The typical magnet is a large solenoid
- The current flowing through the wires of the magnet produces resistive heating
- The produced heat is usually taken away by flowing low conductivity water through the hollow conductors
- Low conductivity water is water that has been purified so that it does not conduct electricity
- These room temperature magnets typically can produce magnetic fields up to 1.5 T
- Room temperature magnets are usually relatively inexpensive to construct but expensive to operate

Superconductivity (2)

- Some applications such as magnetic resonance imaging require the highest possible magnetic field to extract the best signal to noise ratio in the measurements
- A magnet can be constructed using superconducting coils rather than resistive coils
- Such a magnet can produce higher field than a room-temperature magnet
- The disadvantage of a superconducting magnet is that the conductor must be kept at the temperature of liquid helium, which is approximately 4 K
- Thus the magnet must be enclosed in a cryostat filled with liquid helium to keep the coils cold

Superconductivity (3)

- During the last two decades, physicists and engineers have discovered new materials that are superconducting at temperatures much above 4 K
- Temperatures of up to 100 K have been reported in these so-called high-temperature superconductors, which means that they can be made superconducting by cooling them with liquid nitrogen
- Many groups around the world are working hard to find materials that are superconducting at room temperature
- These materials would revolutionize many parts of industry and in particular transportation.

High temperature superconductor